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The inclusive semileptonic B decay lepton spectrum from $B \to Xe\bar{\nu}$

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Abstract. In this talk, we review the QCD calculations of the lepton spectrum from inclusive semileptonic B decay. We compare this prediction to that of the ACCMM model. This latter work was done in collaboration with Csaba Csaki.

In this talk we discuss the calculation of the inclusive semileptonic decay lepton spectrum from $B \to Xe\bar{\nu}$. We first discuss the motivation for renewed interest in this calculation, and discuss the use of the OPE and HQET to determine this spectrum. We then compare this prediction to that of heavy quark models, in particular that of Altarelli, Cabbibo, Corbo, Maiani, and Martinelli [1] (hereafter referred to as ACCMM). I will concentrate on $B \to X_c e\bar{\nu}$ and not the detailed questions of the endpoint of the spectrum which are important for $B \to X_u e\bar{\nu}$.

There are several reasons for renewed interest in this prediction. First of all is the copious production of b quarks at LEP and CLEO. A good understanding of the decay spectrum is necessary for understanding fundamental b quark parameters, which are being precisely studied, such as the b width and the forward-backward asymmetry at LEP and those couplings relevant to the rare decays studied by CLEO. Furthermore, with a good understanding of the spectrum one can do detailed measurements of heavy quark fragmentation, which could give interesting tests of heavy quark theory and QCD predictions [2, 3]. Furthermore, in order to do high statistics measurements, one needs to do inclusive measurements. On the theoretical side, it is of interest to study the inclusive decays because they are under much better control theoretically than predictions for exclusive modes, where one needs to know hadronic matrix elements. An understanding of the lepton spectrum is therefore crucial to extracting fundamental parameters.

In the seminal paper of Chay, Georgi, and Grinstein [4] it was shown that one can treat the decay spectrum with the operator product expansion and heavy quark methods. They showed the leading Λ/m corrections to the free quark result vanish in the matrix elements. Subsequently, Bigi, Blok, Koryakh, Shifman, Uraltsev, and Vainshtein [5] studied $(\Lambda/m)^2$ corrections. They compared the results to a free quark, ACCMM model where they did not see the vanishing of the Λ/m corrections as manifest. Following, there were further calculations of $(\Lambda/m)^2$ suppressed effects by Falk, Luke, Savage [7] in the context of $b \to s\gamma$ and by Manohar and Wise [6] in the context of semileptonic decays. We will borrow heavily from these latter two papers in our review of the QCD calculation.

Defining $x = \frac{2E_e}{m}$, $\epsilon = \frac{m_f^2}{m_b^2}$ and $x_m = 1 - \epsilon$, the free quark decay spectrum is given by

$$\frac{d\Gamma(m_b,E)}{dx} = \frac{G_F^2 m^5}{96\pi^3}\frac{x^2(x_m-x)^2}{(1-x)^3}\left[(1-x)(3-2x)+(1-x_m)(3-x)\right] \tag{1}$$

The idea is then to justify and improve on this free quark result. For the study of the inclusive decay, we sum over all final states with given quantum numbers. The spectrum and rate of the inclusive decay is then governed by short distance physics. Intuitively,the justification is that in the $m \to \infty$ limit, the decay time is much shorter than hadronization time scale. With the OPE and HQET, this can be made precise. The crucial observation is that there is a large range of mass scales in the final state $m_D^2 \le P_X^2 \le m_B^2$ so that the energy flowing through the hadron system scales with the mass of the decaying heavy quark. In the $m \to \infty$ limit, this is much larger than the QCD scale. Away from $P_X^2 \approx m_D^2$, the internal quark is far from mass shell. In this case, we expect the OPE to be useful; it will prove valid except near the endpoint.

The general idea is to relate the square of the matrix element of interest to the imaginary part of the time ordered product of currents. One can then perturbatively compute the time ordered product with the operator product expansion (in the region of phase space where it is valid). However unlike a standard OPE it is useful to expand in the heavy quark mass rather than q^2 in order to apply HQET. The coefficients of the heavy quark operators are then obtained by evaluating the time ordered product between quark and gluon states. The time ordered product is evaluated in the end by taking matrix elements of the operators appearing in the OPE between meson states. One can use the HQET to learn about these matrix elements.

Semileptonic decay is determined from the weak hamiltonian density:

$$H_W = -V_{jb}\,\frac{4G_F}{\sqrt{2}}\,\bar{q}_j\gamma^\mu P_L b\,\bar{e}\gamma_\mu P_L\nu_e = -V_{jb}\,\frac{4G_F}{\sqrt{2}}\,J_j^\mu J_{\ell\mu}, \tag{2}$$

The inclusive differential decay rate for $H_b \to X_{u,c}e\overline{\nu_e}$ (here we will be restricting attention to meson decay) is governed by

$$W_j^{\mu\nu} = (2\pi)^3\sum_X \delta^4\left(p_{H_b}-q-p_X\right)\langle H_b(v,s)|J_j^{\mu\dagger}|X\rangle\,\langle X|\,J_j^\nu\,|H_b(v,s)\rangle \tag{3}$$

where

$$p_{H_b} = M_{H_b}v^\mu = m_b v^\mu + k^\mu \tag{4}$$

The $W^{\mu\nu}$ can be expanded in terms of five form factors

$$W^{\mu\nu} = -g^{\mu\nu}W_1 + v^\mu v^\nu W_2 - i\epsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta W_3 + q^\mu q^\nu W_4 + \left(q^\mu v^\nu + q^\nu v^\mu\right)W_5, \tag{5}$$

