

Conservation of channel spin in transfer reactions

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Abstract. The conservation of channel spin implying that the spin of the initial bound pair coupled to that of the initial free particle should result in the same channel spin as the spin of the final bound pair coupled to the spin of the final free particle, follows as a consequence of three-body theory of transfer reactions with the assumption of separability of two-body t -matrix. To test the validity of this principle we look at the experimental data on stripping reactions on even-even nuclei. We find that although reactions to channels not conforming to channel spin conservation are not altogether ruled out, the cross-sections of reactions violating channel spin conservation are much smaller than those conforming to channel spin conservation.

Keywords. Transfer reactions; three-body theory; channel spin.

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1. Introduction

The three-body formulation of transfer reactions of the type

$$A + a(b + x) = B(A + x) + b,$$

wherein we treat the particles a , b and x as cores (ignoring their structures) and use three-body equations (with separable two-body interaction) to describe the process, a consequence that follows is the conservation of channel spin. Accordingly the spin of the initial bound pair coupled to that of the initial free particle results in the same channel spin as the spin of the final bound pair coupled to the spin of the final free particle. The deduction of this result is presented and to see the validity of this principle experimental data on (d, p) and (d, n) reactions on various closed shell nuclei is observed. It is found that although reactions to channels not conforming to the conservation of channel spin are not altogether ruled out, their cross sections are far smaller than those conforming to this principle.

In § 2 we give the deduction of this principle on the basis of three-body theory and the assumption of separability of two-body t -matrix. In § 3 the experimental data is analyzed in this context and inferences are drawn.

2. Reaction amplitude in terms of the solutions of coupled integral equations

In three-body calculations for transfer reactions, the use of Alt–Grassberger–Sandhas (AGS) [1] version of three-body equations viz.

$$U_{ij}(z) = (1 - \delta_{ij})(z - H_0) + \sum_{k=1}^3 (1 - \delta_{ik}) T_k(z) G_0(z) U_{kj}(z), \quad (2.1)$$

where $G_0(z) = (z - H_0)^{-1}$ is the free resolvent operator and $T_k(x)$ is the two-body transition operator in three-body space, is preferred over other versions of three-body equations because the matrix element of the AGS operator $U_{ij}(z)$ between the initial and final asymptotic states, i.e.

$$\left\langle \mathbf{Q}_i d_i \sum_{L_i S_i} \phi_{(L_i S_i) J_i}^n | U_{ij}(z) | \mathbf{Q}_j d_j \sum_{L_j S_j} \phi_{(L_j S_j) J_j}^n \right\rangle,$$

hereafter referred to as the reaction amplitude, is very simply related to the cross-section of the process $j \rightarrow i$. Here \mathbf{Q}_j and d_j are, respectively, the on-shell momentum and the spin projection of the j th particle and $\sum_{L_j S_j} \phi_{(L_j S_j) J_j}^n$ is the bound state of the j th pair, assumed to be mixed $L_j S_j$ state. (In case the bound state is a pure $L_j S_j$ state, the summation over $L_j S_j$ may be taken to be non-existent). To evaluate this matrix element one has to solve the AGS equations (2.1) choosing a suitable basis of representation. Choosing (i) the angular momentum basis viz.

$$\langle p_i q_i ((L_i S_i) J_i s_i) k_i l_i : JM \rangle \equiv \langle p_i q_i (L_i S_i) \beta_i : JM \rangle \equiv \langle p_i q_i \alpha_i : JM \rangle,$$

where p_i is the magnitude of relative momentum of i th pair and q_i is the momentum of the i th particle in the centre of mass frame and $((L_i S_i) J_i s_i) k_i l_i : JM$ is the angular momentum coupling scheme and (ii) invoking a separable approximation for the two-body t -matrix in three-body space, i.e.

$$\begin{aligned} \langle r_k u_k (L_k S_k) \beta_k : JM | T_k(z) | r'_k u'_k (L'_k S'_k) \beta'_k : JM \rangle \\ = \frac{1}{u_k^2} \delta(u_k - u'_k) \delta_{\beta_i \beta'_i} g_{(L_i S_i) J_i}^n(r_k) \tau_k^n(z - u_k^2) g_{(L_i S_i) J_i}^n(r'_k), \end{aligned} \quad (2.2)$$

we can reduce the AGS equations to one-dimensional coupled integral equations (Mathur and Prasad [2,3], Mathur and Padhy [4]), i.e.

