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Ratios of B and D meson decay constants with heavy quark symmetry

A K GIRI, L MAHARANA and R MOHANTA
Physics Department, Utkal University, Bhubaneswar 751 004, India

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Abstract. $SU(3)$ flavor symmetry allows the decay constants f_{D_s} and f_{D_d} as well as f_{B_s} and f_{B_d} to be equal. But due to $SU(3)$ flavor symmetry breaking the ratios f_{B_s}/f_{B_d} and f_{D_s}/f_{D_d} are deviated from unity. We have estimated these ratios in the heavy quark effective theory and obtained $f_{B_s}/f_{B_d} = 0{\cdot}93$, $f_{D_s}/f_{D_d} = 0{\cdot}94$ and the double ratio $(f_{B_s}/f_{B_d})/(f_{D_s}/f_{D_d}) = 0{\cdot}99$.

Keywords. Decay constants; heavy quark effective theory.

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1. Introduction

In recent years, considerable progress has been made towards a QCD based and model independent description of hadrons containing a heavy quark, the so-called heavy quark effective theory [1–4]. This progress has been achieved by assuming an infinite mass limit for the heavy quark and in this limit, two new symmetries beyond those usually associated with QCD arise. These two symmetries are the spin symmetry and the flavor symmetry. The effective theory of hadrons containing a single heavy quark has been applied to various areas of phenomenology [5–11] and successful results have been obtained for bottom quark and charm quark systems. The idea of the heavy quark symmetry is considered to be effective only for heavy quarks whose masses m_Q be significantly larger than QCD scale Λ_{QCD}. Thus the hadrons containing c-quark and/or b-quark may provide laboratory to test the heavy quark effective theory. The pseudo-scalar decay constant is one of the first physical quantities studied in the context of heavy quark effective theory. The decay constants for D_d and D_s mesons are denoted by f_{D_d} and f_{D_s}, respectively and are equal in the chiral symmetry limit where the up, down and strange quark masses go to zero and $SU(3)$ flavor symmetry for the light quarks is an exact symmetry implying $f_{D_s}/f_{D_d} = 1$ and analogous relations for B meson system, i.e. $f_{B_s}/f_{B_d} = 1$. However in nature, the quark masses $m_q \neq 0$, hence the chiral $SU(3)$ symmetry is broken, its effects on the B and D meson systems are observed by their mass differences [12]

$$m_{B_s} - m_{B_d} \sim m_{D_s} - m_{D_d} \sim 100\,\text{MeV}. \tag{1}$$

Due to the symmetry breaking effect the ratios f_{B_s}/f_{B_d} and f_{D_s}/f_{D_d} deviate from unity. These ratios play a significant role in the possibility of constraining the Cabibbo–Kobayashi–Maskawa matrix. This can be seen from the fact that within the standard model, the mixing between B_s and $\bar{B}_s$ occurs with the parameter $x_s = (\Delta M/\Gamma)_{B_s}$ given by [13]

$$x_s = \frac{G_F^2}{6\pi^2} \tau_{B_s} m_W^2 (f_{B_s}^2 B_{B_s}) \eta_{B_s} |V_{ts}^* V_{tb}|^2 F(m_t^2/m_W^2). \tag{2}$$

Equation (2) shows that the ratio x_s/x_d is independent of the top quark mass m_t, the experimental determination of the ratios implies the ratio $|V_{ts}/V_{td}|$ is known, once $(f_{B_s}^2 B_{B_s}/f_{B_d}^2 B_{B_d})$ and τ_{B_s}/τ_{B_d} have been calculated or known. Several investigations have been done for evaluation of the B and D meson decay constant ratios following various approaches. A summary of earlier studies can be found in ref. [14]. Lattice calculation indicates that the ratio can be calculated much more accurately than either f_{B_s} or f_{B_d} due to cancellation of some systematic uncertainties and lattice calculation of Bernard *et al* [15] yield $f_{B_s}/f_{B_d} \simeq f_{D_s}/f_{D_d} \simeq 1{\cdot}1$. Using QCD sum rules Dominguez [16] has shown that $f_{B_s}/f_{B_d} = 1{\cdot}22$ and $f_{D_s}/f_{D_d} = 1{\cdot}21$. Incorporating heavy quark and chiral perturbation theory, Grinstein *et al* [17] obtained $f_{B_s}/f_{B_d} = 1{\cdot}14$ and $f_{D_s}/f_{D_d} = 1{\cdot}1$. To be more specific the double ratio $(f_{B_s}/f_{B_d})/(f_{D_s}/f_{D_d})$ is very close to unity in all the above calculations as small corrections to both numerator and denominator are cancelled. In a recent letter, Oakes [18], using chiral symmetry and basic quantum mechanical arguments, has shown the double ratio to be 1·004.

In this investigation we calculate the ratios f_{B_s}/f_{B_d} and f_{D_s}/f_{D_d} in heavy quark effective theory and show that the double ratio is very close to unity. Since we have used the basic assumptions of HQET, our results are independent of any model-dependent parameters and depends only on the quark and hadron masses. Hence, the only uncertainties in our calculations are due to the uncertainties present in the quark mass terms.

