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Effect of heavy quark symmetry on the mass difference of B-system in minimal left right symmetric model

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Abstract. An estimation of the mass difference of $B_d^0 - \bar{B}_d^0$ system with heavy quark symmetry formalism is presented. The effective Hamiltonian describing the transition $\bar{h}d \leftrightarrow h\bar{d}$ (where $h = b$ for B_d^0-system) is considered in a manifest left right symmetric (MLRS) model along with contribution from neutral Higgs boson. We use the spin and flavor symmetry for heavy quarks to obtain the transition matrix element $\langle B_d^0 | \mathscr{H}_{\mathrm{eff}}(x) | \bar{B}_d^0 \rangle$ in terms of Isgur–Wise function. Assuming that B_d^0 and $\bar{B}_d^0$ states are at rest, we find that Isgur–Wise function turns out to be unity. However using the experimental values of ΔM_K and ΔM_{B_d} as input, we find that $M_R = 835\,\mathrm{GeV}$ and $M_H \geqslant 2{\cdot}9\,\mathrm{TeV}$.

Keywords. Left right symmetry; gauge bosons.

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1. Introduction

In recent years heavy flavor dynamics has proven to be very useful to obtain model independent information on systems containing heavy quarks [1, 2]. When one or more quarks are heavy compared to hadronic scale, some new symmetries appear in the low energy effective Lagrangian for QCD. In the limit $m_Q \to \infty$ (m_Q being the mass of heavy quark), two additional symmetries beyond those of QCD arise [1]. The first one is the heavy flavor symmetry where mass of the heavy quark is scaled out and the Lagrangian is same for all flavors. Thus there is $SU(N_f)$ symmetry among the heavy quarks. The second symmetry is the spin symmetry. In the limit of infinite heavy quark mass, the spin degrees of freedom of heavy quarks are decoupled and thus $SU(2)$ rotation of the heavy quark spin becomes a symmetry. These additional symmetries allow many interesting predictions. In particular they imply model independent relations between form factors of weak decays. Several relations [3,4] have been recently derived showing that the excitation spectra and form factors are independent of mass and spin of heavy quark. Isgur and Wise [5] showed that in the lowest order in QCD all the weak decay amplitudes are determined in terms of a single function $\xi(v{\cdot}v')$ which is known as Isgur–Wise function. Falk *et al* [6] also showed that the transition amplitudes for inclusive semileptonic B-meson decay are given in terms of the Isgur–Wise function.

In the present investigation we exploit these ideas to evaluate the transition matrix element of $B_d^0 - \bar{B}_d^0$ system in a manifestly left right symmetric model (MLRS) [7] and obtain the mass of the right handed gauge and Higgs bosons. Here we consider effective

Hamiltonian describing the transition $\bar{b}d \leftrightarrow b\bar{d}$ in the MLRS model [7] along with contributions from neutral Higgs boson sector. MLRS model is the extension of standard model with gauge group $SU(2)_L \times SU(2)_R \times U(1)$. It has the merit of allowing the gauge group, P and CP to be broken spontaneously at the same time and thus successfully explain the mass mixing [8] and CP violation [9] for $M^0 - \bar{M}^0$ system. Besides this, it offers the possibility of a great deal of new physics beyond standard model at the energy scale of several hundred GeV. It is therefore crucial to find a lower limit on M_R, the mass of right handed partner of W-boson in this model. In evaluating the mass difference $\Delta M = 2\langle M^0 | \mathscr{H}_{\text{eff}}(x) | \bar{M}^0 \rangle$, for $B_d^0 - \bar{B}_d^0$ system we use heavy quark symmetry and obtain the matrix elements in terms of the Isgur–Wise function $\xi(v \cdot v')$. However Isgur–Wise function turns out to be unity since both B_d^0 and $\bar{B}_d^0$ states are at rest. Thus within the limit of above approximation, the expression for the mass difference contains parameters like M_R and M_H. Evaluating $K^0 - \bar{K}^0$ mass difference in MLRS model and fixing it with the experimental value of ΔM_K and using $m_t = 174 \pm 10$ GeV [10] we obtain M_R which subsequently when used in the expression for ΔM_B yields the lower limit of M_H.

