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Causal dissipative cosmology

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Abstract. The full version of the causal thermodynamics of non-equilibrium phenomena is discussed in the context of the flat Friedmann-Robertson–Walker cosmological model. Power law solutions for the scale factor are shown to exist. It is also shown that the temporal behaviour of the temperature depends on the functional dependence of the coefficient of bulk viscosity on density.

Keywords. Cosmology; bulk viscosity.

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1. Introduction

The role of dissipative effects in the evolution of the universe, particularly during its early stages, is a subject of growing importance. Although cosmological models with a fluid with bulk viscosity have been well addressed (see Barrow [1] and references therein), these models are not satisfactory for several reasons [2]. They violate causality, there is a short wavelength secular instability inherent in them and perturbations do not have a well-posed initial value problem. Most of these models consider only the first order deviations from equilibrium. Earlier attempts to build up causally well-behaved viscous fluid models, such as by Muller in 1967 [4] or Israel in 1976 [3], included the second order terms (for an excellent review, see Maartens [5] and references therein). Effects of these nonlinear theories in the cosmological expansion were discussed by Zakari and Jou [6], Oliviera and Salim [7] and Chimento and Jakubi [8]. Although these second order theories are causal and stable, they may lead to some pathological behaviour during the evolution. As pointed out by Hiscock and Salmonson [9], the drawback of these theories may arise out of the fact that most of them drop certain divergence terms. Hiscock and Salmonson discussed a flat Robertson–Walker cosmology with a viscous fluid where the divergence terms were also taken into account. They integrated the equations governing the model numerically and obtained some interesting results. Subsequently, Zakari and Jou [10] and Maartens [5] discussed this full theory and investigated the possibility of having exponential inflation in this model. Recently Romano and Pavon [11, 12] studied numerically both the full and the "truncated" version (where the divergence terms are dropped) in some anisotropic cosmological models.

In the present work, the possibility of having power law inflation in the flat Robertson–Walker cosmological model in the full version of the theory is explored. The notations used are those as in [5]. It is seen that if one assumes the standard relations connecting the different thermodynamic variables and dissipative properties with the energy density ρ ([5], [10]), a power law solution for the scale factor is possible only when the coefficient of bulk viscosity is proportional to $\rho^{1/2}$. If one allows different behaviour for any of the thermodynamic variables, e.g. temperature, then power law solutions may be obtained for other coefficients of bulk viscosity.

2. Cosmological solutions with a causal viscous fluid

The energy momentum tensor for a fluid with bulk viscosity is given by

$$T_{\mu\nu} = (\rho + p_{\text{eff}})u_\mu u_\nu + p_{\text{eff}} g_{\mu\nu}, \tag{2.1}$$

where ρ is the energy density, u_μ is the velocity four vector and the effective pressure p_{eff} is given by

$$p_{\text{eff}} = p + \pi, \tag{2.2}$$

p being the thermodynamic pressure and π the bulk viscous stress. From the considerations of energy-momentum conservation

$$T^{\mu\nu}{}_{;\nu} = 0,$$

number conservation

$$N^{\mu}{}_{;\mu} = 0,$$

Boltzmann H-theorem

$$S^{\mu}{}_{;\mu} \geqslant 0$$

and the Gibb's equation

$$T\mathrm{d}s = \mathrm{d}(\rho/n) + p\mathrm{d}(1/n)$$

where

$$N^\mu = nu^\mu, \quad S^\mu = sN^\mu - \left(\frac{\tau\pi^2}{2\xi T}\right)u^\mu,$$

n is the number density, s the specific entropy, τ the relaxation time for the bulk viscous stress, ξ the coefficient of bulk viscosity and T the temperature, one can arrive at the evolution equation for the bulk viscosity

$$\pi + \tau\dot{\pi} = -3\xi H - \frac{\varepsilon}{2}\tau\pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right). \tag{2.3}$$

In the above equation, $H = \frac{1}{3}\theta = \frac{1}{3}u^{\mu}{}_{;\mu}$ is the Hubble parameter (for discussions in detail, see [5, 9, 10]).

For $\tau = 0$ one gets back the non-causal theory and the coefficient $\varepsilon = 0$ or 1 for the "truncated" and the "full" causal theory respectively.

In a spatially flat Robertson–Walker cosmological model ($k=0$), i.e. with the metric

$$ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2), \tag{2.4}$$

Einstein's equations $G_{\mu\nu} = T_{\mu\nu}$ (in units where $8\pi G = 1$ and $c = 1$) become

$$3H^2 = \rho, \tag{2.5}$$

$$2\dot{H} + 3H^2 = -p - \pi, \tag{2.6}$$

where $H = 3\dot{R}/R$ and $T_{\mu\nu}$ is given by (2.1) and (2.2). The conservation equation of energy momentum,

$$\dot{\rho} + 3(\rho + p + \pi)H = 0 \tag{2.7}$$

is not an independent equation as it follows from the Einstein equations as a consequence of the Bianchi identity.