$$\begin{aligned} T_{ij}(q_i q'_j \beta_i \beta_j : J) = K_{ij}(q_i q'_j \beta_i \beta_j : J) + \sum_{k, \beta_k} \int u_k^2 du_k K_{ik}(q_i u_k \beta_i \beta_k : J) \\ \tau_k^n(z - u_k^2) T_{kj}(u_k q'_j \beta_k \beta_j : J). \end{aligned} \quad (2.3)$$

Here the Born term K_{ik} (or K_{ij}) is defined by

$$\begin{aligned} K_{ik}(q_i u_k \beta_i \beta_k : J) = (1 - \delta_{ik}) \sum_{L_i S_i} \sum_{L_k S_k} \int \int \frac{g_{(L_i S_i) J_i}^n(p_i) g_{(L_k S_k) J_k}^n(r_k)}{z - p_i^2 - q_i^2} p_i^2 dp_i r_k^2 dr_k \\ \langle p_i q_i (L_i S_i) \beta_i : JM | r_k u_k (L_k S_k) \beta_k : JM \rangle \end{aligned} \quad (2.4)$$

and the quantities $T_{ij}(q_i q'_j \beta_i \beta_j : J)$ occurring in (2.3), are related to the matrix element of AGS operator between the angular momentum basis states as follows:

$$\begin{aligned} T_{ij}(q_i q'_j \beta_i \beta_j : J) = \sum_{L_i S_i} \sum_{L_j S_j} \int \int \frac{g_{(L_i S_i) J_i}^n(p_i) p_i^2 dp_i g_{(L_j S_j) J_j}^n(p'_j) p_j'^2 dp_j'}{(z - p_i^2 - q_i^2)(z - p_j'^2 - q_j'^2)} \\ \langle p_i q_i (L_i S_i) \beta_i : J | U_{ij}(z) | p_j q'_j (L_j S_j) \beta_j : J \rangle. \end{aligned} \quad (2.5)$$

We can use (2.8) and (2.9) in (2.7) to get

$$\begin{aligned}
 & \left\langle \mathbf{Q}_i d_i \sum_{L_i S_i} \phi_{(L_i S_i) J_i M_i}^n | U_{ij}(z) | \mathbf{Q}_j d_j \sum_{L_j S_j} \phi_{(L_j S_j) J_j M_j}^n \right\rangle \\
 &= \sum_{K'_i l'_i} \sum_{K'_j l'_j} \sum_{JM} \left(\frac{1}{4} Q_i Q_j \right)^{1/2} \iint \sum_{L_i S_i} \sum_{L_j S_j} \phi_{(L_i S_i) J_i M_i}^n(p'_i) \phi_{(L_j S_j) J_j M_j}^n(p'_j) JM \\
 & \quad \langle p'_i Q_i ((L_i S_i) J_i S_i) K'_i l'_i : JM | U_{ij}(z) | p'_j Q_j ((L_j S_j) J_j S_j) K'_j l'_j : JM \rangle
 \end{aligned}$$

(2.10)

One can do summation over M by joining the two JM lines (shown by dotted lines) (Elbaz and Castle [5]). Since K'_i, l'_i, K'_j and l'_j are dummy suffixes, one can replace them by K_i, l_i, K_j and l_j , respectively. Using the relation between the bound state wave function in momentum space $\phi_{(L_i S_i) J_i}^n(p'_i)$ and the form factors $g_{(L_i S_i) J_i}^n(p'_i)$ (2.6) and using (2.5) we get

$$\begin{aligned}
 & \left\langle \mathbf{Q}_i d_i \sum_{L_i S_i} \phi_{(L_i S_i) J_i M_i}^n | U_{ij}(z) | \mathbf{Q}_j d_j \sum_{L_j S_j} \phi_{(L_j S_j) J_j M_j}^n \right\rangle \\
 &= \left(\frac{1}{4} Q_i Q_j \right)^{1/2} N_i N_j \sum_{K_i l_i} \sum_{K_j l_j} \sum_J T_{ij}(Q_i Q_j \beta_i \beta_j : J)
 \end{aligned}$$