2. Theory

A convenient framework for systematically analyzing the heavy particle systems is provided by the so-called heavy quark effective theory developed by Georgi [3]. The basic observation is that as the mass of the heavy quark $m_Q \to \infty$, its velocity v becomes a conserved quantity with respect to soft processes. Hence in HQET the effective heavy quark field $h_v^Q(x)$ is related to the original field $Q(x)$ by

$$h_v^Q(x) = \exp(i m_Q v \cdot x) Q(x), \tag{3}$$

and is constrained to satisfy the relation

$$\not{v} h_v^Q(x) = h_v^Q(x). \tag{4}$$

Before turning to the detailed calculations it is important to understand what HQET tells us about the decay constants, since it provides the only results that follow directly from QCD. At leading order in the effective theory the decay constant follow the scaling behavior

$$f_P \propto \frac{1}{\sqrt{m_P}}. \tag{5}$$

In the effective theory the decay constants for the D_d and D_s mesons are defined in terms of hadronic matrix elements as

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 c|D_q(v)\rangle = if_{D_q}\sqrt{m_{D_q}}v^\mu, \tag{6}$$

where $q=(s,d)$ for D_s and D_d mesons respectively. The decay constants for B mesons are defined by equations analogous to (6). These decay constants are equal in the $SU(3)$ flavor symmetry limit. In reality the QCD hamiltonian contains a quark mass term $\mathscr{H}(x)=\Sigma_i m_i \bar{q}_i(x)q_i(x)$, which breaks the symmetry. Therefore the axial vector current is not exactly conserved and the divergence of the axial current can be given as

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5 c) = i(m_c+m_q)\bar{q}\gamma_5 c. \tag{7}$$

The current in the full theory can be expanded in terms of the operators in the effective theory as [19]

$$\bar{q}\Gamma Q = C(\mu)\bar{q}_v\Gamma h_v^Q, \tag{8}$$

where $C(\mu)$ is the short distance coefficient which depends on the renormalization scale μ and at leading order $C(\mu)=1$. Using (8) the modified (7) in the effective theory is given as

$$\partial_\mu(\bar{q}_v\gamma^\mu\gamma_5 h_v^c) = i(m_c+m_q)\bar{q}_v\gamma_5 h_v^c. \tag{9}$$

To obtain the ratio of the decay constants we have to evaluate the matrix elements of the operator present in the above equation. Matrix elements of the operator on the l.h.s. of (9) can be most concisely computed employing a compact trace formalism. For the relevant matrix elements of the operator one can write from ref. [18]

$$\langle 0|\partial_\mu(\bar{q}_v\gamma_\mu\gamma_5 h_v^c)|D_q(v)\rangle = \frac{i}{2}\bar{\Lambda}F(\mu)\mathrm{Tr}(v_\mu\gamma_\mu\mathscr{M}(v)), \tag{10}$$

where $F(\mu)$ is the scale dependent low energy parameter independent of m_Q, denotes the asymptotic value of the scaled decay constant given as $F(\mu)=\sqrt{m_{D_q}}f_{D_q}$ [20], μ is the scale at which the effective current is renormalized. $\bar{\Lambda}$ is a parameter characterizing the properties of the light degrees of freedom defined as [19]

$$\bar{\Lambda} = m_{D_q} - m_c, \tag{11}$$

and

$$\mathscr{M}(v) = -\sqrt{m_{D_q}}\frac{1+\not{v}}{2}\gamma_5, \tag{12}$$

denotes the spin wave function of the D_q meson [20].

Using (11) and (12) we obtain from (10) that

$$\langle 0|\partial_\mu(\bar{q}_v\gamma_\mu\gamma_5 h_v^c)|D_q(v)\rangle = i(m_{D_q}-m_c)f_{D_q}m_{D_q}. \tag{13}$$

Using (9) and (13) one can easily obtain the ratio of the decay constants for D meson as

$$\frac{f_{D_s}}{f_{D_d}} = \frac{m_{D_d}}{m_{D_s}}\left(\frac{m_{D_d}-m_c}{m_{D_s}-m_c}\right)\left(\frac{m_c+m_s}{m_c+m_d}\right)\frac{\langle 0|\bar{s}_v\gamma_5 h_v^c|D_s\rangle}{\langle 0|\bar{d}_v\gamma_5 h_v^c|D_d\rangle}. \tag{14}$$

Analogous relation holds for the ratio of the decay constants for B mesons given as

$$\frac{f_{B_s}}{f_{B_d}}=\frac{m_{B_d}}{m_{B_s}}\left(\frac{m_{B_s}-m_b}{m_{B_s}-m_b}\right)\left(\frac{m_b+m_s}{m_b+m_d}\right)\frac{\langle 0|\bar{s}_v\gamma_5 h_v^b|B_s\rangle}{\langle 0|\bar{d}_v\gamma_5 h_v^b|B_d\rangle}. \tag{15}$$

