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Static and dynamic properties of heavy light mesons in infinite mass limit

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Abstract. We summarize the consequences of the infinite limit of heavy quark mass in the results of form factors, charge radii and decay constants of heavy light mesons within a QCD inspired quark model recently reported.

Keywords. Heavy light mesons; form factors; decay constants; charge radii.

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1. Introduction

In a recent communication [1] referred as I, we have reported the results of form factors, charge radii and decay constants of both light and heavy flavoured pseudoscalar mesons in a QCD inspired quark model. The technique used was the quantum mechanical perturbation theory [2] with plausible relativistic corrections [3, 4].

The present paper reports the results of the same model in infinite mass limit ($m_Q \to \infty$, m_Q being the heavy quark mass). It is well-known that in the limit of infinitely large quark mass, additional symmetries beyond the ones of QCD arise [5–7] which enable one to obtain model independent information on the weak decay matrix elements of heavy mesons. Indeed, in heavy to heavy transitions like $b \to c$ decays, all heavy quark bilinear current matrix elements are described in terms of only one form factor – the so called Isgur–Wise (IW) function in leading order. This result is phenomenologically useful since it allows a model independent determination of the CKM matrix element [8, 9] $|V_{bc}|$ for semileptonic $B \to D$ and $B \to D^*$ decays.

Phenomenological utility of such an infinite mass limit in the study of static and dynamic properties of mesons as a low energy phenomenon is perhaps not so much, as the number of form factors involved are small. However, it is still meaningful to study the consequences of such an extra symmetry as a low energy approximation even for such familiar quantities. It will at least allow one to see if such a limit is close approximation to reality.

The aim of the present paper is to study the static and dynamic properties of the heavy light mesons in the infinite mass limit and estimate the percentage of deviation of such a limit.

In § 2, we present the essential theory and results while § 3 contains the conclusion.

2. Theory and results

2.1 *Analysis with Coulomb potential*

The Coulombic wavefunction in the ground state is given by [2]

$$\psi(r)=\frac{1}{\sqrt{\pi a_0^3}}\mathrm{e}^{-r/a_0} \tag{1}$$

where a_0 is given by eq. (19) of I with

$$\mu=\frac{m_q m_Q}{m_q+m_Q} \tag{2}$$

being the reduced mass of the heavy light mesons. Here m_q and m_Q are the masses of light and heavy quarks respectively.

In Isgur–Wise limit $(m_Q\to\infty)$ [5–7]

$$\mu\simeq m_q \tag{3}$$

and

$$\lim_{m_Q\to\infty} a_0\equiv a_0^\infty=\frac{1}{\frac{4}{3}m_q\alpha_s}. \tag{4}$$

The relation between a_0 and a_0^∞ is

$$a_0=a_0^\infty\left[1+\frac{m_q}{m_Q}\right]. \tag{5}$$

With the introduction of spin effect, a_0^∞ changes to $a_0^{\infty^P}$ corresponding to the modification of α_s to α_s^P for pseudoscalar mesons given by eq. (38) of I.

With the introduction of relativistic effect, the wavefunction eq. (1) modifies to

$$\psi^{\mathrm{Rel}}(r)=\left[\frac{2}{2^{2\varepsilon}\Gamma(3-2\varepsilon)}\right]^{1/2}\frac{\mathrm{e}^{-r/a_0^\infty}}{\sqrt{\pi a_0^{\infty^3}}}\left(\frac{r}{a_0^\infty}\right)^{-\varepsilon} \tag{6}$$

where ε is defined in eq. (26) of I.

The elastic charge form factor, eq. (42) of I with eq. (6) yields

$$eF(Q^2)=\frac{e_q}{\left[1+\frac{a_0^{\infty^2}Q^2}{4}\right]^n}+e_Q \tag{7}$$

for $Q^2\ll m_Q^2$, where $n=2$, 1·25, 1·1 and 1 corresponding to $\varepsilon=0$, 0·25, 0·4 and 0·5 respectively. In eq. (7) above, e_q and e_Q are the charges of the light and the heavy quarks respectively.

