PRAMANA
— journal of
physics

Supplement to Vol. 45
October 1995
pp. 351-364

Testing electroweak vector boson self interactions

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Abstract. We review anomalous triple electroweak vector boson (TEVB) couplings. The anomalous vertex is written in a model independent way and related to the moments of the W boson. Loop-induced bounds on these couplings are derived, including those obtained from the oblique parameters S, T, U. We show that these bounds from oblique parameters in particular, cannot rule out anomalous couplings even if the oblique parameters are in complete agreement with SM. The importance of testing the TEVB vertex beyond LEPI is thereby demonstrated. We examine direct bounds obtainable from the W-radiative and $W\gamma$ production at CDF and D0 collaborations. However, all the bounds on TEVB to date are too weak to limit most models. It is pointed out that future colliders will be needed to probe loop level SM corrections to the TEVB vertex, as well as examine extensions beyond the SM. A detailed study of the process $e^+e^- \rightarrow \ell^+\ell'^-\nu\bar{\nu}'$ is carried out with numerical results pressented for the "Next Linear Collider" (NLC) proposed to run at $\sqrt{s} = 500GeV, 1TeV$. We find that the TEVB couplings can be studied accurately enough to probe regions close the loop level corrections within the SM, and also allow for a possibility of separating the photon coupling from that of the Z using polarized beams at the NLC.

1. Introduction

The $SU(2)_L \otimes U(1)_Y$ theory of electroweak interactions, which is a part of the standard model (SM), has had dramatic confirmations in the last decade. Experiments at LEP are currently able to measure the mass of the Z and its couplings to fermions at a less than 1% level and they agree with the theory considered up to one loop. This success of the SM has generated a feeling that even those of its parameters that have not been tested directly are likely to be in good agreement with observation. While present precision measurements do constrain new physics *vis-a-vis* the parameters of the SM, such constraints need not be as restrictive in some sectors as in others. In fact, the gauge boson couplings of the standard model of electroweak interactions are only just beginning to be directly measured. There has now been observation of the process $p\bar{p} \rightarrow e\nu\gamma X$, presumably representing $W\gamma$ production and radiative W decay, at the Collider Detector at Fermilab (CDF), the D0 collaboration at Fermilab [1, 2, 3] and at UA2 at CERN.[4] Recently a lot of work has been devoted to a global analysis of low energy data and LEP data, in order to extract bounds on the gauge boson couplings.[5]. Extensive studies[6] have also been undertaken to determine the possibilities of detecting such deviations at future colliders.

2. The general triple electroweak vector boson vertex

The vector boson self interactions are uniquely fixed by the gauge structure of the SM. However, new physics as well as higher order corrections will modify these self interactions. The couplings of W bosons to the photon and Z can be described by an effective Lagrangian, the most general form of which may be written down with the minimum requirements– that of Lorentz invariance, global $SU(2)$ and local $U(1)$ symmetry [7, 8] as:

$$\begin{aligned}\mathcal{L}_{eff}^{V} = & - g_V \Big[i g_1^V \left(W^\dagger_{\alpha\beta} W^\alpha - W^{\dagger\alpha} W_{\alpha\beta} \right) V^\beta + i\kappa_V W^\dagger_\alpha W_\beta V^{\alpha\beta} \\ & + i\frac{\lambda_V}{M_W^2} W^\dagger_{\alpha\beta} W^\beta{}_\sigma V^{\sigma\alpha} + i\tilde{\kappa}_V W^\dagger_\alpha W_\beta \tilde{V}^{\alpha\beta} + i\frac{\tilde{\lambda}_V}{M_W^2} W^\dagger_{\alpha\beta} W^\beta{}_\sigma \tilde{V}^{\sigma\alpha} \\ & + g_4^V W^\dagger_\alpha W_\beta (\partial^\alpha V^\beta + \partial^\beta V^\alpha) + g_5^V \epsilon^{\alpha\beta\mu\nu} W^\dagger_\alpha \overleftrightarrow{\partial_\mu} W_\beta V_\nu \Big]\end{aligned} \quad (1)$$

In the above equation V represents the neutral gauge bosons either the photon or the Z, $V_{\alpha\beta} = \partial_\alpha V_\beta - \partial_\beta V_\alpha$, $W_{\alpha\beta} = \partial_\alpha W_\beta - \partial_\beta W_\alpha$ and g_V is the WWV coupling strength in the SM with $g_\gamma = e$ and $g_Z = ec/s$, where $c^2 \equiv 1 - s^2 \equiv M_W^2/M_Z^2$. The above Lagrangian consists of five dimension 4 terms and 2 dimension 6 terms and has the corresponding seven parameters or couplings characterizing the couplings of the W to each of neutral gauge bosons. These seven operators exhaust all possible Lorentz structures if it is assumed that $\partial_\mu W^\mu = 0$ and $\partial_\mu V^\mu = 0$. These conditions are automatic if the bosons are coupled to light fermions. Higher dimension operators can be added by the replacement $V_\mu \rightarrow \Box^n V_\mu$. However, contributions from these operators provides only the momentum dependence to the couplings. The SM values for these couplings are $g_1^Z = \kappa_\gamma = \kappa_Z = 1$, and $\lambda_\gamma, \lambda_Z, \tilde{\kappa}_\gamma, \tilde{\kappa}_Z, \tilde{\lambda}_\gamma, \tilde{\lambda}_Z, g_4^\gamma, g_4^Z, g_5^\gamma, g_5^Z$ are all zero. Electromagnetic gauge invariance fixes g_1^γ to be unity and g_4^γ and g_5^γ to be zero. The other couplings $g_1^Z, g_4^Z, g_5^Z, \kappa_\gamma, \kappa_Z, \lambda_\gamma, \lambda_Z, \tilde{\kappa}_\gamma, \tilde{\kappa}_Z, \tilde{\lambda}_\gamma, \tilde{\lambda}_Z$ have to be determined experimentally. Considering that a large number of parameters have to be determined experimentally some assumptions have often been made to reduce this set by making some assumptions. The couplings g_4^Z and g_5^Z violate custodial SU(2) and hence can be assumed to be small or zero. The number of extra parameters is further reduced by taking g_1^Z, the weak neutral charge of the W, equal to unity.

