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Scaling laws for plasma transport due to η_i-driven turbulence

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Abstract. The scale invariance technique has been employed to discuss the η_i-driven turbulent transport under a new fluid model developed by Kim *et al* [1]. Our analysis reveals that the finite Larmour radius effect plays a decisive role to determine the scaling behaviour of the energy transport under the new fluid model. However, the overall scaling of the transport coefficient remains unchanged as compared to that derived by Connor [2] under the traditional fluid model. The approximations considered by Connor [2] are qualified with additional requirements within the new fluid approach. In the dissipative case, which has not been discussed earlier, additional constraints on the power scaling laws of the transport properties are imposed due to the dissipative mechanisms in the basic governing equations.

Keywords. Scaling laws; invariance technique; similarity transformations; transport coefficient; η_1-driven turbulence.

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1. Introduction

Theoretical and experimental study of anomalous transport in magnetic confinement systems (to determine the physical mechanism for the transport phenomena) has become a subject of main concern for the plasma physicists. Turbulent transport due to ion temperature gradient (or temperature drift) instability has been suggested as one of the physical phenomena that leads to an anomalous ion thermal conduction. This has been proposed as the cause of a deterioration in confinement in the Alcator C experiment at high density when gas puffing produces flat density profile [3]. Many projects, theoretical [4–11] and experimental [12, 13] have been devoted to solving the energy transport due to η_i-modes. Many of them are based on the fluid models [5, 6, 9, 11]. Nonlinear saturation level of the η_i-driven fluctuations predicted by these models are found to be much larger (by at least an order of magnitude) than the levels predicted by the more sophisticated particle simulations [14]. This is generally attributed to the lack of ion Landau damping in the conventional fluid models. Accordingly, Hammett and Perkins [15] incorporated the approximated Landau terms into the basic equations to reduce the saturation level of the η_i-turbulence. However, very recently Kim *et al* [1] have developed a new fluid model for the η_i-driven turbulence by incorporating the complete treatment of the polarization drift due to finite Larmour radius effects in basic equations. Their general belief is that by just including the damping effects into the basic equations may not be enough for bringing down the saturated level for the η_i-driven fluctuations to match with that predicted by numerical simulations [14]. Rather one should very carefully treat the ion polarization drift while working out the energy conservation property. In the existing fluid models most of the authors either completely neglect divergence of polarization drift ($\nabla \cdot \mathbf{v}_p$) term in the heat equation or include only part of it. Consequently, inconsistency arises so far as the contribution of diamagnetic drift to the kinetic energy in conservation law is concerned. They [1] considered this aspect and treated the ion temperature fluctuation while deriving the expression for the polarization drift ($\mathbf{v}_p$). Thus the new set up of fluid equations for the description of η_i-mode turbulence differs from the others due to contribution of ion-diamagnetic drift to the kinetic energy in the energy balance relation.

Once the responsible physical mechanism for the anomalous transport is established, the nonlinear analytical calculation of the transport properties to describe their scaling behaviour becomes quite a tedious and intractable job. To avoid this difficulty, Connor and Taylor [16] were the first to suggest a technique more general than the analytical calculation. It is based on the invariance principle of the basic governing equations under a group of linear transformations which eventually describe the scaling properties of the anomalous transport phenomena. This method has already been successfully applied to various situations of physical mechanisms [2, 11, 16–19]. Based on the traditional fluid model of non-dissipative plasmas, scaling behaviour of transport associated with η_i-turbulence has been discussed by Connor [2]. However, recent development of a new fluid model [1] incorporating the classical dissipations due to collisions warrants a fresh look at the power scaling laws of the transport due to η_i-turbulence in the dissipative and non-dissipative cases. This paper considers various cases of approximations and compares the findings with the earlier results [2] in the non-dissipative domain of the basic equations. In this domain, the overall scaling behaviour of the transport remains unchanged except that the approximations used by Connor [2] are qualified with additional requirements in the light of the new fluid model for the η_i mode. Inclusion of the collisional dissipations and Landau dampings introduce additional constraints on the power scaling laws of the η_i-driven turbulent transport by increasing the number of free indices for describing the functional form of the transport coefficients. Section 2 deals with the description of basic equations developed by Kim *et al* [1] to describe the η_i-driven turbulence. Section 3 includes the derivations of the scaling laws for the diffusion coefficients under various possible approximations of the dissipative and non-dissipative fluid models. Results and discussions form § 4 of this paper.