For $Q^2\gg m_Q^2$, on the other hand,

$$eF(Q^2)=\frac{e_q}{(1+(a_0^{\infty^2}Q^2/4))^n}+\frac{e_Q}{(1+(a_0^{\infty^2}m_q^2Q^2/4m_Q^2))^n}. \tag{8}$$

Equation (7) suggests that in low energy limit $Q^2\ll m_Q^2$, heavy quark component of the form factor is approximately Q^2 independent and measures its charge.

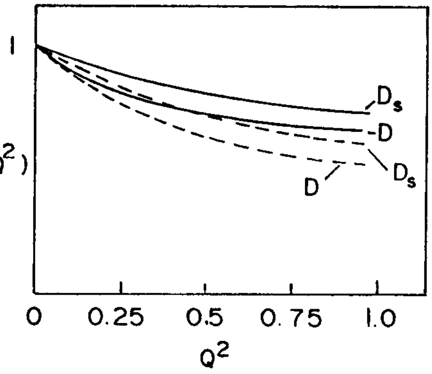

Figure 1. D and D_s meson form factors in the infinite mass limit (solid lines) and without infinite mass limit (dashed lines) taking $n = 1$ in eq. (7) of the text.

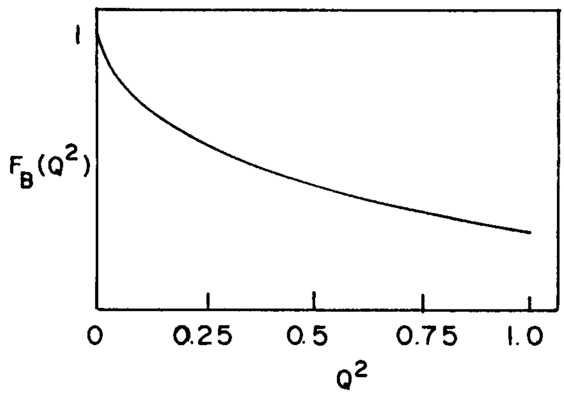

Figure 2. B meson form factor. It shows no appreciable difference in the infinite and finite mass limits.

Using eq. (7), we plot $F(Q^2)$ vs Q^2 for D and D_s mesons (solid lines) in figure 1, while in figure 2, we plot the same quantity for B meson, using $1/Q^2$ behaviour corresponding to $n = 1$ in eq. (7). In the same figures, we also plot (dashed lines) the corresponding functions taking into account the finite quark masses $m_c = 1{\cdot}55$, $m_b = 4{\cdot}95$ GeV. The two results for D and D_s differ by $\sim 7{\cdot}6\%$ to $\sim 10\%$ but the difference is negligible for B meson.

2.2 *Charge radii*

Using eq. (7), the average charge radius defined by eq. (51) of I yields

$$\lim_{m_Q \to \infty} \langle r^2 \rangle \equiv \langle r^2 \rangle^\infty = \frac{3n}{2} a_0^{\infty^2} e_q \tag{9}$$

where n has values as in eqs (7) or (8). In table 1, we record $\langle r^2 \rangle^\infty$ for different pseudoscalar

Table 1. $\langle r^2 \rangle$ in fm^2 for heavy light mesons using eqs (9) and (10) of the text.

Particles	Eq. (10) (finite m_Q)	Eq. (9) (infinite m_Q)	Change in percentage
D^+	0·25	0·23	8
D^0	−0·44	−0·46	4·5
D_s	0·13	0·11	15·4
B	0·46	0·46	∼0
B_d^0	−0·23	−0·23	∼0
B_s^0	−0·11	−0·11	∼0