We organize the paper as follows. In § 2, we give the outlines of minimal left right symmetric model and the effective Hamiltonian we use in our consideration. Section 3 is devoted to the evaluation of hadronic matrix elements and the mass differences for $B_d^0 - \bar{B}_d^0$ and $K^0 - \bar{K}^0$ system. In § 4, we evaluate the Isgur–Wise function. Section 5 contains results and discussion.

2. Minimal left right symmetric model and evaluation of effective Hamiltonian

We review some features of manifest minimal left right symmetric model relevant to our discussion. The Lagrangian is invariant under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In this model one can write the effective Lagrangian for $K^0 - \bar{K}^0$ and $B_d^0 - \bar{B}_d^0$ transition process as [11]

$$\mathscr{L}_{\text{eff}} = \frac{f}{\sqrt{2}}[\bar{p}(x)\gamma^\mu U \text{L} n(x) W_{\text{L}\mu}^\dagger + \bar{p}(x)\gamma^\mu U \text{R} n(x) W_{\text{R}\mu}^\dagger] + \frac{f}{\sqrt{2}M_\text{L}}[\bar{p}(x)(UD_n\text{R} - D_p U\text{L})n(x)S_\text{L}^+] + \frac{f}{\sqrt{2}M_\text{R}}[\bar{p}(x)(UD_n\text{L} - D_p U\text{R})n(x)S_\text{R}^+] + \text{h.c.}, \tag{1}$$

where $p(x)$ and $n(x)$ are p- and n-type quarks defined by

$$p(x) = \begin{pmatrix} u(x) \\ c(x) \\ t(x) \end{pmatrix}, \quad n(x) = \begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix}, \tag{2}$$

with u, c, t, d, s and b being six quark flavors. W_L and W_R are left and right handed charged gauge bosons with mass M_L and M_R whereas S_L^+ and S_R^+ are unphysical charged Higgs bosons [11] (longitudinal components of W_L^+ and W_R^+ respectively). Left right symmetry of the gauge interactions requires that the two $SU(2)$ gauge

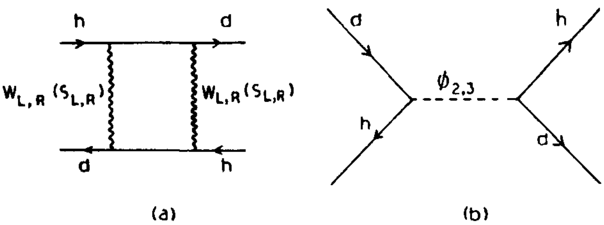

Figure 1(a, b). Feynman diagrams for the $K^0 - \bar{K}^0$ and $B_d^0 - \bar{B}_d^0$ transition amplitudes for $(h = s, b)$. $S_{L,R}$ are the unphysical gauge bosons corresponding to $W_{L,R}$ and $\Phi_{2,3}$ are the flavour changing neutral Higgs bosons of the minimal left right symmetric model.

couplings be equal [12], i.e. $f_L = f_R = f$. The left- and right-handed weak *CKM* mixing matrices are taken to be same in the Lagrangian i.e. $U_L = U_R = U$. The helicity projection matrices for the left- and right-handed ones are denoted by $L = (1 - \gamma_5)/2$ and $R = (1 + \gamma_5)/2$ respectively. D_p and D_n are the mass matrices chosen to be diagonal and are written as

$$D_p = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad \text{and} \quad D_n = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \tag{3}$$