2. Basic governing equations

Under the new fluid model [1], the basic equations for describing the η_i-driven turbulent transport are given as follows:

Continuity equation

$$\frac{\partial}{\partial t}(\phi - \nabla_\perp^2 \psi) + \frac{\partial \phi}{\partial y} + \nabla_\parallel v_\parallel + \left[\frac{\partial p}{\partial x}, \frac{\partial \phi}{\partial x}\right] + \left[\frac{\partial p}{\partial y}, \frac{\partial \phi}{\partial y}\right]$$
$$- [\phi, \nabla_\perp^2 \psi] + \mu_\perp \nabla_\perp^4 \psi = 0, \quad (1)$$

Parallel momentum equation

$$\frac{\partial v_\parallel}{\partial t} + \nabla_\parallel \psi + [\phi, v_\parallel] - v_\perp \nabla_\perp^2 v_\parallel - v_\parallel \nabla_\parallel^2 v_\parallel = 0, \quad (2)$$

Pressure equation

$$\frac{\partial}{\partial t}(p - \tau \Gamma n) + \tau \bar{K} \frac{\partial \Phi}{\partial y} + [\Phi, (p - \tau \Gamma n)] - \chi_\perp \nabla_\perp^2 p - \chi_\parallel \nabla_\parallel^2 p = 0. \quad (3)$$

These are the normalized equations and their derivations and normalization constants are described in paper [1]. $\tau = T_i/T_e$, $\bar{K} = K - \Gamma$, $K = \eta_i + 1, \Gamma$ is the adiabatic gas constant, $\psi = \Phi + p$. The perpendicular dissipative coefficients are given by $\mu_\perp = \tau\delta/4$, $v_\perp = 0{\cdot}3\tau\delta, \chi_\perp = \tau\delta$, where $\delta = (v_i/\omega_{ci})(L_n/\rho_s)$ and L_n, ρ_s and v_i are respectively Larmour radius, density scale length and ion collisional frequency. For the parallel diffusion coefficients, $v_\parallel$ and $\chi_\parallel$ are chosen to model the ion Landau damping. Further $\nabla_\parallel = (\partial/\partial \xi) + sx(\partial/\partial y), \xi = z/L_n$ with $s(= L_n/L_s)$ as the shear parameter. Square bracket [ ] enclosing within it the physical quantities denotes for Poisson bracket defined as $[f, g] = ((\partial f/\partial x)(\partial g/\partial y)) - ((\partial f/\partial y)(\partial g/\partial x))$. Now these equations have been solved under different sets of approximations in the dissipative and non-dissipative cases by applying the concept of the invariance principle to establish the scaling behaviour of thermal transport due to η_i-driven turbulence.

3. Scale invariance

3.1 *Non-dissipative case* ($\mu_\perp = v_\perp = \chi_\perp = v_\parallel = \chi_\parallel = 0$)

Connor [2] has already discussed the power scaling laws of η_i-driven thermal transport under the traditional fluid model of collisionless plasmas. However, in the light of the new fluid model [1], we applied the scale invariance technique to see the effect of the structural changes in the new basic governing equations of η_i-mode on the scaling behaviour of thermal transport.