The parameters $\kappa_{\gamma,Z}$ and $\lambda_{\gamma,Z}$ are unique as they can be related to the multipole moments of the W – which can be defined with respect to both neutral bosons γ/Z as follows

$$\begin{aligned}\mu_W &= \frac{e}{2M_W}(1 + \kappa + \lambda) \\ Q_W &= -\frac{e}{M_W^2}(\kappa - \lambda) \\ d_W &= \frac{e}{2M_W}(\tilde{\kappa} + \tilde{\lambda})\end{aligned}$$

$$\tilde{Q_W} = - \frac{e}{M_W^2}(\tilde{\kappa} + \tilde{\lambda}) \tag{2}$$

where μ_W is the magnetic dipole, Q_W is the electric quadrupole moment, d_W is the electric dipole and $\tilde{Q_W}$ is the magnetic quadrupole of the W boson. Tree level unitarity will restrict the WWV couplings to their SM values at asymptotically high energies. Any deviation from the standard model is thus described by a form factor and depends on a scale for new physics 'Λ'. It is customarily assumed that these form factors are constant for $\sqrt{s} \ll \Lambda$ and start decreasing when Λ is reached or surpassed, in analogy with the nucleon form factors. These form factors will therefore act as regulators for loop corrections.

3. Anomalous couplings in various models

If the W bosons are composite objects, then deviation of the triple gauge boson coupling parameters from their standard model values could be very large indeed; as an example, κ has been calculated to be greater than three in one model[9]. However, within the standard model, upper bounds on the one loop corrections to the tree level values of κ_γ and λ_γ are expected to be [10] $\Delta\kappa_\gamma = 1.7 \times 10^{-2}$ and $\Delta\lambda_\gamma = 0.25 \times 10^{-2}$ for a top quark of about 170GeV. In extensions of the standard model such as those containing extra Higgs doublets, extra heavy fermions [11], the deviations from the tree level standard model values tend to be of about the same order of magnitude as these one loop corrections. For the minimal supersymmetric SM, due to cancellations, the values expected are slightly below the SM ones.Typically, in most renormalizable models, $\kappa_{\gamma,Z}$ are expected to vary by only less than 5% of the SM value. Variations in $\lambda_{\gamma,Z}$ are generally (although not always) further suppressed by an order of magnitude. Hence, variations of, or limits on $\kappa_{\gamma,Z}$ alone are considered usually in literature.

4. Loop-induced constraints on anomalous couplings

Much work has gone into constraining the anomalous couplings in eqn(1),in particular by loop induced processes at low energies. However, questions have been raised on the validity of these bounds. In particular the validity of the Lagrangian itself has been questioned. The Lagrangian is not explicitly gauge invariant, and hence it has been claimed[12] that it is not an valid extension of the SM. However, Burgess and London [13] have argued by an explicit proof that – any Lagrangian that contains W's and Z's which satisfies Lorentz invariance and $U(I)_{em}$ gauge invariance is automatically $SU_L(2) \times U_Y(1)$ gauge invariant–the gauge invariance being realized nonlinearly. Burgess and London, however, agree that most bounds on these couplings are over estimated. The real culprit they point out is not gauge invariance but the improper use of cutoff in estimating the size of the loop diagrams. They point out that the cutoff dependence in low energy theory does not, in general, give an accurate indication of the true dependence on the heavy physics, although it can do so for a logarithmic divergence. The constraints on the anomalous couplings are therefore considerably weakened. Some of the valid bounds –depending at most on the log of the cutoff scale Λ – are considered here. Bounds from tree level unitarity are based on the assumption of a dipole form factor and provide the limits[14]

$$|\kappa - 1| \leq 7.4\frac{\mathrm{TeV}^2}{\Lambda^2}, |\tilde{\kappa}| \leq 35\frac{\mathrm{TeV}^2}{\Lambda^2} \text{ and } |\lambda, \tilde{\lambda}| \leq 4.0\frac{\mathrm{TeV}^2}{\Lambda^2}.$$

Understandably these limits are too weak to bound any model. The muon magnetic moment is used set the bound [15]

$$\left|(\kappa_\gamma - 1)\ln\frac{\Lambda^2}{M_W^2} + \lambda_\gamma/3\right| < 3.7$$

However, it has been argued [16] that contributions from direct terms such as $\overline{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu}$ which are needed to cancel divergences weaken these limits. Analogously the neutron dipole moment is also used to limit the CP violating couplings giving[17]

$$\left|(\tilde{\kappa}_\gamma)\ln\frac{\Lambda^2}{M_W^2} + \tilde{\lambda}_\gamma/3\right| < 10^{-3}$$

