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Signature inversion in the $K = 4^-$ band in doubly-odd ^{152}Eu and ^{156}Tb nuclei: Role of the $h_{9/2}$ proton orbital

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Abstract. The phenomenon of signature inversion in the doubly-odd nuclei ^{152}Eu and ^{156}Tb is understood within the framework of a two-quasiparticle plus rotor model. It is shown that the $h_{9/2}$:1/2[541] proton orbital plays a crucial role in reproducing this phenomenon.

Keywords. Doubly odd deformed nuclei; ^{152}Eu and ^{156}Tb; Coriolis coupling calculations; signature inversion.

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1. Introduction

During the last several years many investigations have been carried out to study the unusual features exhibited by the rotational bands of the odd-odd deformed nuclei [1, 2]. One of the most striking and anomalous features has been the signature inversion phenomenon in the high-K rotational bands of doubly-odd lighter rare-earth nuclei [3–7]. These bands are usually assigned a high-j ($i_{13/2}$ neutron $\otimes$ $h_{11/2}$ proton) configuration. Unlike most of the $K_+ = (\Omega_p + \Omega_n)$ bands in the odd-odd nuclei which display a smooth behaviour, these K_+ bands exhibit a large odd-even effect in their rotational energy spacings implying a dependence on the signature quantum number.

It is pertinent to give here a brief description of the signature quantum number and its origin. The signature quantum number is related to the invariance of the nuclear wave-function under rotation by π about an axis perpendicular to the symmetry axis. At large rotational frequencies, signature and parity are the only two quantum numbers which survive.

A rotation by π can be generated either by acting on intrinsic variables and performing a rotation by the corresponding operator R_i or by acting on the collective variables and performing the rotation by the corresponding operator R_e. Invariance of the system under this rotation implies that [8]

$$R_e = R_i \tag{1}$$

or

$$R_i^{-1} R_e = 1.$$

For $K = 0$, the intrinsic states transform as

$$R_i \phi_{r,K=0} = r \phi_{r,K=0}, \tag{2}$$

where r is the eigenvalue of the operator R_i. It follows that

$$r^2 = 1 \text{ and } r = \pm 1. \tag{3}$$

Also,

$$R_e D^I_{MK=0} = R_e Y^I_M = (-1)^I Y^I_M. \tag{4}$$

From (3) and (4), we get

$$r = (-1)^I. \tag{5}$$

The rotational spectrum for $K = 0$ band therefore gets divided into two parts:

$$I = 0, 2, 4, 6, \ldots, \quad r = +1,$$

$$I = 1, 3, 5, 7, \ldots, \quad r = -1. \tag{6}$$

Whereas only $r = +1$ is possible in the even-even nuclei, both the $r = +1$ and $r = -1$ sets are possible in the odd-odd nuclei.

For $K \neq 0$, the intrinsic states are two-fold degenerate and the corresponding operator is $R_i = \exp(-i\pi J_z)$ which has a value $\exp(-i\pi\alpha)$, where α is the signature quantum number. The square of this operator leaves the wavefunction unchanged for a system having even number of fermions. However an odd numbered system transforms like spinors and consequently changes sign. Thus

$$R_i^2 = (-1)^A \tag{7}$$

where A is the total number of particles in the system. It is therefore clear that the rotational bands of an odd-odd system having $K \neq 0$ can also be classified according to the classification given in (6) for $K = 0$ bands. The $r = +1$ members of the rotational bands correspond to the signature quantum number $\alpha = 0$ whereas the members having $r = -1$ correspond to the signature quantum number $\alpha = 1$.

In general, the wavefunction, which incorporates the π-invariance and also the axial symmetry may be written as

$$\psi^I_{MK} = \left[\frac{2I+1}{16\pi^2}\right]^{1/2} \{D^I_{MK}\phi_K + (-1)^{I+K} D^I_{M-K}\phi^-_K\}, \tag{8}$$

where $\phi_K \equiv |K\alpha_p\rangle = |\rho_p\Omega_p\rangle|\rho_n\Omega_n\rangle$ for an odd-odd nucleus and $\phi^-_K \equiv R_i\phi_K$. Since the rotational Hamiltonian having a Coriolis term breaks the time reversal symmetry, different contributions are obtained for the $\alpha = 0$ and $\alpha = 1$ members of the rotational bands giving rise to an odd-even shift in energy for $K = 0$ bands. This odd-even effect is the prime source of signature dependent features in the odd-odd nuclei. An additional source of odd-even shift is the Newby term arising due to diagonal n–p interaction for $K = 0$ bands. However the contribution of this term is very small as compared to the contribution from the decoupling term in the high-j bands. The Newby term therefore does not appear to play a significant role in the signature inversion phenomenon.

The signature dependent term in the Hamiltonian dictates that the energetically favoured signature in these bands is given by $\alpha_f = 1/2(-1)^{j-1/2}_p + 1/2(-1)^{j-1/2}_n$. However, a signature inversion at lower spins is observed in the K_+ bands having high-j configuration because the unfavoured spins lie lower in energy up to a critical spin I_c. The signature splitting then reverts to the normal signature beyond the critical spin.

We have recently shown [6] that the Coriolis coupling term is sufficient to explain the signature inversion in ^{160}Ho. This is supported by other calculations [5] also. We could show that a Coriolis mixing of the $[(i_{13/2})_n, (h_{11/2})_p]$ orbitals is sufficient to explain the weak signature inversion seen in the $K^{\pi}_{+} = 6^- \{7/2^-[523]_p \otimes 5/2^+[642]_n\}$ band of ^{160}Ho. It essentially represents a transmission of large odd-even shift present in the $K=0$ and $K=1$ bands having the configuration $\{1/2^-[550]_p \otimes 1/2^+[660]_n\}$ through a very high order Coriolis coupling to the $K=6$ band. However the same calculations did not succeed in the other two nuclei namely ^{152}Eu and ^{156}Tb where the signature inversion is more pronounced. We notice that another high-j orbital belonging to $h_{9/2}$ namely 1/2[541] proton orbital lying quite low in energy must also be taken into account. The systematics of the single particle states also [9] suggest that this orbital along with the 1/2[660] neutron orbital is bound to play an important role in the signature inversion phenomenon. In § 2, we give a brief description of the model. In § 3, we present the results of our calculations for ^{152}Eu and ^{156}Tb within the framework of a two-quasiparticle plus axially symmetric rotor model [TQPRM] and demonstrate that the signature inversion is a result of Coriolis mixing between the large number of bands arising from a coupling of the $i_{13/2}$ neutron with the $h_{11/2}$ and the $h_{9/2}$ proton orbitals. The mechanism of the signature inversion is also brought out. In § 4 we summarise the results.