Under the fluid approximation $\nabla_\perp^2 \ll 1$ and the assumption of weak potential fluctuation $\Phi \ll p$, the relative dominance of the terms Φ and $\nabla_\perp^2 \psi$ (inside the paranthesis of (1)) suggests the possibilities of three different cases; $\phi \gg \nabla_\perp^2 p, \phi \sim \nabla_\perp^2 p$ and $\phi \ll \nabla_\perp^2 p$. The self consistent validity conditions of these additional approximations have been derived and verified by calculating the scalings of $\phi, p, \nabla_\perp^2 p, \partial\phi/\partial t$ and $\partial\phi/\partial y$. It is found that the first case is inconsistent whereas the remaining last two cases are valid. Now under the approximations $\phi \ll p, \nabla_\perp^2 \ll 1$ and $\phi \sim \nabla_\perp^2 p$ equation (1) is rewritten as

$$\frac{\partial}{\partial t}(\phi - \nabla_\perp^2 p) + \frac{\partial \Phi}{\partial y} + \nabla_\parallel v_\parallel + \left[\frac{\partial p}{\partial x}, \frac{\partial \Phi}{\partial x}\right] + \left[\frac{\partial p}{\partial y}, \frac{\partial \Phi}{\partial y}\right] - [\Phi, \nabla_\perp^2 p] = 0. \quad (4)$$

Now we seek all the linear transformations of the independent and dependent variables

$$n \to \alpha n, \Phi \to \beta\Phi, p \to \gamma p, v_{\parallel} \to \mu v_{\parallel}, \bar{K} \to \nu\bar{K}, s \to \delta s, x \to l_1 x, y \to l_2 y,$$
$$\xi \to l_3 \xi, t \to l_4 t$$

which leave the basic equations (2–4) invariant. We find only one such transformation

$$\text{T:}\; n \to l^3 n, \phi \to l\phi, p \to l^3 p, v_{\parallel} \to l^2 v_{\parallel}, \bar{K} \to l^2 \bar{K}, s \to l^{-2} s, x \to lx, y \to ly,$$
$$\xi \to l^2 \xi, t \to lt, \quad \text{for} \quad l_1 = l_2 = l.$$

Under this transformation, diffusion co-efficient D should transform as $D \to lD$, since any transport coefficient must scale as $(\Delta r)^2/\Delta t$. Now if D is expressed as [2]

$$D = \frac{\rho_s^2 C_s}{L_n} \hat{D}(s, \bar{K}) \tag{5}$$

where $\rho_s^2 C_s / L_n$ is a normalization factor and $\hat{D}(s, \bar{K})$ a normalized diffusion coefficient, the functional form of $\hat{D}(s, \bar{K})$ can be determined as

$$\hat{D} = s^p \bar{K}^q.$$

The requirement that it remains invariant under the above transformation T, imposes the following restriction on the exponents

$$\tfrac{1}{2} = -p + q$$

so that the functional form of D is restricted to

$$D = \frac{\rho_s^2 C_s}{L_n} \left(\frac{L_s}{L_n}\right)^{1/2} F\left(\frac{L_n^2}{L_s L_T}\right). \tag{6}$$

Following the same technique, the scalings of the normalized ϕ, p, $\nabla_\perp^2 p$, $\partial\phi/\partial t$, $\partial\phi/\partial y$ and $\nabla_\perp^2$ are derived to reveal that $\phi \to s^{-1/2}(s\bar{K})^q$, $p \to s^{-3/2}(s\bar{K})^q$, $\nabla_\perp^2 p \to s^{-1/2}(s\bar{K})^q$, $\partial\phi/\partial t \to (s\bar{K})^q$, $\partial\phi/\partial y \to (s\bar{K})^q$ and $\nabla_\perp^2 \to s(s\bar{K})^q$. Thus the self consistent validity of the approximations requires $s \ll 1$ for $s\bar{K} \sim 1$ up to an unknown function of order unity. The scaling of the diffusion coefficient in this case shows the $s^{-1/2}$ dependence as reported even by Horton *et al* [6].