We observe that the process we consider here can occur through gauge bosons exchange as depicted in figure 1, in the lowest order Feynman diagrams. Thus we obtain the effective interaction Hamiltonian as

$$\mathscr{H}_{\text{eff}}(x) = -\mathscr{L}_{\text{eff}}(x) = \mathscr{H}_{\text{eff}}(\text{LL}) + \mathscr{H}_{\text{eff}}(\text{LR}) + \mathscr{H}_{\text{eff}}(\text{RR}), \tag{4}$$

where $\mathscr{H}_{\text{eff}}(\text{LL})$, $\mathscr{H}_{\text{eff}}(\text{RR})$ and $\mathscr{H}_{\text{eff}}(\text{LR})$ represent the Hamiltonian with the exchange of (W_L, W_L), (W_R, W_R) and (W_L, W_R) gauge bosons respectively in the box diagram which are written as

$$\mathscr{H}_{\text{eff}}(\text{LL}) = \frac{G_F^2 M_L^2}{4\pi^2} \sum_{i,j=u,c,t} \lambda_i \lambda_j ((1 + \tfrac{1}{4} x_i x_j) K_2(x_i, x_j, 1) - 2 x_i x_j K_1(x_i, x_j, 1)) \times (\bar{h}\gamma^\mu \text{L} d)(\bar{h}\gamma_\mu \text{L} d), \tag{5}$$

$$\mathscr{H}_{\text{eff}}(\text{RR}) = \mathscr{H}_{\text{eff}}(\text{LL}), \quad \text{L} \leftrightarrow \text{R}, \tag{6}$$

and

$$\mathscr{H}_{\text{eff}}(\text{LR}) = \frac{G_F^2 M_L^2}{4\pi^2} \sum_{i,j=u,c,t} \lambda_i \lambda_j 2\eta \sqrt{x_i x_j} \times ((4 + \eta x_i x_j) K_1(x_i, x_j, \eta) - (1 + \eta) K_2(x_i x_j, \eta))(\bar{h}\text{L}d)(\bar{h}\text{R}d), \tag{7}$$

where

$$K_n(x_i, x_j, \eta) = \frac{\eta^{-n+2} \ln(1/\eta)}{(1-\eta)(1-x_i\eta)(1-x_j\eta)} + \frac{x_i^n \ln x_i}{(x_i - x_j)(1-x_i)(1-\eta x_i)} + \frac{x_j^n \ln x_j}{(x_j - x_i)(1-x_j)(1-\eta x_j)}. \tag{8}$$

The parameters taken in the above expressions are $\lambda_i = U^*_{ih} U_{id}$, $\eta = (M_L^2/M_R^2)$ and $x_i = (m_i^2/M_L^2)$ where m_i is the mass of the ith quark flavor. In fact the Hamiltonian given in the above expressions indicate a transition for $K^0 - \bar{K}^0$ system with $h = s$ and for $B_d^0 - \bar{B}_d^0$ with $h = b$ flavor. The neutral Higgs boson contribution to the effective Hamiltonian is realized through the exchange of two neutral Higgs bosons Φ_2 and Φ_3 at the tree level is given by

$$\mathscr{H}_{\text{eff}}(H^0) = -\frac{\sqrt{2}G_F}{M_H^2} \sum_{i,j=u,c,t} (m_i \lambda_i)^2 (\bar{h}Ld)(\bar{h}Rd). \tag{9}$$

In the above we have assumed a common Higgs mass M_H for both Φ_2 and Φ_3 [13].

3. Heavy quark symmetry and mass difference for $M^0 - \bar{M}^0$ system

To evaluate the hadronic matrix elements for $M^0 - \bar{M}^0$ system taking the MLRS Hamiltonian we consider the mass matrix $\mathscr{M}$ as

$$\mathscr{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix}, \tag{10}$$

where

$$M_{11} = M_{22} = \langle M^0 | \mathscr{H}_{\text{eff}}(x) | M^0 \rangle = \langle \bar{M}^0 | \mathscr{H}_{\text{eff}}(x) | \bar{M}^0 \rangle, \tag{11}$$

and

$$M_{12} = \langle M^0 | \mathscr{H}_{\text{eff}}(x) | \bar{M}^0 \rangle = \langle \bar{M}^0 | \mathscr{H}_{\text{eff}}(x) | M^0 \rangle. \tag{12}$$

In (11) and (12) $|M^0\rangle$ represents the meson state and $|\bar{M}^0\rangle$ represents corresponding anti-meson state. We diagonalize the mass matrix and obtain the mass difference between M^0 and $\bar{M}^0$ mesons as [14]

$$\Delta M_{M^0} = 2M_{12} = 2\langle M^0 | \mathscr{H}_{\text{eff}}(x) | \bar{M}^0 \rangle, \tag{13}$$

and we use heavy quark effective theory (HQET) to evaluate the above matrix elements for $B_d^0 - \bar{B}_d^0$ system.