(2.11)

Now cutting the diagram across dotted lines we get

$$\begin{aligned}
 & \left\langle \mathbf{Q}_i d_i \sum_{L_i S_i} \phi_{(L_i S_i) J_i M_i}^n | U_{ij}(z) | \mathbf{Q}_j d_j \sum_{L_j S_j} \phi_{(L_j S_j) J_j M_j}^n \right\rangle \\
 &= (Q_i Q_j)^{1/2} N_i N_j \sum_{K_i l_i} \sum_{K_j l_j} \sum_J T_{ij}(Q_i Q_j J_j K_i l_i J_j K_j l_j : J) \delta_{l_i l_j} \delta_{K_i K_j} [l_i]^{-2} [K_i]^{-2}
 \end{aligned}$$

(2.12)

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Here the first diagram stands for the sum

$$\sum_{M_i} (J_i M_i s_i d_i | K_j M_k) (J_j M_j s_j d_j | K_j M_k)$$

while the second and third, respectively, stand for $([l_i]^2 P_l(\cos \theta))/4\pi$ and $\{l_j K_j J_j\}$. Finally, the square of the modulus of the reaction amplitude, summed over final magnetic quantum numbers and averaged over initial magnetic quantum numbers, which in turn is proportional to the differential cross-section of the $j \rightarrow i$ process, is given by

$$\begin{aligned} & \left| \left\langle \mathbf{Q}_i d_i \sum_{L_i S_i} \phi_{(L_i S_i) J_i M_i}^n | U_{ij}(z) | \mathbf{Q}_j d_j \sum_{L_j S_j} \phi_{(L_j S_j) J_j M_j}^n \right\rangle \right|_{\text{avg}}^2 \\ &= \frac{1}{(2s_j + 1)(2J_j + 1)} \frac{1}{3} \left(\frac{1}{4} Q_i Q_j \right) (N_i N_j)^2 \sum_J \sum_{J'} (2J + 1)(2J' + 1) \sum_{l_i=J-1}^{J+1} \sum_{l_i'=J'-1}^{J'+1} \\ & \quad \times T_{ij}(Q_i Q_j J_i K_i l_i J_j K_i l_i'; J) T_{ij}^*(Q_i Q_j J_i K_i l_i' J_j K_i l_i; J') \frac{P_{l_i}(\cos \theta) P_{l_i'}(\cos \theta)}{16\pi^2}. \end{aligned} \quad (2.13)$$

From (2.12) we observe that the reaction amplitude contains a factor $\sum_{M_k} (J_i M_i s_i d_i | K_j M_k) (J_j M_j s_j d_j | K_j M_k)$ and $\delta_{K_i K_j}$. It implies that, for the amplitude to be nonzero, the spin J_j of the initial bound pair coupled to the spin s_j of the initial free particle must give rise to the same channel spin K_j as the spin J_i of the final bound pair coupled to the spin s_i of the final free particle. This inference of conservation of channel spin ($K_i = K_j$) rests, of course, on our assumption that the two-body t -matrix is essentially separable, its non-separable part being negligible and the bound state in the particular channel delineates the interaction in that channel. It is of interest, however, to test the truth of this conjecture on the basis of experimental data.

3. Discussion

Let us consider (d, p) and (d, n) reactions on even-even target nuclei (spin $s_j = 0$). Since the deuteron spin J_j equals one, the initial channel spin $K_j = 1$. According to this principle the final channel spin K_i should also be equal to 1. Since s_i (the spin of the outgoing proton or neutron) is $\frac{1}{2}$ the spin of the residual nucleus J_i should be $\frac{1}{2}$ or $\frac{3}{2}$. Thus, according to this hypothesis, stripping reactions leading to residual nuclei with spins other than $\frac{1}{2}$ and $\frac{3}{2}$ would not be permitted.