Let us now consider the third case i.e. $\phi \ll \nabla_\perp^2 p$ under the approximations of weak potential fluctuation $\Phi \ll p$ and fluid regime $\nabla_\perp^2 \ll 1$. In this limit again only (1) is modified and given as

$$-\frac{\partial}{\partial t}(\nabla_\perp^2 p) + \frac{\partial\Phi}{\partial y} + \nabla_{\parallel} v_{\parallel} + \left[\frac{\partial p}{\partial x}, \frac{\partial\Phi}{\partial x}\right] + \left[\frac{\partial p}{\partial y}, \frac{\partial\Phi}{\partial y}\right] - [\Phi, \nabla_\perp^2 p] = 0. \tag{7}$$

Now we seek again the linear transformations of the independent and dependent variables

$$n \to \alpha n, \quad \Phi \to \beta\Phi, \quad p \to \gamma p, \quad v_{\parallel} \to \mu v_{\parallel}, \bar{K} \to \nu\bar{K}, s \to \delta s, x \to l_1 x, y \to l_2 y,$$
$$\xi \to l_3 \xi, t \to l_4 t$$

which leave the basic equations (2, 3, 7) invariant. We find two such transformations

$$T_1:\ n \to l^3 n, p \to l^3 p, v_\parallel \to l^2 v_\parallel, \bar{K} \to l^2 \bar{K}, s \to l^{-3} s, x \to lx, y \to ly,$$
$$\xi \to l^3 \xi, t \to l^2 t, \quad \text{for} \quad l_1 = l_2 = l$$
$$T_2:\ n \to \beta^{3/2} n, \Phi \to \beta\Phi, p \to \beta^{3/2} p, v_\parallel \to \beta v_\parallel, \bar{K} \to \beta\bar{K}, s \to \beta^{-1/2} s,$$
$$x \to \beta^{1/2} x, y \to \beta^{1/2} y, \xi \to \beta^{1/2} \xi.$$

Under the combined operation of these two transformations diffusion coefficient D should transform as $D \to \beta D$, since any transport coefficient must scale as $(\Delta r)^2/\Delta t$. Now if D is again expressed by (5) with the functional form of $\hat{D}(s, \bar{K})$ as $\hat{D} = s^p \bar{K}^q$, the requirement that it remains invariant under the above transformations $T_1 - T_2$, imposes the following restrictions on the exponents

$$1 = -p/2 + q, \quad 0 = -3p + 2q$$

so that the functional form of D is restricted to

$$D = \frac{\rho_s^2 C_s}{L_s}\left(\frac{L_n}{L_T}\right)^{3/2}. \tag{8}$$

This functional form is the same as that derived by Connor [2] under the approximations $\Phi \ll p, \nabla_\perp^2 \ll 1$ and $(\partial\Phi/\partial t) \ll (\partial\Phi/\partial y)$. Again to verify the validity of the approximations, the scalings of the normalized ϕ, p, $\nabla_\perp^2 p$, $\partial\phi/\partial t$, $\partial\phi/\partial y$ and $\nabla_\perp^2$ are derived to reveal that $\phi \to s\bar{K}^{3/2}$, $p \to \bar{K}^{3/2}$, $\nabla_\perp^2 p \to \bar{K}^{1/2}$, $\partial\phi/\partial t \to (s\bar{K})^2$, $\partial\phi/\partial y \to (s\bar{K})$, $\nabla_\perp^2 \to \bar{K}^{-1}$. This is to note that self-consistent validity of the approximations requires $s\bar{K} \ll 1$ for $s \ll 1$ and $\bar{K} \gg 1$ up to an unknown constant of order unity. This implies that s and $\bar{K}$ should scale as $s \sim \varepsilon^2$, $\bar{K} \sim \varepsilon^{-1}$ for $\varepsilon \ll 1$. Physically it suggests a situation of very weak magnetic shear. In this case the diffusion coefficient scales linearly with the shear parameter s which is the same as reported by Connor [2]. However, the approximation $s\bar{K} \ll 1$ has been questioned by Hamaguchi and Horton [5] based on their numerical simulation. Their criticism lies on the positive footing and is based on the linear property of the η_i mode. In the light of their comments we would like to add that the inertia term completely disappears in the continuity equation of the traditional fluid model for $s\bar{K} \ll 1$ which poses a qualitative problem at the linear level of the η_i mode. Nevertheless, in the new fluid model developed by Kim *et al* [1], the inertia term now arising due to finite Larmour radius effect (polarization drift), still survives for the approximation $s\bar{K} \ll 1$ and determines the explicit form of the diffusion coefficient and other variables. As discussed above, the linear scaling of the D with s holds good for very weak magnetic shear. This implicitly includes the approximation $\partial\phi/\partial t \ll \partial\phi/\partial y$.