In HQET the ground state for pseudoscalar heavy meson containing a heavy quark Q and a light anti-quark $\bar{q}$ is given in terms of interpolating fields [2] as

$$P_i(v) = \bar{q}_v \gamma_5 h_v^i \sqrt{M_P}, \tag{14}$$

where h_v^i is a heavy quark of type 'i' with four velocity v and related to the conventional quark field operator $Q_i(x)$ by

$$Q_i(x) = \exp(-im_Q v \cdot x) h_v^i, \tag{15}$$

and the light quark q, stands for a column vector in flavor $SU(3)$ space as

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}. \tag{16}$$

Thus the pseudoscalar heavy meson transforms as a $SU(3)$ antitriplet. The charge conjugate state $\bar{P}_i(v)$ can be related to $P_i(v)$ state by charge conjugation convention as

$$\bar{P}_i(v) = \mathscr{C} P_i(v) \mathscr{C}^{-1} = c P_i(v), \tag{17}$$

with the charge conjugation matrix for Dirac spinor $c = i\gamma^2\gamma^0$. Hence we obtain

$$\bar{P}_i(v) = -\bar{h}_v^i\gamma_5 q_v\sqrt{M_{\bar{P}}}. \tag{18}$$

Thus the ground states for B_d^0 and $\bar{B}_d^0$ mesons are

$$B_d^0(v) = \bar{d}_v\gamma_5 b_v\sqrt{M_{B_d^0}}, \tag{19}$$

and

$$\bar{B}_d^0(v) = -\bar{b}_v\gamma_5 d_v\sqrt{M_{\bar{B}_d^0}}. \tag{20}$$

In order to estimate the mass matrix elements given in (13), we need the evaluation of the matrix elements of the quark operator contained in the effective Hamiltonian i.e.,

$$\langle M^0(v)|\bar{h}_v\gamma_\mu \mathrm{L} d_{v'}\bar{h}_{v'}\gamma^\mu \mathrm{L} d_v|\bar{M}^0(v')\rangle, \tag{21}$$

and

$$\langle M^0(v)|\bar{h}_v \mathrm{L} d_{v'}\bar{h}_{v'}\mathrm{R} d_v|\bar{M}^0(v')\rangle. \tag{22}$$

Evaluation of the above matrix elements in (21) and (22) are formally done by vacuum saturation method [15]. We present here an explicit evaluation of (21) using the wave functions for $B_d^0(v)$ and $\bar{B}_d^0(v')$ in HQET as given in (19) and (20)

$$\begin{aligned}&\langle M^0(v)|\bar{h}_v\gamma_\mu \mathrm{L} d_{v'}\bar{h}_{v'}\gamma^\mu \mathrm{L} d_v|\bar{M}^0(v')\rangle,\\ &= \langle M^0(v)|\bar{h}_v\gamma_\mu \mathrm{L} d_{v'}|0\rangle\langle 0|\bar{h}_{v'}\gamma^\mu \mathrm{L} d_v|\bar{M}^0(v')\rangle.\end{aligned} \tag{23}$$

Now we consider the first part of eq. (23), with $B_d^0(v)$ state as given in (19)

$$\begin{aligned}\langle M^0(v)|\bar{h}_v\gamma_\mu \mathrm{L} d_{v'}|0\rangle &= \sqrt{M_{B^0}}\langle 0|\bar{d}_v\gamma_5 h_v\bar{h}_v\gamma_\mu \mathrm{L} d_{v'}|0\rangle\\ &= \sqrt{M_{B^0}}\,\mathrm{Tr}\left(\gamma_5\frac{(1+\not{v})}{2}\gamma_\mu \mathrm{L}\langle 0|d_{v'}\bar{d}_v|0\rangle\right)\\ &= \sqrt{M_{B^0}}v^\mu\xi(v\cdot v').\end{aligned} \tag{24}$$

In the above we have used the relations [16]

$$\langle 0|h_v\bar{h}_v|0\rangle = \frac{(1+\not{v})}{2}, \tag{25}$$

and

$$\langle 0|d_v\bar{d}_{v'}|0\rangle = \xi(v\cdot v'), \tag{26}$$

where $\xi(v\cdot v')$ is the Isgur–Wise function.