On examining the data on (d, p) and (d, n) reactions on various even-even nuclei, we find that although reactions leading to residual nuclei with spins other than $\frac{3}{2}$ and $\frac{1}{2}$ are not ruled out, the differential cross-sections of reactions leading to spin states $\frac{3}{2}$ and $\frac{1}{2}$ dominate over all others. From this, one can infer that the two-body interaction is essentially separable and has only a small non-separable component. The data are reproduced in table 1, which shows peak values of $\sigma(\theta)$ for (d, p) or (d, n) reactions on various even-even nuclei, leading to various excited states of the residual nuclei.

Table 1. Peak values of $\sigma(\theta)$ for various excited states of the residual nucleus.

Reaction and deuteron lab. energy	State of residual nucleus (E_x in MeV)	$\sigma_{\text{peak}}(\theta)$ (mb/sr)	References
${}^6\text{Li}(d, p){}^7\text{Li}$ $E_d = 12$ MeV	$E_x = 0.0(p_{3/2})$	9.3	Schiffer <i>et al</i> [6]
	$E_x = 0.98(p_{1/2})$	6.8	
${}^{12}\text{C}(d, p){}^{13}\text{C}$ $E_d = 12$ MeV	$E_x = 0.0(p_{1/2})$	21.3	Schiffer <i>et al</i> [6]
	$E_x = 3.68(p_{3/2})$	21.2	
${}^{16}\text{O}(d, p){}^{17}\text{O}$ $E_d = 5.03$ MeV	$E_x = 0.0(d_{5/2})$	31.1	Exfor [7] (Courtesy IAEA)
	$E_x = 0.87(s_{1/2})$	137.5	
${}^{16}\text{O}(d, p){}^{17}\text{O}$ $E_d = 12$ MeV	$E_x = 0.0(d_{5/2})$	15	Alty <i>et al</i> [8]
	$E_x = 0.87(s_{1/2})$	30	
	$E_x = 5.80(d_{3/2})$	28	
${}^{16}\text{O}(d, n){}^{17}\text{F}$ $E_d = 7.73$ MeV	$E_x = 0.0(d_{5/2})$	60.2	Oliver <i>et al</i> [9]
	$E_x = 0.5(s_{1/2})$	249.0	
${}^{16}\text{O}(d, n){}^{17}\text{F}$ $E_d = 12$ MeV	$E_x = 0.0(d_{5/2})$	50.0	Oliver <i>et al</i> [9]
	$E_x = 0.5(s_{1/2})$	150.0	
${}^{36}\text{Ar}(d, p){}^{37}\text{Ar}$ $E_d = 10.06$ MeV	$E_x = \text{g.s.}(f_{7/2})$	5.21	Fitz <i>et al</i> [10]
	$E_x = 1.27(p_{3/2})$	25.51	
	$E_x = 1.52(d_{3/2})$	0.53	
${}^{36}\text{Ar}(d, p){}^{37}\text{Ar}$ $E_d = 10.86$ MeV	$E_x = \text{g.s.}(f_{7/2})$	5.21	Sen <i>et al</i> [11]
	$E_x = 1.259(p_{3/2})$	25.51	
	$E_x = 1.509(d_{3/2})$	0.53	
${}^{38}\text{Ar}(d, p){}^{39}\text{Ar}$ $E_d = 11$ MeV	$E_x = \text{g.s.}(f_{7/2})$	0.8	Ipson <i>et al</i> [12]
	$E_x = 1.262(p_{3/2})$	1.5	
	$E_x = 2.650(p_{3/2})$	0.9	
${}^{40}\text{Ca}(d, p){}^{41}\text{Ca}$ $E_d = 5$ MeV	$E_x = 0.0(f_{7/2})$	2.30	Leighton <i>et al</i> [13]
	$E_x = 2.0(p_{3/2})$	10.3	
${}^{40}\text{Ca}(d, p){}^{41}\text{Ca}$ $E_d = 11.8$ MeV	$E_x = 0.0(f_{7/2})$	4.0	Schmidt-Rohr <i>et al</i> [14]
	$E_x = 1.97(p_{3/2})$	17.0	
	$E_x = 2.47(d_{3/2})$	10.0	
${}^{48}\text{Ca}(d, p){}^{49}\text{Ca}$ $E_d = 5.5$ MeV	$E_x = 0.0(p_{3/2})$	25.0	Roy and Bogaarde [15]
	$E_x = 2.03(p_{1/2})$	23.0	
${}^{48}\text{Ca}(d, p){}^{49}\text{Ca}$ $E_d = 7.2$ MeV	$E_x = 0.0(p_{3/2})$	both cross-section are comparable ≈ 20	Belole <i>et al</i> [16]
	$E_x = 2.03(p_{1/2})$		
${}^{48}\text{Ca}(d, p){}^{49}\text{Ca}$ $E_d = 13$ MeV	$E_x = 0.0(p_{3/2})$	50.0	Metz <i>et al</i> [17]
	$E_x = 2.03(p_{1/2})$	40.0	
	$E_x = 4.01(f_{5/2})$	11.0	
${}^{52}\text{Cr}(d, p){}^{53}\text{Cr}$ $E_d = 7.5$ MeV	$E_x = \text{g.s.}(p_{3/2})$	10.5	Rao <i>et al</i> [18]
	$E_x = 0.565(p_{1/2})$	3.71	
	$E_x = 1.008(f_{5/2})$	0.73	
${}^{58}\text{Fe}(d, p){}^{59}\text{Fe}$	$E_x = \text{g.s.}(p_{3/2})$	7.29	Klema [19]
	$E_x = 0.287(p_{1/2})$	1.99	
	$E_x = 0.470(f_{7/2})$	1.16	
	$E_x = 0.728(p_{3/2})$	3.79	
	$E_x = 1.026(f_{7/2})$	0.29	

