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Semi-empirical formulae for the Λ and neutron-hole oscillator frequency

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Abstract. We have obtained an expression for the oscillator frequency in inverse powers of the nuclear mass number, by equating the spacing of the outermost levels of a square well, found to be nearly constant to the oscillator spacing for which the spacings are also constant. The formulae for the oscillator frequency obtained here are compared with similar formulae obtained by other authors. A reasonable qualitative agreement is found to exist between our formulae of $\hbar\omega_\Lambda$ and $\hbar\omega_N$ and those given in the standard literature, obtained mainly from size considerations. Our derivation is based only on the assumption that a particle-nucleus potential exists. Any reference to particle-hole states is made purely for a rough comparison of our parameters, otherwise nothing hinges on that description.

Keywords. Semi-empirical; oscillator frequency.

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1. Introduction

The nucleon–nucleus potential that describes several average properties approximately, is expected to have an interior region of constant density with a diffused tail. For medium and heavy nuclei, the Woods–Saxon (W–S) potential offers a good representation of the average nucleon–nucleus potential. However, the harmonic oscillator has been widely employed as the nucleon–nucleus potential in many structure calculations. The most important reason for this choice is that many calculations can be performed analytically using oscillator wave functions. Further, leaving aside matters of detail, the oscillator density is not too bad. These and other reasons may be regarded as justification for the use of the oscillator.

Let us consider the use of the oscillator in the shell model calculations. In most simple calculations, the closed shells and the closed sub-shells are ignored and only nucleons in the one or two outer shells are taken into account. These nucleons are usually described by oscillator wave functions. For these outer nucleons the W–S wave function is not expected to be far too different from the oscillator wave function. When using the oscillator rather than a more realistic potential, one hopes that parameters of the residual interaction will take care of the discrepancies and one would seek greater clarification and elucidation of these matters.

Recently, we carried out calculations [1] of the Λ-hypernuclear energy spectra over a wide range of mass numbers, assuming that the Λ-particle and the neutron-hole move in two seperate oscillator potentials of different oscillator frequencies [2, 3]. Though the description of the spectrum as pure particle-hole states is successful in reproducing the excited states fairly well, it is not so successful in accounting for the cross-sections and angular distributions which require mixed states, the so-called analog and supersymmetric states [4, 5]. Thus, while our discussion with the oscillator cannot be taken to be very realistic, it may not be too far off the mark either. We may, therefore, regard it as a qualitative description. The success of our calculations [1] shows that the oscillator roughly describes not only the outermost neutron-holes but also those in the inner orbits as well as the Λ-particle in its various orbits. One may be surprised because for medium and heavy nuclei, the nucleon–nucleus or Λ-nucleus potential is more like a W–S potential than an oscillator, but a relevant fact is that for slightly large angular momentum ℓ, the upper levels of the square well (we hope this is also true of potentials like W–S) are nearly equally spaced whereas all levels of the oscillator are equally spaced [6]. Thus, we can always equate the spacing of the upper levels of a square well to the spacing of a certain oscillator. This leads to a formula for the oscillator frequency in inverse powers of the mass number. The situation of the inner orbits is not at all crucial for our present purpose. Our central premise is that a particle-nucleus potential exists and it resembles a W–S potential which may be roughly approximated by a square well. Any reference to the particle-hole picture of hypernuclear excitations is made purely for a rough comparison of our parameters and no more.

The dependence of oscillator spacing on the mass number has already been obtained by many authors. A Λ-oscillator spacing formula derived by Lalazissis *et al* [2] is based on the idea of approximating the Λ-nucleus potential in a simple model, to an oscillator-like potential. Using virial theorem [7] for the oscillator potential, $\hbar\omega_\Lambda$ is then expressed in terms of the expectation value of the kinetic energy to establish the dependence of $\hbar\omega_\Lambda$ on A. We may note that the Λ-nucleus potential is assumed to be oscillator-like to begin with.

The nucleon-oscillator frequency ($\hbar\omega_N$) has been expressed in terms of $A^{-1/3}$ and A^{-1} in [3] by equating the r.m.s. radius of an oscillator to that of a W–S form. Recently, the proton- and neutron-oscillator spacing formulae have been obtained by Lalazissis and Panos [8] using the r.m.s. radius of the nucleons derived from the density of the neutrons and protons seperately [9].

The work of other authors [2, 3, 8] is based on size considerations. Our derivation is based mainly on energy level separation considerations, namely that the spacing of the first few levels of the square well are nearly equally spaced and so may be equated to the spacing of an oscillator. We confine ourselves to the square well because, at present, we are not able to carry out the required calculations for a W–S potential. The derivations based on these ideas are presented in the next section. Results and discussion are presented in § 3. The last section is conclusions and summary.

2. Derivation of the formula

For $\ell \ll K_0 R$, the spacing ΔE_ℓ for neighbouring levels of same ℓ for a square well of depth V_0, by nuclear particle, is given by [6]

$$\Delta E_\ell = \frac{\pi V_0}{K_0 R}, \tag{1}$$

where $K_0 = [2mV_0/\hbar^2]^{1/2}$ and R is the nuclear radius which is written as $r_0 A^{1/3}$. Here, we shall take $R = r'_0 A^{1/3} + \Delta$, used extensively in the standard literature [10–12]. We note that the spacing ΔE_ℓ is constant for a given potential as for the harmonic oscillator.