The evaluation of the second part of eq. (23) gives

$$\langle 0|\bar{h}_{v'}\gamma^\mu \mathrm{L} d_v|\bar{M}^0(v')\rangle = \sqrt{M_{\bar{B}^0}}v'^\mu\xi(v\cdot v'), \tag{27}$$

thus we obtain

$$\langle M^0(v)|\bar{h}_v\gamma^\mu \mathrm{L} d_{v'}\bar{h}_{v'}\gamma^\mu \mathrm{L} d_v|\bar{M}^0(v') = 2(v\cdot v')\xi^2(v\cdot v')M_{M^0}. \tag{28}$$

The factor 2 occurs because we can choose the current on the side of M^0 in two different ways. Similarly evaluation of (22) gives

$$\langle M^0(v)|\bar{h}_v \mathrm{L} d_{v'}\bar{h}_{v'}\mathrm{R} d_v|\bar{M}^0(v')\rangle = 2M_{M^0}\xi^2(v\cdot v'). \tag{29}$$

For the evaluation of the four quark operator contained in the effective Hamiltonian for $K^0 - \bar{K}^0$ system we again insert the vacuum state between two currents in all possible ways and use PCAC [15]. Thus we have

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 s|\bar{K}^0(p_k)\rangle = -\langle K^0(p_k)|\bar{d}\gamma^\mu\gamma_5 s|0\rangle = i\sqrt{2}f_K p_k^\mu, \tag{30}$$

where f_K is the K-meson decay constant, related to the pion decay constant by $f_K \sim 1{\cdot}22 f_\pi$ where $f_\pi = 93$ MeV. Using these relations we obtain [9]

$$\langle K^0|\bar{d}\gamma^\mu \mathrm{L} s\bar{d}\gamma_\mu \mathrm{R} s|\bar{K}^0\rangle = \tfrac{4}{3} f_K^2 M_K^2, \tag{31}$$

and [11]

$$\langle K^0|\bar{d}\mathrm{L}s\bar{d}\mathrm{R}s|\bar{K}^0\rangle = \rho\langle K^0|\bar{d}\gamma^\mu \mathrm{L}s\bar{d}\gamma_\mu \mathrm{R}s|\bar{K}^0\rangle, \tag{32}$$

where ρ is given by [17]

$$\rho = \left(\frac{3}{4}\frac{M_K^2}{(m_s + m_d)^2} + \frac{1}{8}\right). \tag{33}$$

With the matrix elements as given in (31) and (32) for $K^0 - \bar{K}^0$ system and in (28) and (29) for $B^0 - \bar{B}^0$ system we estimate their mass differences in subsection A and subsection B respectively.

A. *Mass difference for $K^0 - \bar{K}^0$ system*

Here we estimate the mass matrix elements M_{12} for $K^0 - \bar{K}^0$ system. We neglect the exchange of u-quark in the box diagram (figure 1), since $m_u = 0$. Including the contributions from c- and t-quark exchange, and keeping x_c only up to first order, we obtain

$$\begin{aligned} M_{12}^{\mathrm{LL}} &= \langle K^0|\mathscr{H}_{\mathrm{eff}}(\mathrm{LL})|\bar{K}^0\rangle \\ &= \frac{G_F^2 M_L^2}{4\pi^2} f_{\mathrm{LL}} \frac{4}{3} f_K^2 M_K^2, \end{aligned} \tag{34}$$

with

$$\begin{aligned} f_{\mathrm{LL}} &= \lambda_c^2 x_c + 2\lambda_c\lambda_t x_c\left(-\frac{3}{4}\frac{x_t}{(1-x_t)} + \ln x_t\right) \\ &\quad + \lambda_t^2 \frac{x_t}{(1-x_t)^2}\left(1 - \frac{11}{4}x_t + \frac{1}{4}x_t^2 - \frac{3}{2}x_t^2\frac{\ln x_t}{(1-x_t)}\right), \end{aligned} \tag{35}$$