2. The model and the methodology

The total Hamiltonian of the system in the framework of the TQPRM [1] is divided into two parts, the intrinsic and the rotational.

$$H = H_{\text{intr}} + H_{\text{rot}}. \tag{9}$$

The intrinsic part consists of a deformed axially symmetric average field H_{av}, a short range residual interaction H_{pair}, and a short range neutron-proton interaction V_{np}, so that

$$H_{\text{intr}} = H_{\text{av}} + H_{\text{pair}} + V_{np}. \tag{10}$$

The vibrational part has been neglected in this formulation. For an axially-symmetric reflection-symmetric rotor,

$$H_{\text{rot}} = \hbar^2/2\mathscr{I}(I^2 - I_3^2) + H_{\text{cor}} + H_{\text{ppc}} + H_{\text{irrot}}, \tag{11}$$

where

$$H_{\text{cor}} = -\hbar^2/2\mathscr{I}(I_+ j_- + I_- j_+)$$

$$H_{\text{ppc}} = \hbar^2/2\mathscr{I}(j_{p_+} j_{n_-} + j_{p_-} j_{n_+})$$

$$H_{\text{irrot}} = \hbar^2/2\mathscr{I}[(j_p^2 - j_{pz}^2) + (j_n^2 - j_{nz}^2)]. \tag{12}$$

The particle angular momentum j is given by the sum of angular momentum of the odd proton j_p and the odd neutron j_n. The operators $I_{\pm} = I_1 \pm iI_2$, $j_{\pm} = j_1 \pm ij_2$, $j_{n_\pm} = j_{n_1} \pm ij_{n_2}$, and $j_{p_\pm} = j_{p_1} \pm ij_{p_2}$ are the usual shifting operators. $\mathscr{I}$ is the moment of inertia with respect to the rotation axis.

The set of basis eigenfunctions of $H_{\text{av}} + \hbar^2/2\mathscr{I}(I^2 + I_3^2)$ may be written in the form of the symmetrized product of the rotational wavefunction D^I_{MK} and the intrinsic wavefunction $|K\alpha_\rho\rangle$ as

$$|IMK\alpha_\rho\rangle = \left[\frac{2I+1}{16\pi^2(1+\delta_{K0})}\right]^{1/2} [D^I_{MK}|K\alpha_\rho\rangle + (-1)^{I+K} D^I_{M-K} R_i |K\alpha_\rho\rangle], \tag{13}$$

where the index α_ρ characterizes the configuration ($\alpha_\rho \equiv \rho_p \rho_n$) of the odd neutron and the odd proton. A correct choice of the set of basis functions is very important as all the states which may couple together and influence each others behaviour should be included in the calculations.

Diagonalization of the total Hamiltonian matrix for each value of the angular momentum I gives us the energies $E_{\text{th}}(I, \alpha_\rho \sigma)$ for all the bands built on the two-quasiparticle ($2qp$)configuration, $|K\alpha_\rho\sigma\rangle$ present in the basis set of the eigenfunctions.

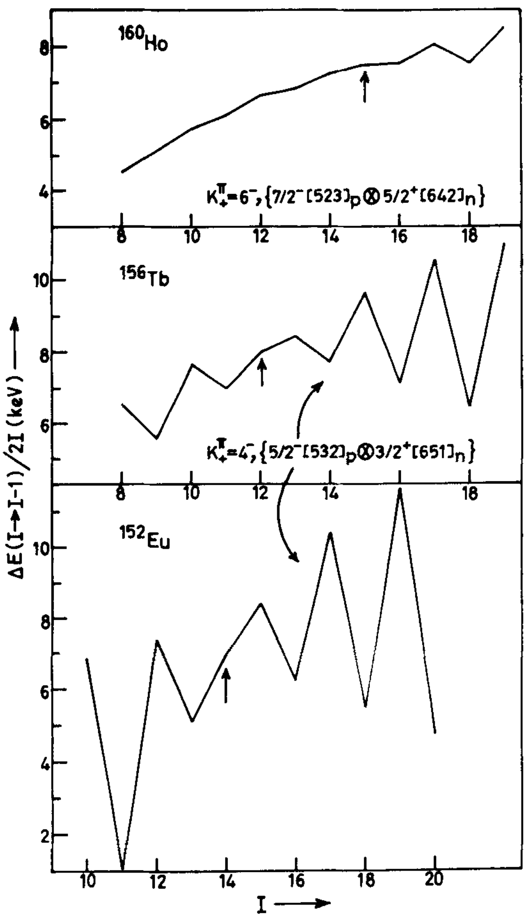

Figure 1. The experimental plots of $\Delta E(I \rightarrow I-1)/2I$ vs. I for the odd-even staggering and signature inversion exhibited by the $K^\pi_+ = 6^-$ band in ^{160}Ho [10] and the $K^\pi_+ = 4^-$ band in ^{152}Eu [11] and ^{156}Tb [3]. The point of inversion is shown by an arrow.

The Newby-shift enters as a parameter along with the other parameters such as the quasiparticle energies E_α, the moment of inertia $\mathscr{J}$ and the single particle matrix elements $\langle j_+ \rangle$. The single particle matrix elements are initially taken from the Nilsson model wavefunctions and some of the important ones are modified during the least square fitting procedure of the band level energies. A complete Coriolis coupling calculation thus requires a knowledge of a large number of $2qp$ states which are often unknown. We have therefore estimated the excitation energies of the important unidentified bands by using a semiempirical formulation [9]. In this formulation, the known properties of the quasiparticle configurations involved are taken from the neighbouring odd-A nuclei.

3. TQPRM calculation in ^{152}Eu and ^{156}Tb

3.1 *Signature inversion in* ^{152}Eu *and* ^{156}Tb: *Empirical data*

In figure 1 we plot the experimental data of $\Delta E(I \to I-1)/2I$ vs. angular momentum I to show the odd-even staggering in energy, and the signature inversion exhibited by the $K^\pi_+ = 6^-$ band in ^{160}Ho [10] and the $K^\pi_+ = 4^-$ band of ^{152}Eu [11] and ^{156}Tb [3]. The suggested Nilsson configuration for the $K^\pi_+ = 6^-$ band in ^{160}Ho is $\{7/2^-[523]_p \otimes 5/2^+[642]_n\}$ and for the $K^\pi_+ = 4^-$ band in both the nuclei ^{152}Eu and ^{156}Tb is $\{5/2^-[532]_p \otimes 3/2^+[651]_n\}$. The critical spin where the inversion occurs is shown by an arrow. The critical spin is defined from the higher spin side; it is the point where the normal behaviour changes into the anomalous behaviour. The pattern of odd-even staggering in all the three nuclei are similar but considerably enhanced signature effects are observed in the $K = 4$ band of ^{152}Eu and ^{156}Tb. A number of distinguishing features as compared to ^{160}Ho can be noted. We observe that the odd-even staggering in ^{160}Ho is much less in magnitude as compared to ^{152}Eu and ^{156}Tb. Also the magnitude of the staggering at lower spins is very large in ^{152}Eu and ^{156}Tb which decreases as the point of inversion is approached; after the inversion the magnitude of the staggering again increases gradually. It appears very natural to explain the large magnitude as a direct consequence of the shifting of Fermi level of both the proton and

Table 1. The experimental data [9] of the one-quasiparticle bands in the neighbouring odd-A nuclei used to estimate the band energies of the unidentified bands (given in the first row) included in the TQPRM calculations of ^{152}Eu. The fitted values are given in brackets.