5. Bounds from oblique parameters on anomalous couplings

New physics beyond the SM can be usefully constrained in terms of the "oblique" electroweak correction [18, 19] parameters $\widetilde{S}, \widetilde{T}$, and $\widetilde{U}$ [21] (or $\Delta\epsilon_1, \Delta\epsilon_2$, or $\Delta\epsilon_3$). These are linearly related to $\widetilde{\Pi}_{WW}(q^2)$, $\widetilde{\Pi}_{ab}(q^2)$ (where $a, b = \gamma, Z$), which are the contributions from new physics to the electroweak vector boson self–energies as follows:

$$\begin{aligned} \alpha M_W^2 T &= \left[\Pi_{WW}(0) - s^2\Pi_{ZZ}(0)\right], \\ \alpha M_Z^2 S &= 4cs\left[cs\left\{\delta\Pi_{ZZ}(M_Z^2) - \Pi_{\gamma\gamma}(M_Z^2)\right\} + (s^2 - c^2)\Pi_{\gamma Z}(M_Z^2)\right] \\ \alpha M_W^2 U &= 4s^2\left[\delta\Pi_{WW}(M_W^2) - s^2\left\{s^2\Pi_{\gamma\gamma}(M_Z^2) + c^2\delta\Pi_{ZZ}(M_Z^2)\right.\right. \\ &\quad \left.\left. +2\,cs\,\Pi_{\gamma Z}(M_Z^2)\right\}\right] \end{aligned} \tag{3}$$

where $\delta\Pi(q^2) \equiv \Pi(q^2) - \Pi(0)$. In order to constrain the WWV couplings, we relate them to these oblique parameters and use the known constraints on the latter. The approach presented here is from Choudhury, Roy and Sinha([22]). Analysis of the type discussed here have also been done by Hagiwara *et al.* ([23]) and Burgess *et al.* ([24]), with similar conclusions. Hagiwara *et al.* deal with a linearly realized Lagrangian whereas we and Burgess *et al.* deal with a nonlinearly realised Lagrangian. The major difference between our analysis and that of Burgess *et al.* is that they include vertex corrections ignored by us and include more than the three oblique parameters. However, Burgess *et al.* ignore κ^2, λ^2 and $\kappa\lambda$ terms which have been included by us.

If the anomalous vector boson couplings constitute the sole source of new physics, $\widetilde{T}$ and $\widetilde{U}$ will measure the weak isospin breaking induced by them, while $\widetilde{S}$ arises from their $SU(2)_L \otimes U(1)_Y$ breaking aspect through the involvement of the longitudnal vector boson modes. One can work out the divergent contributions to the $\widetilde{\Pi}$-functions from the nonstandard WWV vertex by using (1) and dimensional regularization. Defining $\widehat{\Pi}(q^2)$ to be the coefficient of the usual $(2/\epsilon - \gamma_E)$[13] term in $\widetilde{\Pi}(q^2)$, we compute

$$\widehat{\Pi}_{ab}(q^2) = -\frac{g_a g_b q^2}{192\pi^2} \begin{array}{l} [(\eta_a + \eta_b)(36 - 4r - r^2) + \eta_a\eta_b r(2 - r) \\ +(3\lambda_a + 3\lambda_b + \eta_a\lambda_b + \eta_b\lambda_a)(24 - 4r) \\ +\lambda_a\lambda_b(36 + 8r - 2r^2)], \end{array} \tag{4}$$

where $r \equiv q^2/M_W^2$ and $\eta_{a,b} \equiv 1 - \kappa_{a,b}$. For compactness, we introduce the notation:

$$\begin{array}{lll} \mathcal{C}_1 \equiv 36c^2 - 4 - c^{-2}, & & \mathcal{C}_2 \equiv 24c^2 - 4, \\ \mathcal{C}_3 \equiv 2 - c^{-2}, & & \mathcal{C}_4 \equiv 36c^2 + 8 - 2c^{-2} \end{array}$$

With $\widehat{T}$, $\widehat{S}$ and $\widehat{U}$ defined analogously in terms of the $\widehat{\Pi}$'s and denoting $\langle F\rangle \equiv s^2F_\gamma + c^2F_Z$ for any F, we have

$$\widehat{S} = -\frac{1}{12\pi}[(\eta_Z - \eta_\gamma)\{\mathcal{C}_1 + \langle\eta\rangle\mathcal{C}_3 + \langle\lambda\rangle\mathcal{C}_2\} + (\lambda_Z - \lambda_\gamma)\{\mathcal{C}_2(3 + \langle\eta\rangle) + \langle\lambda\rangle\mathcal{C}_4\}] \tag{5}$$

Since $\widehat{\Pi}_{ab}(0) = 0$, only $\widehat{\Pi}_{WW}$ contributes to $\widehat{T}$ and we can write

$$\widehat{\Pi}_{WW}(q^2) = \alpha M_W^2\left[\widehat{T} - \frac{r}{48\pi s^2}\mathcal{R}(r)\right], \tag{6}$$

$$\widehat{T} = -\frac{3}{16\pi}\left[\{4 + \eta_\gamma\}\eta_\gamma + \frac{\eta_Z}{c^2s^2}\{2(2c^4 + 2c^2 - 1) + (c^4 + c^2 - 1)\eta_Z\}\right], \tag{7}$$

$$\begin{array}{rcl} \mathcal{R}(r) & \equiv & (4 - 2r)\{5\langle\eta\rangle + \langle\eta^2\rangle\} + 28\eta_Z + 5\eta_Z^2 + 8(3 - r)\{3\langle\lambda\rangle + \langle\lambda\eta\rangle\} \\ & & +24\lambda_Z\{3 + \eta_Z\} + 2(6 + 2r - r^2)\langle\lambda^2\rangle + 4(3 + 3c^{-2} + r)\lambda_Z^2. \end{array} \tag{8}$$

Consequently,

$$\widehat{U} = -\frac{1}{12\pi}\mathcal{R}(1) + \frac{1}{12\pi c^2}[2\mathcal{C}_1\langle\eta\rangle + \mathcal{C}_3\langle\eta\rangle^2 + 2\mathcal{C}_2\langle\lambda\rangle(3 + \langle\eta\rangle) + \mathcal{C}_4\langle\lambda\rangle^2]. \tag{9}$$

Since $\widehat{T}$ depends only on $\widehat{\Pi}_{WW}(0)$ and $\widehat{\Pi}_{ZZ}(0)$ and not on their q^2 variations it is unaffected by dimension 6 operators and hence is independent of λ_γ and λ_Z. Also, terms in $\widehat{S}$ are proportional to either $\eta_Z - \eta_\gamma$ or $\lambda_Z - \lambda_\gamma$ as they should, since $\widehat{S}$ originates from the mixing between weak hypercharge (Y) and the third component of weak isospin – the WWY vertex being linear in these differences.