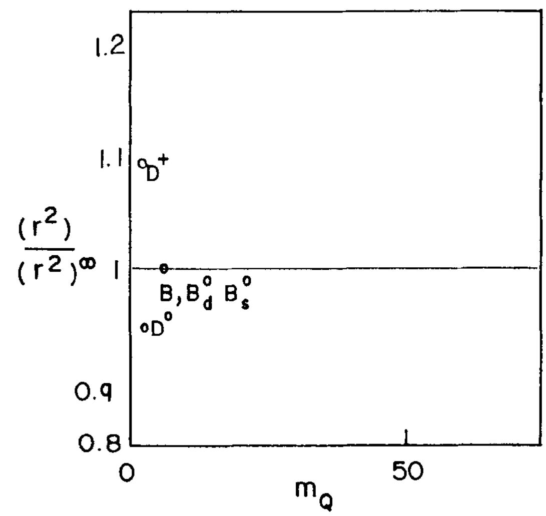

Figure 3. $\langle r^2 \rangle / \langle r^2 \rangle^\infty$ vs m_Q for charmed and bottomed mesons.

mesons for $n = 1$ and compare with the values obtained with the expression

$$\langle r^2 \rangle = \frac{3n}{2} a_0^2 \left[\frac{e_q}{(1 + (m_q/m_Q))^2} + \frac{e_Q}{(1 + (m_Q/m_q))^2} \right] \tag{10}$$

for the same mesons with finite m_Q. Equation (10) corresponds to eq. (52) of I with $g_{\text{conf}} = 1$.

From eq. (9), the following symmetry relations are obtained:

$$\langle r^2 \rangle_{D^+}^\infty = \langle r^2 \rangle_{B^0}^\infty = \tfrac{1}{2} \langle r^2 \rangle_{D^0}^\infty = \tfrac{1}{2} \langle r^2 \rangle_{B^+}^\infty \tag{11}$$

and

$$\langle r^2 \rangle_{D_s}^\infty = \langle r^2 \rangle_{B_s}^\infty. \tag{12}$$

The relation between $\langle r^2 \rangle$ and $\langle r^2 \rangle^\infty$ is

$$\langle r^2 \rangle = \langle r^2 \rangle^\infty \left[1 + \left(\frac{e_Q}{e_q} \right) \left(\frac{m_q}{m_Q} \right)^2 \right]. \tag{13}$$

In figure 3, we plot $\langle r^2 \rangle / \langle r^2 \rangle^\infty$ vs m_Q for various heavy light mesons. Table 1 and figure 3 suggest that infinite mass limit $m_Q \to \infty$ is nearly true for b quark but not so for c quark.

2.3 *Analysis with confinement*

(i) *Form factors*: The form factor eq. (44) of I in the infinite mass limit can be rewritten as

$$\begin{aligned} eF(Q^2) = {} & \frac{1}{\left(1 - 3m_q b a_0^{\infty^3} + \frac{45}{8} m_q^2 b^2 a_0^{\infty^6}\right)} \Bigg[e_q \Bigg(\frac{1}{(1 + (a_0^{\infty^2} Q_i^2/4))^2} \\ & - 3m_q b a_0^{\infty^3} \frac{(1 - (a_0^{\infty^2} Q_i^2/4))}{(1 + (a_0^{\infty^2} Q_i^2/4))^4} - \frac{15 m_q^2 b^2 a_0^{\infty^6}}{8(1 + (a_0^{\infty^2} Q_i^2/4))^4} \\ & \times \left\{ 3 - \frac{4 a_0^{\infty^2} Q_i^2}{(1 + (a_0^{\infty^2} Q_i^2/4))} - \frac{a_0^{\infty^4} Q_i^4}{(1 + (a_0^{\infty^2} Q_i^2/4))^2} \right\} \Bigg) \\ & + e_Q \left(1 - 3m_q b a_0^{\infty^3} + \frac{45}{8} m_q^2 b^2 a_0^{\infty^6} \right) \Bigg] \end{aligned} \tag{14}$$

for $Q^2 \ll m_Q$ where Q_i is defined in eq. (22) of I. With relativistic effect ($\varepsilon = 0{\cdot}5, \alpha_s = 0{\cdot}65$), eq. (14) becomes