and

$$\begin{aligned} M_{12}^{\mathrm{LR}} &= \langle K^0|\mathscr{H}_{\mathrm{eff}}(\mathrm{LR})|\bar{K}^0\rangle \\ &= \frac{G_F^2 M_L^2}{4\pi^2} f_{\mathrm{LR}}(2\eta)\left(\frac{M_K^2}{(m_s+m_d)^2} + \frac{1}{6}\right) f_K^2 M_K^2, \end{aligned} \tag{36}$$

with

$$\begin{aligned} f_{\mathrm{LR}} &= \lambda_c^2 x_c[4(1+\ln x_c) + \ln\eta] \\ &\quad + 2\lambda_c\lambda_t\frac{\sqrt{x_c x_t}}{1-x_t}[(4-x_t)\ln x_t + (1-x_t)\ln\eta] \\ &\quad + \lambda_t^2\frac{x_t}{(1-x_t)^2}[(4-x_t)(1-x_t) + (4-2x_t)\ln x_t + (1-x_t)^2\ln\eta], \end{aligned} \tag{37}$$

where we have kept only terms of order η. Similarly the mass matrix element for the Higgs sector is given as

$$M_{12}^{\text{Higgs}} = -\frac{\sqrt{2}G_F}{M_H^2}(m_t^2\lambda_t^2 + m_c^2\lambda_c^2 + m_t m_c\lambda_t\lambda_c)\left(\frac{M_K^2}{(m_s+m_d)^2}+\frac{1}{6}\right)f_K^2 M_K. \quad (38)$$

Substituting these expressions for M_{12} in (13) we obtain

$$\Delta M_K = \frac{G_F^2 M_L^2}{2\pi^2} f_{LL}\frac{4}{3}f_K^2 M_K^2 \times\left[1+\frac{3}{4}\left(\frac{M_K^2}{(m_s+m_d)^2}+\frac{1}{6}\right)\times\left(2\eta\frac{f_{LR}}{f_{LL}} - \frac{4\sqrt{2}\pi^2}{G_F M_H^2}\frac{(m_t^2\lambda_t^2 + m_c^2\lambda^2 + m_t m_c\lambda_t\lambda_c)}{M_L^2 f_{LL}}\right)\right]. \quad (39)$$

B. *Mass difference for $B_d^0 - \bar{B}_d^0$ system with heavy quark symmetry*

For $B_d^0 - \bar{B}_d^0$ system, we also neglect the exchange of c-quark as its mass is much smaller that the b-quark mass of the external line. Hence considering the contribution only from virtual t-quark along with the QCD correction factors and using (28) and (29) we obtain the expressions for M_{12} as

$$M_{12}^{LL} = \frac{G_F^2 M_L^2}{2\pi^2} f_{LL} M_{B_d^0}(v\cdot v')\xi^2(v\cdot v')\eta_{QCD}, \quad (40)$$

with

$$f_{LL} = \lambda_t^2\frac{x_t}{(1-x_t)^2}\left(1 - \frac{11}{4}x_t + \frac{1}{4}x_t^2 - \frac{3}{2}x_t^2\frac{\ln x_t}{(1-x_t)}\right), \quad (41)$$

and

$$M_{12}^{LR} = \frac{G_F^2 M_L^2}{2\pi^2} f_{LR}(2\eta) M_{B_d^0}\xi^2(v\cdot v'), \quad (42)$$

with

$$f_{LR} = \lambda_t^2\left[\eta_1\left(\frac{x_t(4-x_t)}{(1-x_t)} + \frac{(4-2x_t+x_t^2)}{(1-x_t)^2}x_t\ln x_t\right) + \eta_2 x_t \ln\eta\right], \quad (43)$$

and

$$M_{12}^{\text{Higgs}} = -\frac{\sqrt{2}G_F}{M_H^2} m_t^2\lambda_t^2 M_{B_d^0}\xi^2(v\cdot v'). \quad (44)$$