It should be noted that, unlike in the SM and its straightforward extensions, the oblique parameters are not finite quantities here. This is a consequence of the presence of nonrenormalizable terms in the Lagrangian of (1). In a cut–off dependent regularization scheme, this fact would manifest itself through a non-trivial functional dependence on the cut–off scale [23]. As a matching condition between two effective theories [25], we identify $\mu = \Lambda$, the scale at which new physics becomes manifest (assumed to be $\sim 1TeV$). Using the $\overline{\text{MS}}$ scheme of renormalization, we are lead to

$$\widetilde{S}(\Lambda^2) = \widetilde{S}(M_Z^2) + \widehat{S}\ln\frac{\Lambda^2}{M_Z^2} \tag{10}$$

and similar equations for $\widetilde{T}$ and $\widetilde{U}$. Observed bounds on $\widetilde{S}$,$\widetilde{T}$ and $\widetilde{U}$ can now be translated into $\widehat{S}$, $\widehat{T}$ and $\widehat{U}$.

We use [26] $\widetilde{S} = -0.31 \pm 0.49$, $\widetilde{T} = -0.12 \pm 0.34$ and $\widetilde{U} = -0.11 \pm 0.92$, though our results are insensitive to the central values. As pointed out, $\widetilde{T}$ depends only on η_γ and η_Z, being expressed as a sum of quadratic functions of theirs. Thus the measurement of $\widetilde{T}$ alone constrains these parameters to lie on an elliptic band in the $\eta_\gamma - \eta_Z$ plane, the width of the band being given by the errors on M_W/M_Z and $\widetilde{T}$. $\widetilde{S}$ and $\widetilde{U}$ then reduce the allowed region into smaller part of the elliptic band. Since $\widetilde{S}$ is proportional to the differences of $\gamma-$ and Z–couplings, constraints on it generally (though not always) tend to make those converge. Rather unexpectedly, $\widetilde{U}$ plays a significant role in constraining these anomalous couplings. In conjunction with $\widetilde{S}$ and $\widetilde{T}$, it serves to exclude a large part of the parameter space.

We obtain the (95% C.L.) bounds $-6.1 \lesssim \eta_\gamma \lesssim 4.1$, $-6.0 \lesssim \lambda_\gamma \lesssim 4.5$, $-2.0 \lesssim \eta_Z \lesssim 0.3$ and $-4.5 \lesssim \lambda_Z \lesssim 1.9$, earlier *UA2* analysis [4] had yielded $-8.4 \lesssim \eta_\gamma \lesssim 12.1$ (for arbitrary λ_γ) and $-8.5 \lesssim \lambda_\gamma \lesssim 6.5$ (for arbitrary η_γ). The allowed regions represent solutions to polynomial equations, which are curves that thicken into bands on account of experimental error bars on the coefficients. As the errors shrink to zero, the allowed parameter space collapses into the curves still permitting wide ranges of values for the anomalous couplings. This observation holds even for the SM point *viz.* $\widetilde{S} = \widetilde{T} = \widetilde{U} = 0$, being a consequence of cancellations between various contributions to the $\widetilde{\Pi}$–functions.

The bounds that we have obtained above are still large as compared to the ratio v^2/Λ^2, where v is the electroweak VEV [$v = 2M_W/(e\,s)$]. Arguments based on Naive Dimensional Analysis [27] suggest, for instance, that $\kappa_V - 1 \gtrsim v^2/\Lambda^2$. However, any approximate equality $\kappa_V - 1 \sim v^2/\Lambda^2$ is only a matter of conjecture. This makes phenomenological investigations of these bounds, as performed here, rather important.

6. Direct bounds on anomalous couplings

The bounds mentioned above are to some extent model dependent, in the sense that they are obtained by loop level processes, where 'other new physics' can also contribute. However, there has now been the observation of $p\bar{p} \to e\nu\gamma X$, presumably representing $W\gamma$ production and radiative W decay, at the Collider Detector Facility (CDF) [1] at the Tevatron ($\sqrt{s} = 1.8$ TeV) at Fermilab and at UA2 ($\sqrt{s} = 630$ GeV) at CERN [4]. Such measurement allows for the possibility of directly testing the gauge boson couplings by tree level processes. The best way to detect possible nonstandard gauge boson couplings is to make use of the radiation amplitude zero[28] which occurs in the angular distribution of $d\bar{u} \to W\gamma$ as well as in $W \to d\bar{u}\gamma$.The presence of the zero will be explicitly borne out in equations (12) and (14) below by the factors Z contained therein. The dip which persists in $p\bar{p} \to W\gamma X$, is very sensitive to deviations from the standard model gauge boson couplings.

Preliminary results from CDF[1] quote a limited number of events (6 events corresponding to an integrated luminosity ($\mathcal{L}$) of 4.3pb^{-1}) [29] UA2 has also reported their final results and quote 10 events with an integrated luminosity of 13pb^{-1}.

With such few events no angular distribution can not be obtained. One can nevertheless obtain the first direct bounds[30] on κ, from the total number of $W\gamma$ events, as well as the number of radiative W decays, $W^+ \to e^+\nu\gamma$ and $W^- \to e^-\bar{\nu}\gamma$.