P / N	~ 50·0 keV (systematics) 5/2[532]		~ 600 keV (systematics) 3/2[541]		~ 800 keV (systematics) 7/2[523]		~ 900 keV (systematics) 1/2[541]		~ 1000 keV (systematics) 1/2[550]	
345 keV 3/2[651]	400 (175)	500 (250)	945 (945)	1045 (1045)	1145 (1145)	1245 (1245)	1245 (1150)	1345 (1250)	1345 (1245)	1445 (1345)
355·7 keV 1/2[660]	405 (405)	505 (505)	955 (955)	1055 (1055)	—		1255 (1000)	1355 (1100)	1355 (1150)	1455 (1250)
~ 400 keV systematics 5/2[642]	450 (450)	550 (550)	1000 (950)	1100 (1050)	—		1300 (1300)	1400 (1400)	1400 (1400)	1500 (1500)

Table 2. Theoretically calculated bandhead energies for all the interacting bands in ^{152}Eu. The experimental data of yrast band is only known [11]. Also given are the parameter values of E_α, $\hbar^2/2\mathscr{I}$, E_N and those values of $\langle j_+ \rangle$ which were adjusted along with the Nilsson model values in the parentheses. The deformation was taken as $\varepsilon_2 = 0{\cdot}18$ and $\varepsilon_4 = -0{\cdot}03$.

Configuration Proton ⊗ Neutron	K^π, I	E_{exp} (keV)	E_{cal} (keV)	E_α (keV)	$\hbar^2/2\mathscr{I}$ (keV)	E_N (keV)
5/2[532] ⊗ 1/2[660]	$3^-,3$		349·3	405·0	9·5	
5/2[532] ⊗ 1/2[660]	$2^-,2$		455·0	505·0	9·0	
1/2[550] ⊗ 1/2[660]	$1^-,1$		1284·3	1150·0	7·2	
1/2[550] ⊗ 1/2[660]	$0^-,0$		1355·8	1250·0	7·2	4·0
3/2[541] ⊗ 1/2[660]	$2^-,2$		852·5	955·0	9·5	
3/2[541] ⊗ 1/2[660]	$1^-,1$		843·2	1055·0	9·0	
5/2[532] ⊗ 3/2[651]	$4^-,5$	180·6	146·8	175·0	7·0	
5/2[532] ⊗ 3/2[651]	$1^-,1$		253·1	250·0	7·0	
1/2[550] ⊗ 3/2[651]	$2^-,2$		1378·1	1245·0	9·5	
1/2[550] ⊗ 3/2[651]	$1^-,1$		1363·2	1345·0	9·0	
3/2[541] ⊗ 3/2[651]	$3^-,3$		874·7	945·0	9·5	
3/2[541] ⊗ 3/2[651]	$0^-,0$		1190·1	1045·0	9·0	0·0
1/2[550] ⊗ 5/2[642]	$3^-,3$		1757·5	1400·0	9·5	
1/2[550] ⊗ 5/2[642]	$2^-,2$		1556·2	1500·0	9·0	
3/2[541] ⊗ 5/2[642]	$4^-,4$		956·8	950·0	11·9	
3/2[541] ⊗ 5/2[642]	$1^-,1$		1055·1	1050·0	11·6	
5/2[532] ⊗ 5/2[642]	$5^-,5$		418·6	450·0	12·0	
5/2[532] ⊗ 5/2[642]	$0^-,0$		488·8	550·0	11·6	0·0
7/2[523] ⊗ 3/2[651]	$5^-,5$		1263·3	1145·0	9·5	
7/2[523] ⊗ 3/2[651]	$2^-,2$		1268·9	1245·0	9·0	
1/2[541] ⊗ 1/2[660]	$0^-,0$		746·2	1000·0	12·9	−40·0
1/2[541] ⊗ 1/2[660]	$1^-,1$		1135·2	1100·0	12·5	
1/2[541] ⊗ 3/2[651]	$1^-,1$		1159·9	1150·0	11·9	
1/2[541] ⊗ 3/2[651]	$2^-,2$		1564·6	1250·0	11·6	
1/2[541] ⊗ 5/2[642]	$2^-,2$		1275·3	1300·0	11·9	
1/2[541] ⊗ 5/2[642]	$3^-,3$		1025·4	1400·0	11·6	

$\langle 1/2[550] | 1/2[550] \rangle_p = 5{\cdot}32(5{\cdot}79)$ $\quad\langle 5/2[532] | 3/2[541] \rangle_p = 4{\cdot}55(5{\cdot}55)$
$\langle 1/2[541] | 1/2[541] \rangle_p = -4{\cdot}32(-3{\cdot}64)$ $\quad\langle 7/2[523] | 5/2[532] \rangle_p = 4{\cdot}14(5{\cdot}14)$
$\langle 3/2[541] | 1/2[541] \rangle_p = 4{\cdot}75(0{\cdot}18)$ $\quad\langle 1/2[660] | 1/2[660] \rangle_n = -3{\cdot}74(-6{\cdot}69)$
$\langle 3/2[651] | 1/2[660] \rangle_n = 3{\cdot}69(6{\cdot}66)$
$\langle 5/2[642] | 3/2[651] \rangle_n = 3{\cdot}52(6{\cdot}52)$

the neutron from $7/2[523]_p$ to $5/2[532]_p$ and $5/2[642]_n$ to $3/2[651]_n$. The dependence of the magnitude of staggering on the Nilsson orbitals occupied by the odd-neutron and the odd-proton is also observed in other cases. However, by using a physically meaningful set of parameters, we have been unable to reproduce the signature inversion in these two nuclei within the basis space of the $[(h_{11/2})_p(i_{13/2})_n]$ orbitals only; it is indeed very surprising that our calculations fail in ^{156}Tb and ^{152}Eu. It therefore seems very natural to explore the role of the 1/2[541] proton state belonging to the $h_{9/2}$ orbital. This orbital lies low in energy and when combined with an $i_{13/2}$ neutron orbital will give rise to a $K = 0$ band which has a phase opposite to the normal odd-even staggering.