The QCD correction factors are taken [8] to be $\eta_{QCD} = 0{\cdot}83$ and $\eta_1 = \eta_2 = 1{\cdot}8$. Thus we obtain

$$\Delta M_{B_d} = \frac{G_F^2 M_L^2}{\pi^2} f_{LL} M_{B_d^0}\xi^2(v\cdot v')\eta_{QCD} \times\left[(v\cdot v') + 2\eta\frac{f_{LR}}{f_{LL}}\frac{1}{\eta_{QCD}} - \frac{2\sqrt{2}\pi^2}{G_F M_H^2}\frac{(m_t^2\lambda_t^2)}{M_L^2 f_{LL}\eta_{QCD}}\right]. \quad (45)$$

4. Quark model and Isgur–Wise function

Since we are dealing with two-body transition amplitude the product $(v \cdot v')$ is fixed by kinematics and hence the Isgur–Wise function has to be taken at this point. Here we consider the transition of heavy quark b_v between B_d^0 and $\bar{B}_d^0$. The quark b_v with four velocity v makes a transition from B_d^0 to $\bar{B}_d^0$ state. Assuming both B_d^0 and $\bar{B}_d^0$ states are in the rest frame where $v^\mu = (1,0,0,0)$, we obtain $(v \cdot v') = 1$. We have used the GISW quark model [18] to determine the Isgur–Wise function. In the context of this model the IW function may be extracted from the overlap integral

$$I(v') = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \tilde{\Phi}_{\mathrm{F}}^*(\mathbf{p} + \Lambda \mathbf{v}') \tilde{\Phi}_{\mathrm{I}}(\mathbf{p})$$

$$= \int \mathrm{d}^3 x \Phi_{\mathrm{F}}^*(\mathbf{x}) \Phi_{\mathrm{I}}(\mathbf{x}) \exp(-i\Lambda \mathbf{v}' \cdot \mathbf{x}) \tag{46}$$

where the labels I and F denote wave function of the initial and final meson respectively. The "inertia parameter Λ" corresponds to the mass of light degrees of freedom. We shall use for Λ the expression [19],

$$\Lambda = \frac{M_{B_d^0} \cdot m_d}{m_b + m_d}, \tag{47}$$

which accounts for the kinetic effects of heavy quark. The quark masses are taken as $m_d = 330\,\mathrm{MeV}$ and $m_b = 5{\cdot}12\,\mathrm{GeV}$. The wavefunctions are chosen to be the eigen functions of orbital angular momentum l, where both the initial and final mesons i.e. M^0 and $\bar{M}^0$ will have $l = 0$ and thus the wave functions are given by

$$\Phi_{\mathrm{I}}(\mathbf{x}) = Y_0^0\left(\frac{\mathbf{x}}{|\mathbf{x}|}\right) \phi_{\mathrm{I}}(|\mathbf{x}|), \tag{48}$$

and

$$\Phi_{\mathrm{F}}(\mathbf{x}) = Y_0^0\left(\frac{\mathbf{x}}{|\mathbf{x}|}\right) \phi_{\mathrm{F}}(|\mathbf{x}|), \tag{49}$$

with normalization

$$\int \mathrm{d}^3 x \Phi_{\mathrm{I/F}}^*(\mathbf{x}) \Phi_{\mathrm{I/F}}(\mathbf{x}) = \int r^2 \mathrm{d}r \phi_{\mathrm{I/F}}^*(r) \phi_{\mathrm{I/F}}(r) = 1. \tag{50}$$

Inserting the wavefunctions as given in (48) and (49) into the overlap integral (46) and choosing the quantization axis of orbital angular momentum in the direction of velocity, the Isgur–Wise function is given as

$$\xi(v \cdot v') = \int r^2 \mathrm{d}r \phi_{\mathrm{F}}^*(r) \phi_{\mathrm{I}}(r) j_0(\Lambda r \sqrt{(v \cdot v')^2 - 1}). \tag{51}$$