The energy levels of the three-dimensional harmonic oscillator are given by

$$E_{n\ell} = (2n + \ell + \tfrac{3}{2})\hbar\omega,$$

where the symbols have their usual meaning. It follows that neighbouring levels of the same ℓ arise from the change of n by unity. Thus, ΔE_ℓ can be equated to $2\hbar\omega$,

$$2\hbar\omega = \frac{\pi V_0}{K_0 R}.$$

Substituting in the above, R given in terms of A and Δ, and neglecting higher powers of Δ, we have

$$\hbar\omega = \left[\frac{\pi V_0}{2K_0 r'_0}\right] A^{-1/3} - \left[\frac{\pi V_0}{2K_0 r'_0}\right]\left(\frac{\Delta}{r'_0}\right) A^{-2/3}. \tag{2}$$

The dependences of ω on A is of the same form as given for Λ in the standard literature [2]. It is slightly different for the case of the nucleon [3]. However, we see in the next section that in actual practice, the difference is rather small. The values of the coefficients of $A^{-1/3}$ and $A^{-2/3}$ depend upon the particular choice of V_0 and r'_0. As such, too much emphasis cannot be placed on quantitative agreement especially as we are using a square well.

3. Results and discussion

From low-energy scattering experiments, it is well-known [10] that $V_0 R^n = \text{constant}$, where n lies between 2 and 3. It is also known [13] that the level at zero energy, i.e. a just bound level, in a square well potential, remains unaltered if $V_0 R^2 = \text{constant}$.

For a Λ-particle, taking $V_{0\Lambda} = 21{\cdot}7$ MeV and $r_0 = 1{\cdot}30$ fm, as given by Walecka [14] for a square well potential, and using the simpler formula $R = r_0 A^{1/3}$, we find the constant in the equation $V_0 R^2 = \text{constant}$. Then, taking for $V_{0\Lambda}$, the more reasonable value of 30 MeV, the value of Λ-well depth in infinite nuclear matter [7], we get the new value of $r_0 = 1{\cdot}11$ fm for the equivalent square well potential. The value of Δ, the additional constant term in the expression of R, is subject to variations depending upon the way the nuclear radius is defined or measured. We may choose $\Delta = 0{\cdot}70$ fm, a value obtained from early optical model analyses [11], for a rough calculation of the coefficients in the formula for $\hbar\omega_\Lambda$. The value of r'_0 is then obtained from least square fitting of the radius $R = r'_0 A^{-1/3} + \Delta$, with $R = r_0 A^{-1/3}$, over a large range of mass numbers, taking r'_0 as the adjustable parameter and taking $r_0 = 1{\cdot}11$ fm as found above. The value of r'_0 is found to be $0{\cdot}97$ fm. With these choices, the coefficients of $A^{-1/3}$ and $A^{-2/3}$ are $37{\cdot}05$ MeV and $26{\cdot}74$ MeV, respectively. The energy spacings obtained here and with those given in [2], differ by about 1 MeV in the low mass number region. This difference becomes even less in the heavier mass number region. More quantitative agreement is not justified. The values of these two coefficients, as found from χ^2-fit of the excitation data [1], are $38{\cdot}68$ MeV and $34{\cdot}41$ MeV, respectively. The coefficient of

$A^{-1/3}$, as obtained from our formula and from χ^2-fit of the excitation data are quite close. This was only for a rough comparison.

In a similar manner, we obtain the coefficients of $A^{-1/3}$ and $A^{-2/3}$ applying expression (2) for neutron oscillator spacing ($\hbar\omega_N$). On plausible grounds, taking V_{0N}, the depth of the neutron-nucleus square well potential to be roughly 10 MeV more than the depth of the Λ-nucleus square well potential, i.e. the neutron-nucleus depth to be about 40 MeV and taking r_0 and Δ to be same as obtained for Λ, the value of the coefficients of $A^{-1/3}$ and $A^{-2/3}$ are 46·63 MeV and 33·65 MeV, respectively. We may take the same to apply for the neutron-hole. Now, our formula cannot be directly compared with that given in [3], obtained by equating the r.m.s. radius of an oscillator to a Fermi distribution

$$\hbar\omega_N = 38{\cdot}87\,A^{-1/3} - 23{\cdot}24\,A^{-1}, \tag{3}$$

as the power of A in the second term is not same as given in (2). However, the level spacings using the formula obtained here and that given in [3] do not differ much. This kind of agreement may be considered sufficient for our purpose.

One may hope that better values of the coefficients would be achieved, in both the cases of $\hbar\omega_\Lambda$ and $\hbar\omega_N$, for medium and heavy nuclei, if a W–S or some other similar potential is employed and a better search of the potential parameters is carried out. Here, we were interested in the qualitative dependence of the oscillator frequency on the nuclear mass number. The discussion provides one more justification of use of the harmonic oscillator.

4. Conclusions and summary

Here, we have banked mainly on energy considerations as the basis of obtaining the formula for $\hbar\omega_\Lambda$ and $\hbar\omega_N$. The other authors [2, 3, 8] have banked on size considerations. Our main drawback is use of the square well for the Λ as well as the nucleon or the nucleon-hole. Our simple derivation of the formulae does not rest in any way on the assumption of Λ-hypernuclear excitation arising from particle-hole states. That picture has been referred to solely for a rough comparison of our parameters.

Due to difference in the nuclear potential depths of the Λ-, Σ- and Ξ-particles, one expects that the level spacings for these different spectra could be quite different. The observed spectrum would, therefore, provide information on the nuclear potential depths for the different particles.

Incidentally, the work here provides some additional justification for using oscillator wave functions in many nuclear calculations.

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