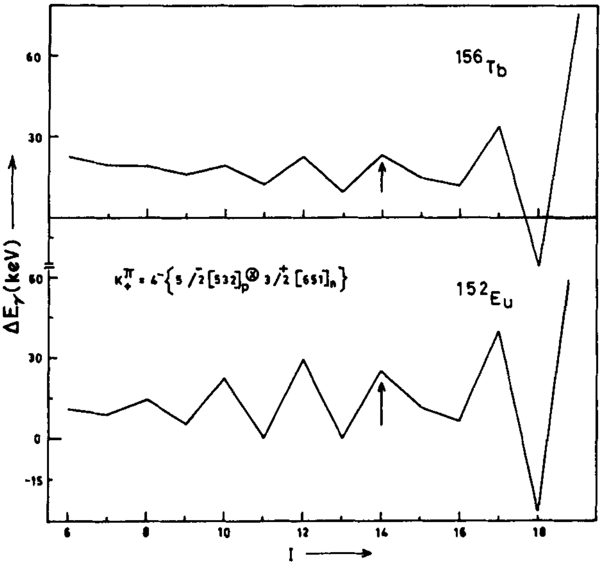

Figure 2(a). The $\Delta E_\gamma \equiv \{\Delta E(I \to I-1) - \Delta E(I-1 \to I-2)\}$ vs. I for the $K^\pi_+ = 4^-$ band in ^{152}Eu and ^{156}Tb from our calculations; the odd-even staggering as well as signature inversion is well reproduced.

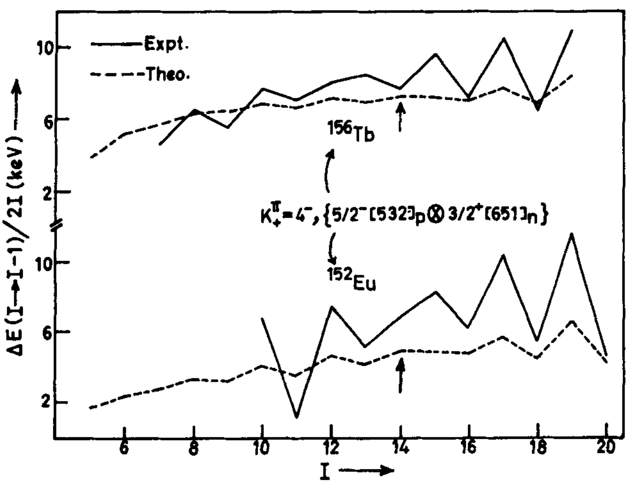

Figure 2(b). Comparison of the experimental data on odd-even staggering for the $K^\pi_+ = 4^-$ band in ^{152}Eu and ^{156}Tb with the TQPRM calculations.

3.2 *Signature inversion in* ^{152}Eu: *Calculations*

A total of 26 $2qp$ rotational bands were included in the [TQPRM] calculations of ^{152}Eu. The positions of all the 25 unknown bands were estimated by using the experimental data of the single particle states in the neighbouring odd-A nuclei [9]. In table 1 we have summarized the estimated values of the bandhead energies as E_α which

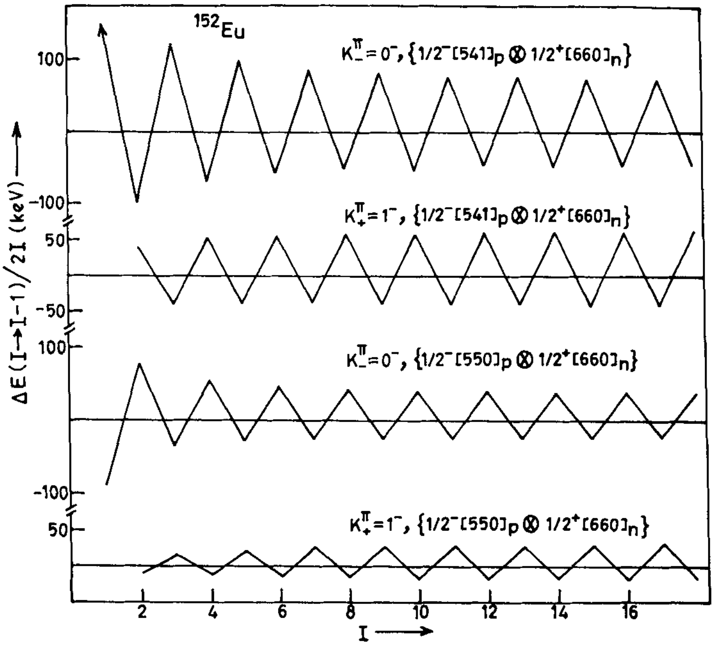

Figure 3. The behaviour of the unperturbed $K^{\pi}_{-} = 0^{-}$ and $K^{\pi}_{+} = 1^{-}$ bands used in the calculation of ^{152}Eu.

represent a reasonably good first order estimate within 100–200 keV. The splitting between two GM partners was uniformly assumed to be 100 keV. Besides the $[(h_{11/2})_p(i_{13/2})_n]$ configurations, 6 bands belonging to the $[1/2[541]_p \otimes i_{13/2}$ neutron] configuration were also included in these calculations. Positions of 12 bands belonging to 6 configurations were adjusted during the fitting procedure; the variation did not exceed 250 keV in any case. These 12 bands were seen to play the most important role in reproducing the signature inversion and form the most important chain for transmitting signature effect. The moment of inertia parameters for the K_+ and the K_- bands were chosen to be 9·5 and 9·0 keV respectively; 14 of these were adjusted during the fitting procedure. We also find that the results are not very sensitive to the values of the Newby-shift E_N; a variation of this parameter up to 100 keV did not produce any significant change in the results. The values of E_N given in table 2 are therefore not well determined from these calculations. The matrix elements were again taken from the Nilsson model. The $i_{13/2}$ neutron matrix elements were attenuated as usual. Of the remaining matrix elements the most significant adjustment was made in the $\langle 3/2[541]|j_+|1/2[541]\rangle_p$ matrix element; it was increased from its Nilsson value 0·18 to 4·75. It indicates the need for a strong coupling of the $1/2[541]_p$ orbital. In table 2 we summarise the final parameters arrived at after the fitting of the $K = 4$ band. The results of our calculation for ^{152}Eu are shown in figures 2(a) and 2(b) where a clear signature inversion can be seen at spin $I = 14$.