To calculate the above integral we insert the orbital wave function of harmonic oscillator in the form

$$\phi(r) = \left(\frac{\beta^2}{\pi}\right)^{3/4} \exp(-\beta^2 r^2/2), \tag{52}$$

with strength $\beta_B = 0{\cdot}41\,\text{GeV}$ for B meson [18]. With $(v{\cdot}v') = 1$ we obtain the Isgur–Wise function for $B_d^0 - \bar{B}_d^0$ system to be

$$\xi(v{\cdot}v') = 1. \tag{53}$$

5. Results and discussion

Here we estimate the masses of W_R and Higgs bosons. To do this we take the constituent quark masses as $m_d = 330\,\text{MeV}$, $m_s = 550\,\text{MeV}$, $m_c = 1{\cdot}8\,\text{GeV}$ and $m_b = 5{\cdot}12\,\text{GeV}$ in addition to the experimentally observed masses of K and B mesons as $M_{K^0} = 497{\cdot}67\,\text{MeV}$ and $M_{B^0} = 5279\,\text{MeV}$. The experimental values for G_F and M_L are taken to be $G_F = 1{\cdot}16637 \times 10^{-5}\,\text{GeV}^{-2}$ and $M_L = 80{\cdot}22\,\text{GeV}$ [20]. The CKM matrices involved in our calculations are taken as their central values [20]. Next assuming the Higgs contribution to be negligible for K-system and taking the experimentally measured value of $\Delta M_K = 3{\cdot}51 \times 10^{-15}\,\text{GeV}$ eq. (39) yields $M_R = 835\,\text{GeV}$. Then using this value of M_R along with the experimental value $\Delta M_{B_d} = 3{\cdot}35 \times 10^{-13}\,\text{GeV}$ [20], we obtain from (45) $M_H \geqslant 2{\cdot}9\,\text{TeV}$.

We have attempted here to predict the masses of right handed gauge boson M_R and Higgs boson M_H basing on heavy quark symmetry formalism. In doing so we have considered the effective Hamiltonian for the system describing $B_d^0 - \bar{B}_d^0$ transition in MLRS model along with the contributions from neutral Higgs boson sector and the hadronic matrix elements for $B_d^0 - \bar{B}_d^0$ system which depends only on the Isgur–Wise function. However the Isgur–Wise function can be evaluated with GISW quark model which is completely determined by considering the kinematics of the system. Thus the estimated expression of the mass difference for $K^0 - \bar{K}^0$ system in left right symmetric model with vacuum saturation method gives the value of M_R which subsequently when used in the expression for ΔM_{B_d}, yields a lower limit of M_H. However in the earlier investigations Beall *et al* [17] have derived a lower bound on W_R mass to be $M_R > 1{\cdot}6\,\text{TeV}$ by demanding $\Delta M_K > 0$ and neglecting the contribution from t- quark. Mohapatra *et al* [11] included the effect of t-quark and considered the effects of Higgs simultaneously with those of gauge bosons, obtained $M_R > 200\,\text{GeV}$ for $M_H = 100\,\text{GeV}$. But the present experimental limit on M_R and M_H are beyond their estimations. Considering the $K_L - K_s$ mass difference in MLRS model Ecker *et al* [9] get lower bounds such as $M_R \geqslant 2{\cdot}5\,\text{TeV}$ and $M_H \geqslant 10\,\text{TeV}$. Donoghue and Holstein [21] have analyzed the non-leptonic $\Delta S = 1$ weak decays and concluded that $M_R > 300\,\text{GeV}$ assuming left right mixing to be the same. Neglecting the t-quark effect Maharana [22] in a field theoretic quark model obtained $M_R > 715\,\text{GeV}$ whereas for inclusion of the effect of t quark Maharana *et al* [23] found that $M_R = 1650\,\text{GeV}$ for $m_t = 162\,\text{GeV}$. However the present investigation has considered $m_t = 174 \pm 10\,\text{GeV}$ as an input [10] to obtain $M_R = 835\,\text{GeV}$ and $M_H \geqslant 2{\cdot}5\,\text{TeV}$. Nevertheless, our result with recent experimental values of m_t, ΔM_B, ΔM_K and *CKM* matrix elements may have better reliability in its predictions over the earlier investigations.

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