This system of equations is not closed as it has two independent equations (2.5) and (2.6) and six unknowns namely ρ, p, R, ξ, τ and T. The popular practice is to assume the *ad hoc* equations

$$p = (\gamma - 1)\rho, \quad \xi = \alpha\rho^q \text{ and } \tau = \frac{\xi}{\rho}. \tag{2.8}$$

Zakari and Jou [10] and also Maartens discussed exponential inflation with the choice

$$T = \beta\rho^r. \tag{2.9}$$

With the help of (2.3), (2.5) and (2.6), it is easy to construct the following evolution equation for H,

$$\tau\ddot{H} + \frac{3}{2}\tau(2\gamma + \varepsilon)H\dot{H} + \dot{H} + \frac{1}{2}\varepsilon\tau\left(\frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right)\dot{H} + \frac{9}{4}\varepsilon\gamma\tau H^3$$
$$+ \frac{3}{4}\varepsilon\gamma\tau\left(\frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right)H^2 + \frac{3}{2}\gamma H^2 - \frac{3}{2}\xi H = 0, \tag{2.10}$$

which in view of (2.8) and (2.9), yields

$$\ddot{H} + \frac{3}{2}[\varepsilon + (2 - \varepsilon - \varepsilon r)\gamma]H\dot{H} + 3^{1-q}\alpha^{-1}H^{2-2q}\dot{H} - \varepsilon(1 + r)H^{-1}\dot{H}^2$$
$$+ \frac{9}{4}(\varepsilon\gamma - 2)H^3 + \frac{1}{2}3^{2-q}\alpha^{-1}\gamma H^{4-2q} = 0. \tag{2.11}$$

As discussed by Maartens [5], this equation is consistent with exponential inflation, where $H = H_0 = \text{constant}$.

But as we shall see this equation admits a power law solution for the scale factor R only if $q = \frac{1}{2}$. If we choose

$$R = At^a \tag{2.12}$$

where A and a are constants, then

$$H = \frac{a}{t}, \quad \dot{H} = -\frac{a}{t^2}, \quad \ddot{H} = \frac{2a}{t^3}. \tag{2.13}$$

Equation (2.11) now becomes

$$A_1 t^{-3} + A_2 t^{2q-4} = 0, \tag{2.14}$$

where

$$A_1 = [2a - \tfrac{3}{2}\{\varepsilon + (2 - \varepsilon - \varepsilon r)\gamma\}B^2 - \varepsilon(1+r)B + \tfrac{9}{4}(\varepsilon\gamma - 2)B^3]$$

and

$$A_2 = \tfrac{1}{2}3^{2-q}\alpha^{-1}\gamma B^{4-2q} - 3^{1-q}\alpha^{-1}B^{3-2q}.$$

Equation (2.14) is an algebraic equation in powers of t and the constant coefficients of the different powers of t should be separately zero so as to make it valid for all t. If $q \neq \frac{1}{2}$, both A_1 and A_2 should be zero and it is easy to check that $A_2 = 0$ yields $3\gamma B = 2$ while this along with $A_1 = 0$ yields $B = 0$ which are clearly inconsistent. But if $q = \frac{1}{2}$, i.e. $\xi \sim \rho^{1/2}$, (2.14) becomes

$$(A_1 + A_2)t^{-3} = 0.$$

In this case $A_1 + A_2 = 0$ and it will be possible to get a power law solution for R.

In this connection, it is worthwhile to mention that the choices made for functional dependence of ξ, τ and T are *ad hoc*. If any one of them is left arbitrary to start with, the power law solution leads to a different functional form of that variable. In what follows, we shall leave the temperature arbitrary to start with and see what form it may take for a power law expansion of the universe.

With the choice (2.12) for the solution of R, the energy density, given by (2.5) becomes

$$\rho = \frac{3a^2}{t^2}. \tag{2.15}$$

Using this and the relation $p = (\gamma - 1)\rho$, one can obtain the expression for π from (2.6) as

$$\pi = \frac{a(2 - 3\gamma a)}{t^2}, \tag{2.16}$$

where $3\gamma a \neq 2$. For $3\gamma a = 2$, π becomes zero and we get the perfect fluid solution. Using (2.16) and its derivatives in (2.3), one obtains the following equation

$$\frac{a_0 t}{\tau} - 2a_0 = -\frac{3at^2}{\tau}\xi - \frac{3\varepsilon a_0 a}{2} - \frac{\varepsilon a_0 \dot{x} t}{2}, \tag{2.17}$$

where

$$a_0 = a(2 - 3\gamma a) = \text{constant} \tag{2.18}$$

and

$$\dot{x} = \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}. \tag{2.19}$$

Using the expressions for ξ and τ from (2.8) and that for ρ from (2.15), it is easy to obtain from (2.17) the result

$$a_1 t^{2q-2} + a_2 \dot{x} = a_3/t, \tag{2.20}$$

where

$$a_1 = a_0 \alpha^{-1}(3a)^{1-q},$$

$$a_2 = \varepsilon a_0/2,$$

$$a_3 = 2a_0 - 9a^3 - \frac{3\varepsilon a_0 a_1}{2}. \tag{2.21}$$

When $q = \frac{1}{2}$, (2.20) after integration will yield an expression for T as $T \sim \rho^r$ where r is a constant. This is similar to the choice of T as given in [5, 10]. But if one has $q \neq \frac{1}{2}$, (2.20) will yield a complicated expression for the temperature T,

$$T = \frac{1}{a_4} t^{(2a_2 - a_3)/a_2} \exp(a_5 t^{2q-1}), \tag{2.22}$$

where a_4 is a constant of integration and $a_5 = a_1/(a_2(2q-1))$.

The temporal behaviour of the temperature will depend upon the values of the constants a_2, a_3 and a_5. These values should be such that T actually decreases with time.

3. Conclusions

In this work some power law solutions for the scale factor R have been found in the Friedmann–Robertson–Walker cosmological model with a causal viscous fluid. It is observed that the temperature is a simple power function of ρ only when $q = \frac{1}{2}$, i.e. when the coefficient of bulk viscosity ξ is proportional to $\rho^{1/2}$. But for other choices of ξ as $\xi(\rho)$, the temperature is a complicated function of time t.

After the present work had been carried out, we were made aware of similar investigations by Maartens and Kgathi [13] and also by Coley *et al* [14].

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