The mechanism of the signature inversion in ^{152}Eu appears to be more complicated than in ^{160}Ho [6]. The signature effects for the $K = 4$ band are seen to follow from the two $K^{\pi}_{-} = 0^{-}\{1/2^{-}[550]_p \otimes 1/2^{+}[660]_n\}$ and the $K^{\pi}_{-} = 0^{-}\{1/2^{-}[541]_p \otimes 1/2^{+}[660]_n\}$ bands. Here the Newby-shifts for the $K = 0$ bands do not play an important role as the

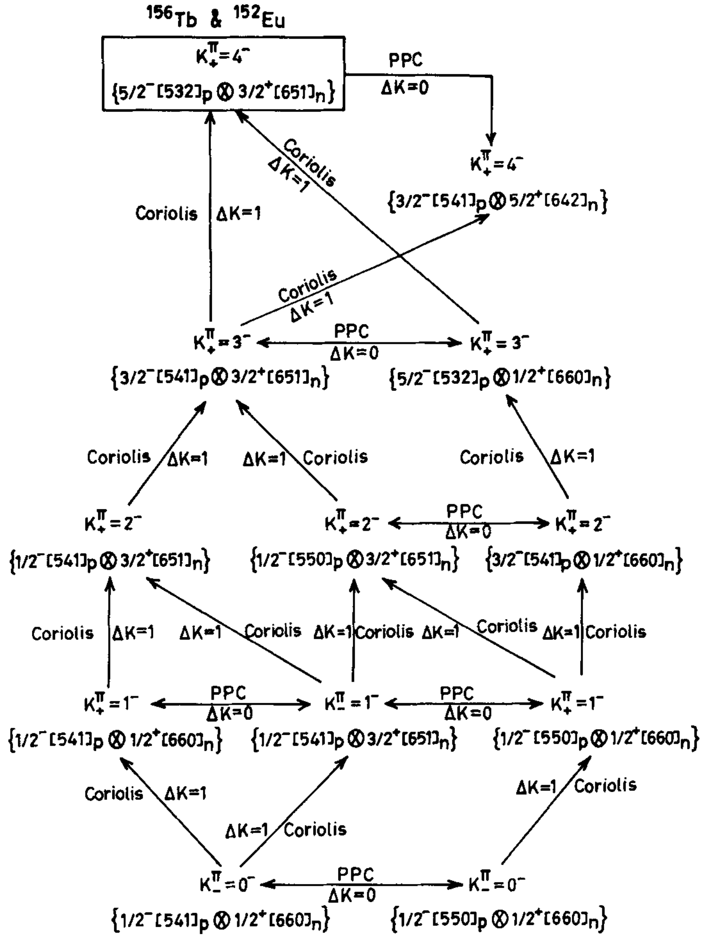

Figure 4. The chain diagram of the K_+ and the K_- bands showing the transmission of the odd-even effect which is responsible for the signature inversion in the $K^\pi_+ = 4^-$ band of ^{152}Eu and ^{156}Tb.

variation of this parameter up to 100 keV has almost no effect on the results. When all the mixing terms except the decoupling parameters a_p and a_n for the proton and the neutron orbitals respectively have been taken to be zero we obtain the results as shown in figure 3. It may be noted that the two $K = 0$ bands exhibit a phase opposite to each other. The corresponding $G - M$ partners having $K = 1$ also exhibit opposite signature dependence. The opposing phases are easily understood from the sign of the decoupling parameters involved. However the K_+ and the K_- bands can couple directly in special situation where Ω_p or $\Omega_n = 1/2$; if $\Omega_p = \Omega_n = 1/2$ then an extra diagonal term of the form of $(-1)^{I+1}\hbar^2/2\mathscr{I}\; a_p \cdot a_n$ contributes to the odd-even shift of the $K = 0$ band. For the $K^\pi_- = 0^-$ $\{1/2^-[550]_p \otimes 1/2^+[660]_n\}$ band both the decoupling parameters are opposite in sign; this favors odd spins in the $K = 0$ band; the GM partner $K = 1$ band couples with $K = 0$ band by a coupling term [1, 12]

$$(-\hbar^2/2\mathscr{I})[I(I+1)]^{1/2}\{a_p + (-1)^{I+1}a_n\}.$$

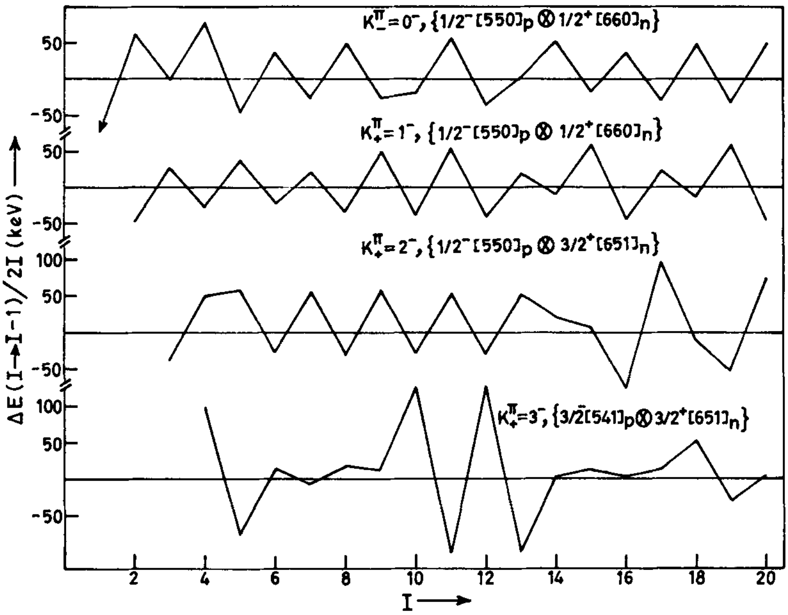

Figure 5(a). The staggering plots of the perturbed K_- and K_+ bands in ^{152}Eu belonging to the most important chain which couples the $K^\pi_+ = 4^-$ band to the $K^\pi_- = 0^-$ band are shown in this figure.

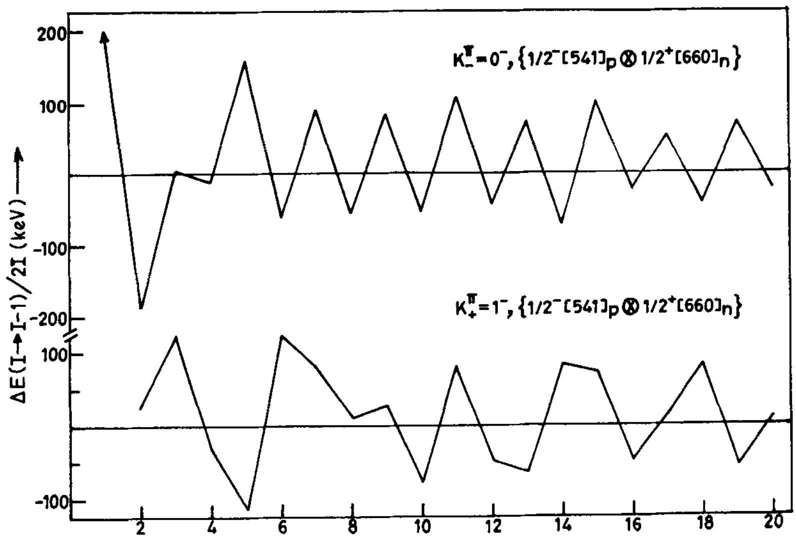

Figure 5(b). Same as in figure 5(a).