The formula for the cross section of $W\gamma$ production is given by

$$\begin{aligned}\sigma(P\bar{P} \quad &\rightarrow \quad W^-\gamma X) = \frac{1}{3}\sum_{i=d,s}\int\int dx_A dx_B [P_i^P(x_A)P_{\bar{u}}^{\bar{P}}(x_B) \\ &\hat{\sigma}(q_i\bar{q_u} \rightarrow W^-\gamma) + P_{\bar{u}}^P(x_A)P_i^{\bar{P}}(x_B)\hat{\sigma}(q_i\bar{q_u} \rightarrow W^-\gamma)_{\hat{u}\leftrightarrow\hat{t}}] \end{aligned} \tag{11}$$

where,

$$\begin{aligned}&\hat{\sigma}(q_i(k_1)\bar{q_j}(k_2) \rightarrow W^-(P)\gamma(k)) = \frac{4\pi\alpha}{\hat{s}}\frac{M_W^2 G_F}{\sqrt{2}}V_{ij}^2\int_{PhaseSpace} \\ &\left(Z^2\frac{\hat{t}^2+\hat{u}^2+2\hat{s}M_W^2}{\hat{u}\hat{t}} - \eta Z\frac{\hat{u}-\hat{t}}{\hat{u}+\hat{t}} + \frac{\eta^2}{2(\hat{t}+\hat{u})^2}[\hat{t}\hat{u} + (\hat{t}^2+\hat{u}^2)\frac{\hat{s}}{4M_W^2}]\right)\end{aligned} \tag{12}$$

and where $\hat{s} = (k_1+k_2)^2, \hat{t} = (P-k_1)^2, \hat{u} = (P-k_2)^2$ with $\hat{s}+\hat{t}+\hat{u} = M_W^2$, and V_{ij} is the KM matrix element. $Z = (Q_i + \frac{\hat{u}}{\hat{t}+\hat{u}})$ is the zero factor, Q_i is the electric charge of the quark q_i, and $\eta = \kappa - 1$. For this section the subscript γ is supressed, *kappa* is meant to imply κ_γ.

For the radiative W decay case, one obtains the result

$$\begin{aligned}&\sigma(P\bar{P} \rightarrow e^-\bar{\nu}\gamma X) = \frac{1}{3}\sum_{i=d,s}\int\int dx_A dx_B \delta(sx_Ax_B - M_W^2)\left[P_i^P(x_A)P_{\bar{u}}^{\bar{P}}(x_B)\right. \\ &\left.\hat{\sigma}(q_i(k_1)\bar{q_u}(k_2) \rightarrow e\nu\gamma) + P_{\bar{u}}^P(x_A)P_i^{\bar{P}}(x_B)\hat{\sigma}(q_i(k_2)\bar{q_{\bar{u}}}(k_1) \rightarrow e^-\bar{\nu}\gamma)\right].\end{aligned} \tag{13}$$

where we obtain $\hat{\sigma}$, using the zero-width approximation as,

$$\begin{aligned}\hat{\sigma}(q_i(k_1\bar{q_j}(k_2) \quad &\rightarrow \quad W^-(q) \rightarrow e^-(p_1)\bar{\nu}(p_2)\gamma(k)) = \frac{48\pi^4\alpha^2}{\hat{s}\,sin^2\theta_w}V_{ij}^2 \\ &B(W^- \rightarrow e\bar{\nu})\int_{PhaseSpace}[Z'^2 A - Z'\eta B + \eta^2 C]\end{aligned} \tag{14}$$

where $Z' = (\frac{Q_i}{Q} - \frac{P_1.k}{q.k})$ is the zero factor, and

$$\begin{aligned}A &= \frac{4}{(P_1.k)(P_2.k)}[(P_1.k_2)^2 + (P_2.k_1)^2] \\ B &= \frac{1}{4(P_1.k)(q.k)}\left[M_W^2(4k_2.P_1 - 2k_1.P_2 - M_W^2) + 4(P_2.k)^2\right. \\ &+4(k_1.P_1)(k_1.P_1 + 2P_2.k) - 8(P_2.k)(k_1.P_2)(\frac{P_2.k}{M_W^2} - 1) \\ &+(1 - \frac{2P_2.k}{M_W^2})[4k_1.P_1\,k_1.P_2 + 2P_1.k\,M_W^2] \\ &-4(k_1.P_1)(k.P_1)(\frac{2k_2.P_1}{M_W^2} + 1) \\ &\left.-\frac{2k_1.P_2\,k.P_1}{P_2.k}(2\,k.k_2 + \frac{4(P_2.k)(P_2.k_1)}{M_W^2} - \frac{4(P_1.k_1)(P_1.k)}{M_W^2})\right]\end{aligned}$$

$$
\begin{aligned}
C &= \frac{1}{4(q.k)^2}\left[2(P_2.k)(M_W^2 - 2(P_2.k))\right. \\
&- \frac{4(P_2.k)}{M_W^2}((P_1+P_2).k_1)(2k_2.P_1 - M_W^2) \\
&\left. -4(P_2.k)(k_1.(2P_1+P_2)) + \frac{8(P_2.k_1)(P_1.k)(k.k_2)}{M_W^2}\right].
\end{aligned} \tag{15}
$$

To obtain the total cross sections we perform the integrations over the parton fractions and over the phase space, using Vegas Monte Carlo routines and EHLQ[31] parton distribution functions. The following cuts were also imposed, to agree with the cuts imposed by CDF: (1) the transverse photon energy $E_{T\gamma} < 5$ GeV; (2) the photon pseudo-rapidity $|\eta_\gamma| < 3.0$; (3) the electron-photon angular separation $\sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} > 0.3$