Accordingly it may be argued that under the approximation of weak potential $\Phi \ll p$, the dominance of finite Larmour radius correction term $\nabla_\perp^2 \psi$ in (1) (arising due to the polarization drift) can explicitly determine the explicit functional form of D. This possibility is ruled out under the traditional fluid model within the validity of fluid approximation. Furthermore, the anisotropic distribution of the turbulence $\partial/\partial x \gg \partial/\partial y$ or $\partial/\partial x \ll \partial/\partial y$ does not affect the scaling properties of transport coefficients and other quantities. This is true even in the case of dissipative fluid model.

Our analysis predicted the same functional forms for thermal diffusion and other quantities as those described by Connor [2]. Based on these conclusions we speculate that the contribution of the ion-diamagnetic drift to the kinetic energy in the energy balance relation, does not qualitatively alter the η_i-driven transport mechanism and consequently the power scaling laws of the transport coefficient associated with the same. This is in good agreement with the analytical conclusions of Kim *et al* [1] which report only quantitative changes (decrease) in the fluctuation levels of the η_i-driven turbulence.

3.2 *Dissipative case* $(\mu_\perp, \nu_\perp, \chi_\perp, \nu_\parallel, \chi_\parallel \neq 0)$

Since the earlier description of the energy transport due to η_i-driven turbulence [2] was limited to the non-collisional fluid model of plasmas, the restrictions on the functional forms of thermal coefficients and other quantities could not be representative of those plasma systems in which dissipations play an important role. Equations (1–3) derived by Kim *et al* [1] include the classical dissipation due to particle collisions and Landau damping terms. This section deals with the scaling behaviour of thermal transport coefficients in the presence of dissipative terms in basic governing equations. For simplicity we have approximated $\nabla_\perp^2 \sim (\partial^2/\partial y^2)$ and $\nabla_\parallel \approx sx(\partial/\partial y)$. This is to note again that the approximation $\phi \gg \nabla_\perp^2 p$ as discussed in the nondissipative case is a non-valid approximation even in the dissipative system. The self-consistency of this statement relies upon the assumption that the scalings of ϕ, p, $\nabla_\perp^2 p$, $\partial\phi/\partial t$ $\partial\phi/\partial y$ and $\nabla_\perp^2$ are unchanged up to an unknown function (due to dissipative effect) of the same order. Now under the approximations $\Phi \ll p$, $\nabla_\perp^2 \ll 1$ and $\Phi \sim \nabla_\perp^2 p$, we again seek all the linear transformations of the dependent and independent variables

$$n \to \alpha n, \Phi \to \beta\Phi, p \to \gamma p, v_\parallel \to \mu v_\parallel, \bar{K} \to \nu\bar{K}, s \to \gamma s, x \to l_1 x, y \to l_2 y, \xi \to l_3 \xi,$$
$$t \to l_4 t, \mu_\perp \to l_5 \mu_\perp, \nu_\perp \to l_6 \nu_\perp, \chi_\perp \to l_7 \chi_\perp, \nu_\parallel \to l_8 \nu_\parallel, \chi_\parallel \to l_9 \chi_\parallel \tag{9}$$

so that equations (1–3) be invariant. We find only one such transformation

$$T_3\colon\ n \to l^3 n, \Phi \to l\Phi, p \to l^3 p, v_\parallel \to l^2 v_\parallel, \bar{K} \to l^2 \bar{K}, s \to l^{-2} s, x \to lx, y \to ly,$$
$$t \to lt, \mu_\perp \to l\mu_\perp, \nu_\perp \to l\nu_\perp, \chi_\perp \to l\chi_\perp, \nu_\parallel \to l^3 \nu_\parallel, \chi_\parallel \to l^3 \chi_\parallel. \tag{10}$$