(Continued)

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Table 1. (Continued)

Reaction and deuteron lab. energy	State of residual nucleus (E_x in MeV)	$\sigma_{\text{peak}}(\theta)$ (mb/sr)	References
$^{58}\text{Ni}(d, p)^{59}\text{Ni}$ $E_d = 12$ MeV	$E_x = g.s(p_{3/2})$	11.37	Chowdhury and Sen Gupta [20]
	$E_x = 0.465(d_{5/2})$	1.176	
	$E_x = 0.881(s_{1/2})$	4.437	
$^{84}\text{Kr}(d, p)^{85}\text{Kr}$ $E_d = 12$ MeV	$E_x = g.s(g_{9/2})$	0.2	Detorie <i>et al</i> [21]
	$E_x = 0.305(s_{1/2})$	0.4	
	$E_x = 1.430(s_{1/2})$	5.0	
$^{120}\text{Sn}(d, p)^{120}\text{Sn}$ $E_d = 15$ MeV	$E_x = g.s(d_{3/2})$	3.17	Schneid [22]
	$E_x = 0.05(s_{1/2})$	1.93	
	$E_x = 0.93(g_{7/2})$	0.276	
	$E_x = 1.91(p_{3/2})$	0.125	
	$E_x = 2.06(f_{7/2})$	0.047	
$^{120}\text{Sn}(d, p)^{121}\text{Sn}$ $E_d = 17$ MeV	$E_x = g.s(d_{3/2})$	3.696	Bechara and Dletzseh [23]
	$E_x = 0.058(s_{1/2})$	1.303	
	$E_x = 1.700(d_{5/2})$	0.110	
	$E_x = 1.857(p_{3/2})$	0.157	
$^{128}\text{Te}(d, p)^{129}\text{Te}$ $E_d = 7.5$ MeV	$E_x = g.s(d_{3/2})$	0.35	Haidenbauer [24]
	$E_x = 0.106(h_{11/2})$	0.065	
	$E_x = 0.179(s_{1/2})$	0.30	
$^{130}\text{Ba}(d, p)^{131}\text{Ba}$ $E_d = 12$ MeV	$E_x = g.s(s_{1/2})$	2.40	Ehrenstein <i>et al</i> [25]
	$E_x = 0.105(d_{3/2})$	0.56	
	$E_x = 0.364(s_{1/2})$	0.08	
	$E_x = 1.100(p_{3/2})$	2.1	
	$E_x = 1.162(f_{7/2})$	0.80	

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