Since a_p and d_n are opposite in sign, even spins are favoured in the $K = 1$ band. On the other hand, the $K^\pi_- = 0$, $\{1/2^- [541]_p \otimes 1/2^+ [660]_n\}$ band has decoupling parameters of the same sign; this favours even spins in the $K = 0$ band and odd spins in the $K = 1$

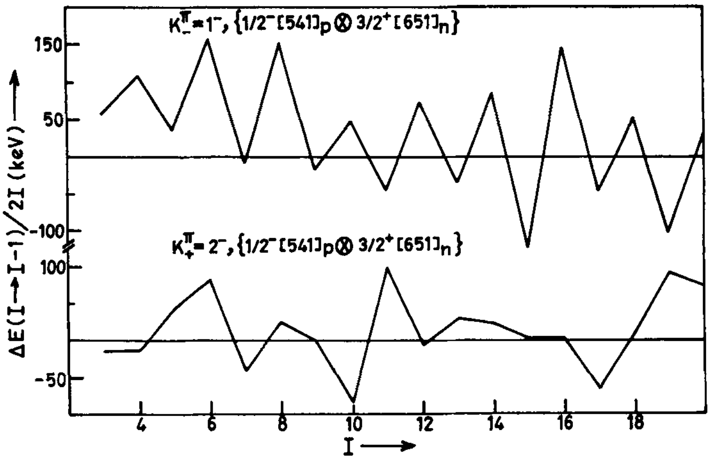

Figure 5(c). Same as in figure 5(a).

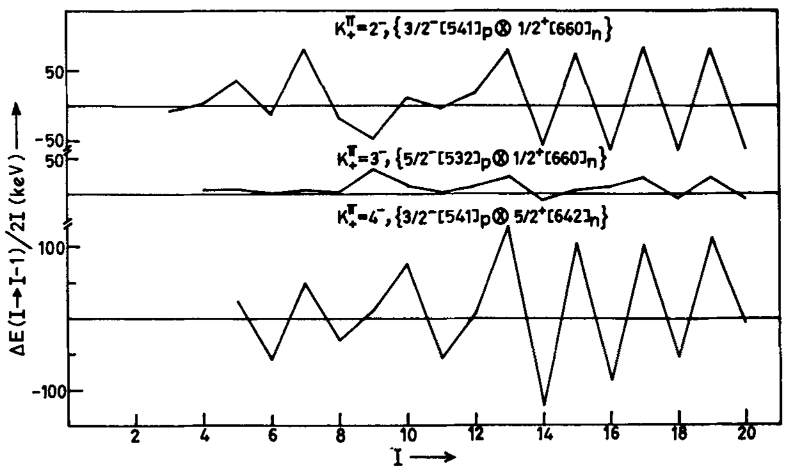

Figure 5(d). Same as in figure 5(a).

Table 3. The experimental data [9] on the one-quasiparticle bands in the neighbouring odd-A nuclei used to estimate the band energies of the unidentified bands (given in the first row) included in the TQPRM calculations of ^{156}Tb. The fitted values are given in the parentheses.

P N	227 keV 5/2[532]		545·3 keV 7/2[523]		863 keV 1/2[541]		891·1 keV 3/2[541]		∼ 1100 keV (systematics) 1/2[550]	
86·5 keV	330	430	650	750	949	1049	1000	1100	1200	1300
3/2[651]	(250)	(350)	(550)	(650)	(850)	(950)	(1000)	(1100)	(1200)	(1300)
266·6 keV	500	600	—		1129	1229	1100	1200	1350	1450
5/2[642]	(500)	(600)	—		(1090)	(1200)	(1100)	(1200)	(1450)	(1550)
720·6 keV	950	1050	—		1583	1683	1600	1700	1800	1900
1/2[660]	(900)	(1000)	—		(1400)	(1500)	(1550)	(1650)	(1700)	(1800)

Table 4. Theoretically calculated bandhead energies for all the interacting bands in ^{156}Tb. The experimental data of yrast band is only known [3]. Also given are the parameter values of E_α, $\hbar^2/2\mathscr{J}$, E_N and those values of $\langle j_+ \rangle$ which were adjusted along with the Nilsson model values in the parentheses. The deformation was taken as $\varepsilon_2 = 0{\cdot}22$ and $\varepsilon_4 = -0{\cdot}02$.

Configuration Proton	Neutron	K^π, I	E_{exp} (keV)	E_{cal} (keV)	E_α (keV)	$\hbar^2/2\mathscr{J}$ (keV)	E_N (keV)
5/2[532] ⊗	3/2[651]	4^-,6	379·0	258·8	250·0	11·85	
5/2[532] ⊗	3/2[651]	1^-,1		356·5	350·0	10·0	
3/2[541] ⊗	3/2[651]	3^-,3		802·2	1000·0	10·0	
3/2[541] ⊗	3/2[651]	0^-,0		1036·9	1100·0	10·0	0·0
7/2[523] ⊗	3/2[651]	5^-,5		696·5	550·0	10·0	
7/2[523] ⊗	3/2[651]	2^-,2		680·2	650·0	10·0	
1/2[550] ⊗	3/2[651]	2^-,2		1153·4	1200·0	10·0	
1/2[550] ⊗	3/2[651]	1^-,1		1257·4	1300·0	10·0	
5/2[532] ⊗	1/2[660]	3^-,3		1065·6	900·0	10·0	
5/2[532] ⊗	1/2[660]	2^-,2		956·5	1000·0	10·0	
1/2[550] ⊗	1/2[660]	1^-,1		1876·2	1700·0	9·5	
1/2[550] ⊗	1/2[660]	0^-,0		1913·3	1800·0	9·5	40·0
3/2[541] ⊗	1/2[660]	2^-,2		1601·5	1550·0	10·0	
3/2[541] ⊗	1/2[660]	1^-,1		1514·2	1650·0	10·0	
5/2[532] ⊗	5/2[642]	5^-,5		476·6	500·0	12·0	
5/2[532] ⊗	5/2[642]	0^-,0		581·0	600·0	12·0	0·0
3/2[541] ⊗	5/2[642]	4^-,4		1054·6	1100·0	12·0	
3/2[541] ⊗	5/2[642]	1^-,1		1217·7	1200·0	11·0	
1/2[550] ⊗	5/2[642]	3^-,3		1475·5	1450·0	10·0	
1/2[550] ⊗	5/2[642]	2^-,2		1583·5	1550·0	10·0	
1/2[541] ⊗	1/2[660]	0^-,0		1378·0	1400·0	12·0	−40·0
1/2[541] ⊗	1/2[660]	1^-,1		1397·8	1500·0	12·0	
1/2[541] ⊗	3/2[651]	1^-,1		855·3	850·0	12·0	
1/2[541] ⊗	3/2[651]	2^-,2		1015·3	950·0	12·0	
1/2[541] ⊗	5/2[642]	2^-,2		1189·3	1090·0	12·0	
1/2[541] ⊗	5/2[642]	3^-,3		1121·9	1200·0	12·0	