With these cuts, the Wγ production $(W^+ + W^-)$ cross section is computed to be

$$\sigma_{W\gamma} = [32.8 - 0.600\eta + 1.42\eta^2]pb. \tag{16}$$

To calculate the number of $p\bar{p} \to W\gamma X \to e\nu\gamma X$ events to be expected at CDF for any given η, we use the the electron and photon acceptances and their detection efficiencies to be $\mathcal{A}_e = 0.41, \mathcal{E}_e = 0.67, \mathcal{A}_\gamma = 0.54$ and $\mathcal{E}_\gamma = 0.50$ respectively, as given by CDF[1]. Also including the Drell-Yan correction factor $K_{DY} = 1.3$, we get $n = \sigma_{W\gamma}\mathcal{L}B(W \to e\nu)K_{DY}\mathcal{A}_e\mathcal{E}_e\mathcal{A}_\gamma\mathcal{A}_\gamma$. Using $B(W \to e\nu) = 0.11$ we get $n = 1.49 - 0.0273\eta + 0.0647\eta^2$. Of the total of six events that are reported by CDF,three events have a large $\eta_\phi \sim 2.4-2.6$ whereas the other three have $\eta_\phi \sim 0.8$ thus they can be clearly classified as $W\gamma$ and W radiative decays.Assuming 3 $W\gamma$ events we get a loose bound

$$-8.6 \le \kappa \le 11.0(95\%CL). \tag{17}$$

The cross section for $p\bar{p} \to WX \to e\nu\gamma X$ is

$$\sigma(W^\pm e\nu\gamma) = (18.8 - 0.0758\eta + 0.108\eta^2)pb. \tag{18}$$

Using the same cuts, acceptances, and efficiencies as before we obtain the quadratic equation in η for the number n of $p\bar{p} \to WX \to e\nu\gamma X$ events: $n = 7.79 + 0.0315\eta + 0.0447\eta^2$. For 3 events this gives a bound

$$-2.4 \le \kappa \le 3.7(96\%CL). \tag{19}$$

If the events are not separated into decay and radiative ones, the total number of events is $n = 9.28 + 0.0042\eta + 0.109\eta^2$, which gives $-3.9 \le \kappa \le 4.8$ (95% CL). UA2 have also set a bound on κ without separating the events to be $-5.4 \le \kappa \le 7.9$.

The very loose bounds that one gets in $P\bar{P} \to WX \to e\nu\gamma X$ case compared to the $P\bar{P} \to W\gamma X \to e\nu\gamma X$ considered earlier can be traced directly to the differences in the production mechanisms for the W in the two cases, and to the effect of the cuts imposed on the SM term relative to the non-SM term in the two cases. Note that in Eq.(8), there is a δ function involving parton fractions x_A, x_B which is not there in Eq.(1), whose effect is to reduce the cross section obtained from Eq.(8) relative to that obtained from Eq.(1). The effect of the cut imposed on the SM term in the $P\bar{P} \to W\gamma X \to e\nu\gamma X$ case is such as to decrease it enormously relative to the η and η^2 terms. Had there been no cut, then the SM term in $P\bar{P} \to W\gamma X \to e\nu\gamma X$ case would have been many times larger.

Clearly the bounds placed on the gauge coupling by direct tests is very poor [32] when it comes to limiting most models. Direct studies of the TEVB vertices will be possible at Fermilab, and upcoming LEP II or proposed future collider like $\gamma\gamma$ collider or the NLC. New machines will be necessary to probe regions much closer to the SM prediction.

7. Testing TEVB at future colliders

With the Next Linear Collider (NLC) in mind we have investigated[33] a set of processes of four lepton production in e^+e^- collisions with respect to their sensitivity to gauge boson coupling parameters. The processes are all of the general form

$$e^+e^- \to \ell^+\ell'^-\nu\bar{\nu}'. \tag{20}$$

we include all possible charged lepton combinations, specifically these are $\mu\tau$, μe (τe), $\mu\mu$ ($\tau\tau$), and ee. The channels given in brackets have the same set of Feynman diagram contributions as their corresponding unbracketed channel and we will henceforth drop reference to them as distinct processes.

For all our processes, we do include the full gauge invariant set of diagrams. the $\mu^+\tau^-$ state can be produced via 9 different diagrams; the μ^+e^- state receives contributions from 18 diagrams. The $\mu\mu$ process has a total of 28 contributing diagrams with most of the extras being γ or Z 'bremsstrahlung' from the initial or final state leptons. The ee process goes via 56 diagrams.. For the $\mu\mu$ and ee final states, in some of the diagrams, all ν species can appear. These diagrams are added incoherently in the calculation. However, for the purpose of counting the number of diagrams, we regard all the ν final states as contributing to a single diagram.

It is important to realize that one needs the full calculation that we discuss here as opposed to the calculations of W pair or single W production. At LEP II energies it is appropriate to calculate W pair production, the processes being dominated by the nearly on-shell W propagators. This is not the case, however, at higher energies unless one is dealing with a final state which allows for the experimental reconstruction of W's. We are considering here the purely leptonic final states. For this case, the W's can be reconstructed (up to a two-fold ambiguity) only under the assumption that they are on-shell. [8] Neither W pair nor single W production can be isolated by experimental cuts for the purely leptonic case. Consequently, we must fully calculate the $\ell^+\ell'^-\nu\bar{\nu}'$ production, as opposed to W pair production only and, in doing so, we aim to unearth a more realistic picture of the sensitivity to the couplings in question.