To determine the functional form of $\hat{D}(s, \bar{K}, \mu_\perp, \nu_\perp, \chi_\perp, \nu_\parallel, \chi_\parallel)$ let us express

$$\hat{D} = s^p \bar{K}^q \mu_\perp^r \nu_\perp^s \chi_\perp^t \nu_\parallel^u \chi_\parallel^v. \tag{11}$$

The requirement of the invariance of $\hat{D}$ under the operation of scale transformation T_3, imposes the following restrictions on the exponents

$$p = -1/2 + q + q'/2 + 3(u+v)/2 \quad \text{where } q' = r + s + t.$$

Now the general form of D is restricted to

$$D = C\frac{\rho_s^2 C_s}{L_n}\left(\frac{L_s}{L_n}\right)^{1/2} F_1\left(\frac{L_n^2}{L_T L_s}, \tau\frac{\nu_i}{\omega_{ci}}\frac{L_n}{\rho_s}\left(\frac{L_n}{L_s}\right)^{1/2}, \nu_\parallel\left(\frac{L_n}{L_s}\right)^{3/2}, \chi_\parallel\left(\frac{L_n}{L_s}\right)^{3/2}\right). \tag{12}$$

Here C is a constant coefficient arising due to functional dependence of $\mu_\perp$, $v_\perp$ and $\chi_\perp$ on $\tau\delta$.

Now if we further specify the transport mechanism by applying the approximation $\Phi \ll \nabla_\perp^2 p$ and retaining the other approximations as described above, only (1) is modified to give

$$-\frac{\partial}{\partial t}\nabla_\perp^2 p + \frac{\partial \Phi}{\partial y} + \nabla_\| v_\| + \left[\frac{\partial p}{\partial x}, \frac{\partial \Phi}{\partial x}\right] + \left[\frac{\partial p}{\partial y}, \frac{\partial \Phi}{\partial y}\right] - [\Phi, \nabla_\perp^2 p] + \mu_\perp \nabla_\perp^4 p = 0. \tag{1i}$$

Again applying the invariance technique, we can find the transformations which will leave (1i), (2–3) invariant. Accordingly we get two such transformations

$$T_4\colon\ n \to l^3 n, p \to l^3 p, v_\| \to l^2 v_\|, \bar{K} \to l^2 \bar{K}, s \to l^{-3} s, x \to lx, y \to ly,$$
$$t \to l^2 t, v_\| \to l^4 v_\|, \chi_\| \to l^4 \chi_\| \quad \text{for} \quad l_1 = l_2 = l$$
$$T_5\colon\ n \to \beta^{3/2} n, \Phi \to \beta\Phi, p \to \beta^{3/2} p, v_\| \to \beta v_\|, \bar{K} \to \beta \bar{K}, s \to \beta^{-1/2} s,$$
$$x \to \beta^{1/2} x, y \to \beta^{1/2} y, \mu_\perp \to \beta\mu_\perp, v_\perp \to \beta v_\perp, \chi_\perp \to \beta\chi_\perp, v_\| \to \beta v_\|,$$
$$\chi_\| \to \beta\chi_\|.$$