$\langle 1/2[550] | 1/2[550] \rangle_p = 3{\cdot}74(5{\cdot}74)$ $\langle 7/2[523] | 5/2[532] \rangle_p = 3{\cdot}12(5{\cdot}12)$
$\langle 1/2[541] | 1/2[541] \rangle_p = -2{\cdot}74(-3{\cdot}47)$ $\langle 1/2[660] | 1/2[660] \rangle_n = -3{\cdot}60(-6{\cdot}60)$
$\langle 3/2[541] | 1/2[550] \rangle_p = 3{\cdot}69(5{\cdot}69)$ $\langle 3/2[651] | 1/2[660] \rangle_n = 3{\cdot}58(5{\cdot}58)$
$\langle 3/2[541] | 1/2[541] \rangle_p = 3{\cdot}52(0{\cdot}22)$ $\langle 5/2[642] | 3/2[651] \rangle_n = 2{\cdot}47(6{\cdot}47)$
$\langle 5/2[532] | 3/2[541] \rangle_p = 3{\cdot}51(5{\cdot}51)$

band. These opposing signature effects of the $K = 0$ and $K = 1$ bands are transmitted to the $K = 4$ band through a coupling of bands in a chain which is shown in figure 4. When all the mixing terms are turned on, the various bands of the chain are perturbed as shown in figure 5(a–d). The final band i.e. $K^\pi_+ = 4^- \{5/2^- [532]_p \otimes 3/2^+ [651]_n\}$ is observed to be highly mixed in nature. We find that the odd-spin members are favored in the low spin region mainly because of the $1/2^-[541]$ orbital present in the calculations. The $1/2^-$ [550] orbital contributes more effectively in the high spin region and leads to the favoring of even-spins. From this we may conclude that a band-crossing like phenomenon is taking place in this nucleus. The $K^\pi_+ = 3^- \{3/2^- [541]_p \otimes 3/2^+ [651]_n\}$ band seems to play a crucial role in the transmission of the odd-even shift

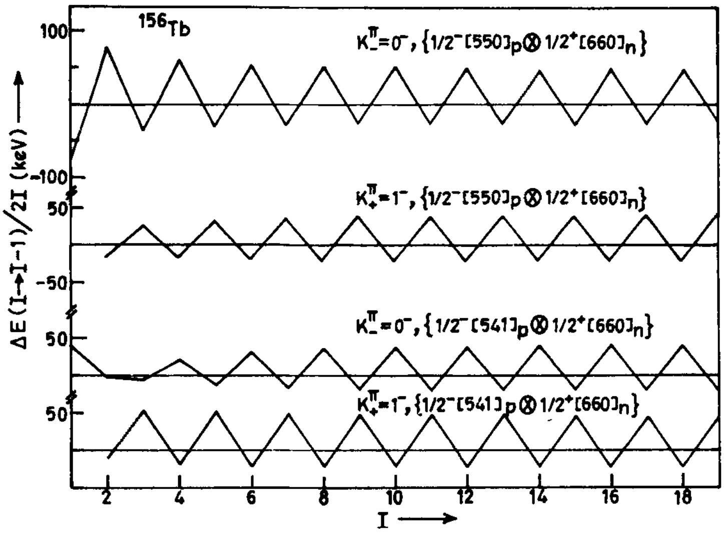

Figure 6. The behaviour of the unperturbed $K^{\pi}_{-} = 0^{-}$ and $K^{\pi}_{+} = 1^{-}$ bands used in the calculation of ^{156}Tb.

to higher-K bands. This band has location in ^{160}Ho such that it breaks the smooth transmission of the odd-even shift of the $\{1/2^{-}[541]_{p} \otimes 1/2^{+}[660]_{n}\} K^{\pi}_{\pm} = 1^{-}, 0^{-}$ bands to the main band. On the other hand, this band is favorably placed in ^{152}Eu and ^{156}Tb to allow the transmission of the odd-even effect coming from the $h_{9/2}$ orbital. Also, the matrix element of $\langle 3/2[541] | j_{+} | 1/2[541] \rangle_{p}$ needs to be considerably enhanced in ^{152}Eu and ^{156}Tb indicating the importance of the $h_{9/2}$ proton orbital in these two nuclei.

3.3 *Signature inversion in* ^{156}Tb: *Calculations*

Calculations for ^{156}Tb were carried out by using the same set of the 26 bands as used in ^{152}Eu. The positions of 25 unknown bands were estimated by using a similar method. In table 3, we summarise the estimated values of the band energies; these changed values reflect the shift in Fermi energy in going from ^{152}Eu to ^{156}Tb [9]. The positions of 16 bands were adjusted within 100 keV during the fitting procedure. The moment of inertia parameters $\hbar^{2}/2\mathscr{J}$ were uniformly chosen to be 10 keV for all the bands; it was adjusted in 13 cases during the fitting procedure. The matrix elements for the $i_{13/2}$ orbital were reduced as usual. Among the other matrix elements the largest variation was again made in $\langle 3/2[541] | j_{+} | 1/2[541] \rangle_{p}$ matrix element; it was changed from 0·22 to 3·52. The 1/2[541] decoupling parameter was also reduced from the Nilsson value of $-3{\cdot}47$ to $-2{\cdot}74$. The final set of parameters arrived at after the fitting are listed in table 4. We must emphasize here the fact that the quality of the fitting in ^{156}Tb as well in ^{152}Eu is not as good as in ^{160}Ho. With our limited resources and time we have not attempted an extensive fitting of the bands. It is our belief that fitting of the data can be considerably improved in both the cases. The final results of our calculations are shown in figures 2(a) and 2(b) where a signature inversion can be observed at $I = 14$.