In order to deal easily with the large number of Feynman diagrams and to readily retain helicity information, we have written the amplitude for each process in the CALKUL helicity formulation [34]. We assume massless spinors describe the fermions although we do retain fermion masses in the propagators; this amounts to neglecting terms proportional to m_f, a good approximation. The matrix element squared for each process is embedded in a Monte Carlo algorithm for integration over the final state four body phase space to yield the cross sections and various distributions. We sum and average over initial spins and sum over final spins. We use $M_Z = 91.196\,GeV$, $\Gamma_Z = 2.534\,GeV$, $M_W = 80.6\,GeV$, $\Gamma_W = 2.25\,GeV$, $m_e = 0.511\,MeV$, $m_\mu = 0.1057\,GeV$, $m_\tau = 1.7841\,GeV$, and $\sin^2\theta_W = 0.23$.

The experimental signature for the processes under consideration is a clean one, an oppositely charged lepton pair and missing transverse momentum and energy due to the neutrinos. We have made some fairly simple cuts as described below to account for detector acceptance and potential backgrounds. For all the processes, we require a cut on the angle of each of the charged leptons relative to the beam such that $-0.95 \leq \cos\theta_{\ell\pm} \leq 0.95$. This is the only cut we impose for the $\mu\tau$ and μe final states. This angular cut is experimentally motivated however it also serves to regulate the t-channel photon poles, allowing us to neglect the terms proportional to m_f. These diagrams contribute to the μ^+e^- and e^+e^- final states, as described above.

One potential background is τ pair production with each of the τ's decaying leptonically. At $\sqrt{s}$ of $200\,GeV$, each of the four lepton processes and the τ pair production, multiplied by the branching ratios of τ into e or μ of 17.8% each [35], yield about the same rate. At higher energies, the τ pair production cross section is falling like $1/s$ while the cross section for our processes remains large. In addition, the τ pair process should have substantially greater missing energy with four neutrinos in the final state. It seems that this source of background is manageable.

The four lepton processes with one or more τ's in the final state ($\mu\tau$ and $\tau\tau$) could feed down as a background to the processes without any τ if the τ(s) decays leptonically. However, factoring in the τ decay branching ratio and accounting for the higher missing energy keeps this background under control. Another potential background comes from two photon processes with the e^+ and e^- undetected near the beam. This is relevant to the $\mu\mu$ and ee processes and we make a cut on missing transverse momentum to eliminate two photon events as a background source; we require total visible $p_T > 10\,GeV$. We also require for these two processes that each charged lepton carry a minimum energy, $E_\ell > 10\,GeV$. Finally, again for the $\mu\mu$ and ee processes we make a cut on the invariant mass of the charged lepton pair; we require $m_{\ell^+\ell^-} > 25\,GeV$ in order to eliminate the low invariant mass dileptons corresponding to the photon pole in these processes.

We evaluate the cross sections as a function of $\sqrt{s}$ for the processes $e^+e^- \to \ell^+\ell'^-\nu\bar{\nu}'$ for $\ell^+\ell'^-$ equal to $\mu^+\tau^-$, μ^+e^-, $\mu^+\mu^-$, e^+e^-, respectively, with the cuts as described above imposed. The sensitivity to κ_V increases with increasing center of mass energy. The $\mu\tau$ process exhibits the most sensitivity to κ_V, as might be expected since it has the least number of extraneous contributing diagrams; however, it also has the smallest cross section. Thus, it is useful to consider all the processes.

We make our study of κ_V dependence at two center of mass energies, $500\,GeV$ and $1\,TeV$, motivated by the possibility of future high energy e^+e^- colliders. For each of the four types of four lepton processes, at each of the two energies, we vary κ_γ alone from 0.9 to 1.1, κ_Z alone over the same range, and κ_γ constrained to equal κ_Z over the same range.

At $\sqrt{s}$ of $500\,GeV$, each process is more sensitive to deviations of κ_V below the standard model value of 1 than above it; however at the higher center of mass energy of $1\,TeV$, the sensitivity to κ_V is considerably more symmetric about 1.

Varying either κ_γ or κ_Z separately or setting them equal, the amplitude at each energy for each process can be expressed as $M = \alpha + \beta\kappa$; we have in each case fit a parabola for the cross section as a function of κ and solved for the cross section as a function of the two parameters κ_γ and κ_Z as

$$\sigma \sim |M|^2 = a + b\kappa_\gamma + c\kappa_Z + d\kappa_\gamma\kappa_Z + e\kappa_\gamma^2 + f\kappa_Z^2. \tag{21}$$

We turn these results into limits on the detection of deviations of κ_V from 1 by assuming an integrated luminosity of $50\, fb^{-1}$ for a proposed collider[36]. ¿From the total cross section of the individual processes, we find the following 2σ limits on measurements of κ_γ and κ_Z. At $\sqrt{s}$ of $500\, GeV$, κ_γ could be measured within $-2.5\%\,(\mu\tau)$ to $+9.5\%\,(ee)$ and κ_Z within the range $-6\%\,(\mu\tau,\ \mu e,\ \mu\ mu)$ to $+8\%\,(\mu e)$. At $1\, TeV$, the corresponding limits on κ_γ are $-1\%\,(\mu\tau)$ to $+3.5\%\,(\mu\tau,\ \mu e)$ and on κ_Z we find limits of $-1.5\%\,(\mu\tau)$ to $+2.5\%\,(\mu\tau)$. The channels given in brackets with each limit indicate which of the processes supplies the best bound. These particular limits simply represent the outer bound of the 2σ contour for the various processes. If one makes some assumptions about the relationship of κ_γ and κ_Z, such as that $\kappa_\gamma = \kappa_Z$ or that $\Delta\kappa_\gamma = \frac{2\cos^2\theta_W}{\cos^2\theta_W - \sin^2\theta_W}\Delta\kappa_Z$ [23], better bounds are obtained. In addition, combining the statistics from all the processes considered here would improve the bounds. In fact, one could also combine these four lepton processes with the similar jet channels such as $e^+e^- \to q\bar{q}'\ell\nu$. Combined bounds would necessitate inclusion of detector acceptances and efficiencies for the various particle types. We emphasize that even the bounds quoted above from the cross sections of individual processes are, indeed, approaching the very interesting realm of probing κ_V to within a few per cent of the standard model value. We note that it is particularly important to go to the higher energy in order to probe values of κ_V larger than 1.