As described earlier the expression for D is restricted to

$$D = C\frac{\rho_s^2 C_s}{L_s}\left(\frac{L_n}{L_T}\right)^{3/2} F_2\left(\tau\frac{v_i}{\omega_{ci}}\frac{L_s}{\rho_s}\left(\frac{L_T}{L_n}\right)^{3/2}, v_\|\frac{L_n}{L_s}\left(\frac{L_T}{L_n}\right)^{1/2}, \chi_\|\frac{L_n}{L_s}\left(\frac{L_T}{L_n}\right)^{1/2}\right). \tag{13}$$

Note that in deriving the functional expressions for $D, \bar{K}$ has been approximated as $\bar{K} \approx L_n/L_T$. Further, the expressions for other quantities can also be determined following the procedure outlined here. This, being a trivial exercise, has not been included in the paper. Furthermore, the dominance of the finite Larmour radius effect determines the explicit functional form of the diffusion coefficients and other quantities for the non-dissipative case and reduces the number of unknown free indices by one for the dissipative case.

It is remarkable to add that the possibility of the approximation $\bar{K}s \gg 1$ (i.e. $\partial\phi/\partial t \gg \partial\phi/\partial y$ as discussed by Connor [2]) within the fluid model $\nabla_\perp^2 \ll 1$ of weak potential fluctuation $\phi \ll p$ requires $\phi \sim \nabla_\perp^2 p$ and $s < \bar{K}$ within the new fluid model [1] of η_i-driven turbulence. The functional form of D in this case remains the same [2]. In the dissipative case it reads as

$$D = C\frac{\rho_s^2 C_s}{L_T}\left(\frac{L_T}{L_s}\right)^{1/4} F_3\left(\tau\frac{v_i}{\omega_{ci}}\frac{L_T}{\rho_s}\left(\frac{L_s}{L_T}\right)^{1/4}, v_\|\frac{L_n}{L_s}\left(\frac{L_T}{L_s}\right)^{1/4}, \chi_\|\frac{L_n}{L_s}\left(\frac{L_T}{L_s}\right)^{1/4}\right). \tag{14}$$

4. Results and discussion

Invariance technique of theoretical analysis is complementary to analytical formulation. It provides a more general framework for solving the basic equations governing any physical mechanism to discuss the associated transport coefficients and saturated fluctuation level. The more we specify the natural mode of transport mechanism, the lesser the number of free exponents in the power scaling laws of the associated transport coefficients and other quantities. The specification of the mechanism is correlated with the approximations employed in describing the linear dispersion characteristics of the mode responsible for the energy transport. However, only those approximations are important which bring about the qualitative changes in the natural mode of transport process. The non-determinism of the constant coefficients is a limitation of this mathematical model of analysis.

From the analysis, one can notice that in the non-dissipative case as discussed in §3.1, the new model equations predict the explicit functional form of the transport coefficients for longer wavelength ($\nabla^2 \ll 1$) of the η_i-driven fluctuation even without imposing the condition $(\partial\phi/\partial t) \ll (\partial\phi/\partial y)$ as considered by Connor [2]. Possibly the finite Larmour radius correction term in (1) (for $\phi \ll \nabla^2 p$ and weak electrostatic potential $\phi \ll p$) plays a predominant role to decide the scaling behaviour of the transport in non-dissipative and dissipative cases of the η_i-driven fluctuation. This puts a restriction on the upper limit of the fluctuation scale size of the nonlinear spectrum due to η_i-driven turbulence within fluid approximation ($\nabla^2 \ll 1$). However, the inclusion of the dissipative mechanisms in the basic governing equations allows further restriction on the power scaling of the transport coefficients by introducing additional free exponents. General forms of the diffusion coefficients for η_i-driven turbulence have been derived which may hopefully provide an input to the tokamak physicists to determine the scaling laws for energy confinement. Further extension of the analysis in the toroidal geometry may be carried out to formulate more realistic power scaling laws for the plasma experiments where dissipative mechanisms are supposed to affect the energy transport. Finally, the validity of the approximation $s\bar{K} \ll 1$ demands very weak magnetic shear within the new fluid model of the η_i mode.

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