$$\begin{aligned} eF(Q^2) = {} & \frac{1}{(1 - \frac{3}{2} m_q b a_0^{\infty^3} + \frac{15}{8} m_q^2 b^2 a_0^{\infty^6})} \Bigg[e_q \Bigg\{ \frac{1}{(1 + (a_0^{\infty^2} Q_i^2/4))} \\ & - \frac{3 m_q b a_0^{\infty^3}}{2(1 + (a_0^{\infty^2} Q_i^2/4))} + \frac{15 m_q^2 b^2 a_0^{\infty^6}}{8(1 + (a_0^{\infty^2} Q_i^2/4))^3} \Bigg\} \\ & + e_Q (1 - \tfrac{3}{2} m_q b a_0^{\infty^3} + \tfrac{15}{8} m_q^2 b^2 a_0^{\infty^6}) \Bigg]. \end{aligned} \tag{15}$$

In figure 4, we plot the representative form factor for D meson using (15) with $b = 0$ and $0{\cdot}05\ \mathrm{GeV}^2$ (solid lines). For comparison, we also plot for the same meson without infinite mass limit (dashed lines).

(ii) *Charge radii*: The expression of charge radii with confinement effect in the $m_Q \to \infty$ limit is

$$\langle r^2 \rangle^\infty = \frac{3n}{2} a_0^{\infty^2} g_{\mathrm{conf}}^\infty e_q \tag{16}$$

where the confinement factor g_{conf}^∞ is defined in eq. (53) of I, with

$$g_{\mathrm{conf}}^\infty \equiv \lim_{m_Q \to \infty} g_{\mathrm{conf}}(\mu, a_0) = g_{\mathrm{conf}}(m_q, a_0^\infty). \tag{17}$$

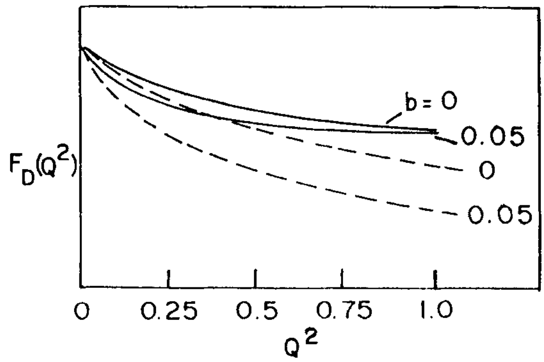

Figure 4. Representative form factor for D meson in infinite mass limit (solid lines) and without infinite mass limit (dashed lines) for confinement parameter $b = 0$ and 0·05 GeV^2.

Table 2. Decay constants in MeV for heavy light pseudoscalar mesons in infinite mass limit.

Mesons	Results with finite m_Q	Results with $m_Q \to \infty$	Change in percentage
D	$\leqslant 209$	$\leqslant 124$	40·67
D_s	$\leqslant 237$	$\leqslant 262$	10·55
B	$\leqslant 107$	$\leqslant 104$	2·8

Unfortunately, unlike pion or kaon, there is no data on charge radii of heavy light mesons and hence eq. (16) or its counterpart eq. (52) of I without infinite mass limit do not yield any phenomenological information on string constant b.

(iii) *Decay constants*: Using eq. (4) in (37) and (54) of I, we obtain values of decay constants of heavy light pseudoscalar mesons in infinite mass limit and record them in table 2. In the same table, we also compare the results without such infinite mass limit. For charmed mesons, they differ by $\sim 40\%$, while for bottom, the difference is negligible.

3. Conclusion

In this paper, we have made an analysis of a QCD inspired quark model for heavy light pseudoscalar mesons in the limit of infinite heavy quark mass.

The simplicity of the infinite mass limit does not guarantee that it is a close approximation to reality. This should be determined by an analysis of the correction to the limit which is the motivation of this paper. Our analysis has shown that while for mesons with b quark, the infinite mass limit is nearly exact, for c quark, it can differ even by $\sim 40\%$. For top quark ($m_t \sim 174$ GeV) the limit would have been almost exact, but absence of bound states of top quark makes this observation rather academic.

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