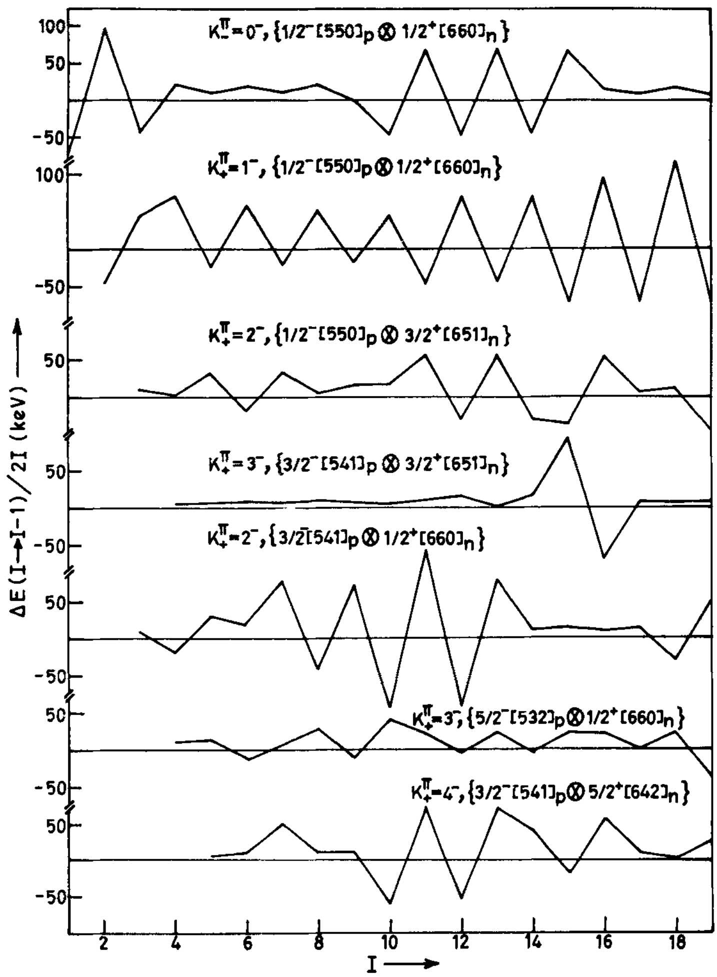

Figure 7(a). The staggering plots of the perturbed K_- and K_+ bands in ^{156}Tb belonging to the most important chain which couples the $K_+^\pi = 4^-$ band to the $K_-^\pi = 0^-$ band are shown in this figure.

The mechanism of signature inversion in ^{156}Tb is identical to that in ^{152}Eu. The signature effects in the main band again follow from the two $K = 0$ bands shown in figure 6. We plot both the $K = 0$ bands and their $G-M$ partners for the situation where all the mixing terms except a_p and a_n are zero. It is rather interesting to note that the two $K = 0$ bands are seen to exhibit the same signature dependence whereas in ^{152}Eu they show opposite phases. Normally we would have expected the $K_-^\pi = 0^-$, $\{1/2^-[541]_p \otimes 1/2^+[660]_n\}$ band to favor even-spin members. However in this case the mixing between the odd spin members of the $K = 0$ and the $K = 1$ band of $\{1/2^-[541]_p \otimes 1/2^+[660]_n\}$ configuration is such that the wavefunctions have almost 50–50% admixture of both the bands. In such a situation it is difficult to label one of the two odd spins as belonging to either $K = 0$ or the $K = 1$ band. Even a slight increase in the admixture in favor of either $K = 0$ or $K = 1$ band puts that member as belonging to

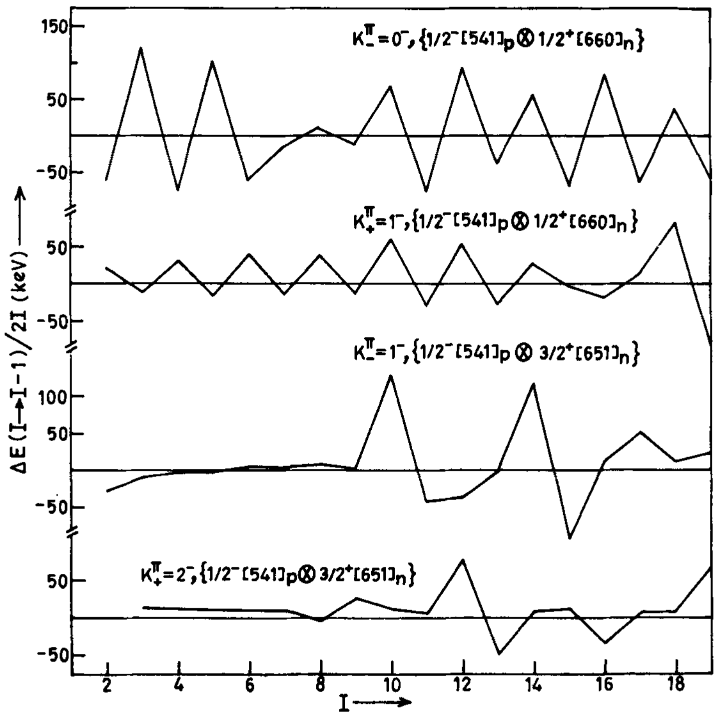

Figure 7(b). Same as in figure 7(a).

one of the two. Thus effectively the odd-spin members of the $K = 0$ and $K = 1$ band have been interchanged in the calculations; this is the reason for the observed behavior. However, this apparent change does not affect the final results.

The signature inversion in the $K = 4$ band of ^{156}Tb also occurs through a coupling of the same chain as for ^{152}Eu which is already shown in figure 4. The behavior of the various bands of the chain, when all the mixing terms are introduced, is shown in figures 7(a–b). The signature inversion in ^{156}Tb therefore has a similar origin as in ^{152}Eu.

5. Conclusions

In conclusions, we find that the phenomenon of signature inversion may be understood within the framework of TQPRM. We could reproduce the general trends of the signature inversion in ^{156}Tb and ^{152}Eu; however we could not obtain the correct magnitude of staggering which seems to require a further refinement of our parameters and fitting procedure. The reproduction of the inversion phenomenon in ^{152}Eu and ^{156}Tb required the inclusion of $h_{9/2}$:1/2[541] proton orbital indicating its importance. The location of the $K_+^\pi = 3^-\{3/2^-[541]_p \otimes 3/2^+[651]_n\}$ band appears to play a crucial role in the relative importance of the $h_{9/2}$:1/2[541] orbital in ^{152}Eu and ^{156}Tb nuclei vis-a-vis ^{160}Ho nucleus where the very weak signature inversion was reproduced without including the $h_{9/2}$ orbital [6]. It would be of considerable interest to be able to explain the whole systematics of signature inversion phenomenon in these nuclei on the basis of the inputs identified in the present study to be of importance. Work is in progress in this direction.

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