To evaluate the matrix elements consistent with Lorentz invariance and heavy quark spin symmetry, we introduce the interpolating fields for the heavy mesons as

$$P(v)=\bar{q}_v\gamma_5 h_v^Q\sqrt{m_P}, \tag{16}$$

where $\bar{q}_v$ is a light antiquark which combines with a heavy quark h_v of velocity v to form the appropriate meson. The current given in the matrix element is the interpolating current with the quantum numbers of the heavy meson, which can induce the generation of a heavy meson out of vacuum. Hence we can immediately obtain

$$\begin{aligned}\langle 0|\bar{q}_v\gamma_5 h_v^Q|P(v)\rangle &= \langle 0|\bar{q}_v\gamma_5 h_v^Q\bar{h}_v^Q\gamma_5 q_v|0\rangle\sqrt{m_P}\\ &= -\operatorname{Tr}\left(\langle 0|\gamma_5\frac{\not{v}+1}{2}\gamma_5|0>M\right)\sqrt{m_P},\end{aligned} \tag{17}$$

where $M=\langle 0|q_v\bar{q}_v|0\rangle$ is a 4×4 matrix [21] and we have used the heavy quark propagator as $\langle 0|h_v^Q\bar{h}_v^Q|0\rangle=(1+\not{v})/2$. Lorentz invariance implies that

$$M=A(v^2)I+B(v^2)\not{v}, \tag{18}$$

where $A(v^2)$ and $B(v^2)$ are functions of the scalar variable v^2. Since $v^2=1$ the functions

$$A(1)=A \quad \text{and} \quad B(1)=B\,\text{(say)}, \tag{19}$$

are universal constants. Substituting the value of M from (18) in (17) we obtain

$$\langle 0|\bar{q}_v\gamma_5 h_v^Q|P(v)\rangle=-2(A-B)\sqrt{m_P}. \tag{20}$$

Using (20) we obtain from (14) and (15) the exact expressions for the ratios of the decay constants as

$$\frac{f_{D_s}}{f_{D_d}}=\sqrt{\frac{m_{D_d}}{m_{D_s}}}\left(\frac{m_{D_s}-m_c}{m_{D_s}-m_c}\right)\left(\frac{m_c+m_s}{m_c+m_d}\right) \tag{21}$$

and

$$\frac{f_{B_s}}{f_{B_d}}=\sqrt{\frac{m_{B_d}}{m_{B_s}}}\left(\frac{m_{B_s}-m_b}{m_{B_s}-m_b}\right)\left(\frac{m_b+m_s}{m_b+m_d}\right). \tag{22}$$

We take the current quark masses as $m_d=10\,\text{MeV}$, $m_s=150\,\text{MeV}$, $m_c=1\cdot 3\,\text{GeV}$, $m_b=4\cdot 3\,\text{GeV}$ and the masses of B and D mesons are $M_{D_d}=1869$ MeV, $M_{D_s}=1968\cdot 5$ MeV, $M_{B_s}=5375$ MeV and $M_{B_d}=5279$ MeV from ref. [12]. With these values we obtain the ratios to be

$$\frac{f_{B_s}}{f_{B_d}}=0\cdot 93 \tag{23}$$

and

$$\frac{f_{D_s}}{f_{D_d}}=0\cdot 94. \tag{24}$$

The double ratio is given as

$$(f_{B_s}/f_{B_d})/(f_{D_s}/f_{D_d}) = 0{\cdot}99. \tag{25}$$

3. Discussion

We have tried to calculate the ratio of B and D meson decay constants using heavy quark effective theory. Considering the $SU(3)$ flavor symmetry breaking in the light quark sector, we obtain the exact expressions for the ratios of the decay constants. The matrix element contained in the expression is calculated in HQET, consistent with the Lorentz invariance and heavy quark spin symmetry. Thus we obtain the ratios in terms of quark and hadron masses and substitution of the masses yield the results $f_{B_s}/f_{B_d} = 0{\cdot}93$ and $f_{D_s}/f_{D_d} = 0{\cdot}94$ and the double ratio $(f_{B_s}/f_{B_d})/(f_{D_s}/f_{D_d}) = 0{\cdot}99$. It has been argued by Grinstein [22] using both heavy quark and chiral symmetry that the double ratio be equal to unity with sizable corrections for the light and heavy quark sector. The double ratio in our case is very close to unity with a correction factor of 1%. Since a direct measurement of f_B through the leptonic decay will be extremely challenging because of the very small branching ratio and difficult signature, a measurement of f_D is much more feasible, so a precise measurement of f_{D_s}/f_{D_d} will determine the value of f_{B_s}/f_{B_d}, which is a factor in determining the relative strengths of $B_s - \bar{B}_s$ and $B_d - \bar{B}_d$ mixing. These mixings give valuable information on the elements of Cabibbo–Kobayashi–Maskawa matrix, i.e. from the measured value of both these mixings one can extract $|V_{ts}/V_{td}|$ from their ratio.

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