We have also generated a number of distributions; these include the differential cross sections with respect to the angle of each charged lepton relative to the beam, the angle between the charged leptons, the energy and transverse momentum of each charged lepton, the total visible energy and transverse momentum and the invariant mass of the charged lepton pair. The charged lepton angular distributions tend to be strongly peaked along the beam line due to the t- channel neutrino exchange; they are generally enhanced somewhat away from the beam direction for nonstandard κ_V values. The energy and transverse momentum distributions of the individual particles tend to be enhanced over most of their range. The total visible transverse momentum is preferentially enhanced where the differential cross section is largest.

It now remains to determine whether we can pinpoint the values of κ_γ and κ_Z individually. There have been a number of approaches proposed for discriminating between deviations of κ_γ and κ_Z. One suggestion is to study processes which only involve one or the other of the γWW and ZWW vertices. The associated production of a W with either a γ or a Z boson, radiative W decay [28], and $e\gamma$ processes such as $e\gamma \to W\nu$ [37] fall into this category. Another suggestion is to make cuts which isolate one of the vertices. For instance,[38] has studied the $\mu^+\mu^-$ production process which we also consider here and have focussed on the ZWW vertex by requiring that the invariant mass of the $\mu^+\mu^-$ pair fall within $5\, GeV$ of M_Z. Here, we emphasize instead the potential usefulness of the helicity structure in providing a determination of κ_γ and κ_Z.

We observe that the $(-++-)$ [39] amplitude is suppressed at $\sqrt{s} \gg M_Z$ for $\kappa_\gamma = \kappa_Z$ as a direct result of the general form of this amplitude, which is given below.

$$M_{(-++-)} = \left[\frac{\kappa_\gamma + 1}{2s} - \frac{\kappa_Z + 1}{2(s - M_Z^2)}\right] A + B \tag{22}$$

Here A and B denote the κ_V dependent and independent factors, respectively, of the amplitude. For large center of mass energies, the cancellation of the κ_γ and κ_Z terms results in a $(-++-)$ helicity contribution of less than about one per cent of the total cross section for the standard model and for $\kappa_\gamma = \kappa_Z$ in general. On the other hand, for nonequal values of κ_γ and κ_Z, this contribution can be as much as 30% of the total. Thus, polarized beams accessing the individual helicity contributions could differentiate between the $\kappa_\gamma = \kappa_Z$ case and the nonequal case. Apart from the general observation described above regarding the case of κ_γ and κ_Z equal, experimental results on the cross sections for the four types of processes we consider with polarized and unpolarized beams could provide a characteristic 'fingerprint' for a $(\kappa_\gamma, \kappa_Z)$ pair. As an example of how this might work,for the $\mu\tau$ process at $1\,TeV$; we note that, for instance, $(\kappa_\gamma, \kappa_Z) = (0.945, 0.945), (1.07, 1.07), (1, 1.095), (1, 0.92)$, and $(0.92, 1)$ all have approximately the same total cross section. The percentage of the cross section supplied by the $(-++-)$ helicity is less than 1% for the two cases quoted with $\kappa_\gamma = \kappa_Z$; it is 3.6% for $(1, 1.095)$, 18% for $(1, 0.92)$, and 27% for $(0.92, 1)$. Since the total cross section for unpolarized beams corresponds to about 1000 events, these cases can possibly be discriminated providing reasonable polarization can be achieved. Similar results from the four types of processes can be combined to narrow in on the actual values of κ_γ and κ_Z, individually. The detailed study of distributions will also be useful as mentioned. We note that the processes considered here offer a very clean experimental signature for excellent sensitivity to κ_γ and κ_Z at a high energy e^+e^- collider.

8. Conclusions

The anomalous electroweak vector boson couplings have just started getting tested. The vector boson vertex can be written in a model independent way and related to the moments of the W boson. We have examined some direct and indirect (loop-induced) bounds on the TEVB couplings. However, these bounds are too weak to limit most models and probe loop level corrections to the standard model. There will be better bounds coming from CDF and D0 collaborations as well as from LEPII when it becomes operational. Bounds obtained from oblique parameters, obtained from LEPI data cannot rule out anomalous couplings even if the oblique parameters are in complete agreement with the SM. Future colliders will be needed to probe modifications to the electroweak vector bosons vertex from loop level corrections within the SM, as well as from most extensions of the SM. In this connection it is found that the Next Linear Collider running at $500 GeV$ and $1 TeV$ can bound these couplings accurately to probe at this level (less than 5%) and also allow for the possibility of separating the photon couplings from that of the Z, using polarized beams.

9. Acknowledgements

Due to time and space constraints justice cannot be done to the vast area of anomalous TEVB coupling in such a short writeup. I have chosen just a few topics that I have been of working on. Several important developments, especially in the area of searches for anamolous TEVB at other future Colliders and HERA have been left out. I would like to thank Nita Sinha for collaborating on most of the work presented here and for reading this manuscript I also thank my several other collaborators with whom parts of the work discussed here have been done. I would like to thank the National/International organizing committee of WHEPP-3 for their kind invitation to present this talk.

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