One then relates this to a time ordered product via Im $T^{\mu\nu} = -\pi W^{\mu\nu}$ where

$$\begin{aligned} T^{\mu\nu} &= -i\int d^4x\; e^{-iq\cdot x}\sum_s\langle H_b(v,s)|T\left(J^{\mu\dagger}(x)\,J^\nu(0)\right)|H_b(v,s)\rangle \\ &= -g^{\mu\nu}T_1 + v^\mu v^\nu T_2 - i\epsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta T_3 + q^\mu q^\nu T_4 + \left(q^\mu v^\nu + q^\nu v^\mu\right)T_5. \end{aligned}$$

Let us consider the analytic structure. There are cuts corresponding to the decay of interest, to the process $e\bar{\nu}H \to X$, and to $e^+\nu_e H \to X$. The idea is then to do a perturbative calculation away from the physical cut, which is always possible away from the endpoint.

As an example,we find the leading order result. This is done explicitly in [6]. The matrix element of the time Fourier transformed, time ordered product of currents

$$-i\int d^4x e^{-iq\cdot x}T(J^{\mu\dagger}J^{\nu}) \tag{6}$$

is

$$\frac{1}{(m_b v-q+k)^2-m_j^2+i\epsilon}\,\overline{u}\,\gamma^{\mu}\,P_L\,\left(m_b\not{v}-\not{q}+\not{k}+m_j\right)\,\gamma^{\nu}\,P_L\,u, \tag{7}$$

Using

$$\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu}=g^{\mu\alpha}\gamma^{\nu}+g^{\nu\alpha}\gamma^{\mu}-g^{\mu\nu}\gamma^{\alpha}+i\epsilon^{\mu\nu\alpha\beta}\gamma_{\beta}\gamma_5. \tag{8}$$

one finds the order k^0 term is

$$\frac{1}{\Delta_0}\quad\overline{u}\Big\{\,(m_b v-q)^{\mu}\,\gamma^{\nu}+(m_b v-q)^{\nu}\,\gamma^{\mu}-(m_b\not{v}-\not{q})\,g^{\mu\nu}$$
$$-i\epsilon^{\mu\nu\alpha\beta}\,(m_b v-q)_{\alpha}\,\gamma_{\beta}\,\Big\}P_L\,u, \tag{9}$$

where

$$\Delta_0=(m_b v-q)^2-m_j^2+i\epsilon \tag{10}$$

By replacing the matrix element with the b quark operator which yields this amplitude when evaluated between b quark states, one obtains

$$\frac{1}{\Delta_0}\quad\Big\{(m_b v-q)^{\mu}\,g^{\nu\lambda}+(m_b v-q)^{\nu}\,g^{\mu\lambda}-(m_b v-q)^{\lambda}\,g^{\mu\nu}$$
$$-i\epsilon^{\mu\nu\alpha\lambda}\,(m_b v-q)_{\alpha}\Big\}\,\overline{b}\,\gamma_{\lambda}P_L\,b.$$

Because it is a current, one has

$$\langle H_b(v,s)|\,\overline{b}\,\gamma^{\lambda}\,b\,|H_b(v,s)\rangle=v^{\lambda}, \tag{11}$$

since b quark number is an exact symmetry.

So we can evaluate the leading order contribution *exactly*. By taking the matrix element of the OPE between H states, we get

$$T_1^0 = \frac{1}{2\Delta_0}(m_b-q\cdot v)$$
$$T_2^0 = \frac{1}{\Delta_0}m_b$$
$$T_3^0 = \frac{1}{2\Delta_0}$$

So to get the amplitude $W^{\mu\nu}$ which determines the spectrum, we need the imaginary part of $T^{\mu\nu}$. This is readily obtained from

$$\frac{1}{\Delta_0} = \delta((m_b v-q)^2-m^2) \tag{12}$$
$$\frac{1}{\Delta_0^2} = -\delta'((m_b v-q)^2-m^2) \tag{13}$$
$$\frac{1}{\Delta_0^3} = \frac{1}{2}\delta''((m_b v-q)^2-m^2) \tag{14}$$

(Here we only use the first one for the leading order term)

The mass shell condition above when integrated over phase space leads to just the free quark decay distribution!

However, with these methods, one can also consider higher order corrections. The important result will be that Λ_{QCD}/m corrections vanish and only two matrix elements, one of which is known, are required to obtain $(\Lambda_{QCD}/m)^2$ corrections

We first introduce heavy quark fields and the heavy quark effective theory. Recall that in the heavy quark limit, the heavy quark mass and spin decouple from the soft degrees of freedom. The heavy quark state looks like a static point source of charge. The b quark is almost on mass shell $p = m_b v + k$.

Define the heavy quark field

$$b_v = \frac{(1+\not{v})}{2} e^{im_b v\cdot x} b + \ldots, \tag{15}$$

with leading order lagrangian

$$\mathcal{L} = \bar{b}_v\,(iv \cdot D)\, b_v + \ldots. \tag{16}$$

Now expand in terms of HQET operators. We follow [7] who give the leading term in terms of the standard quark field, but the mass suppressed operators in terms of heavy quark fields.

$$T\{O^\dagger, O\} \overset{\text{OPE}}{=} \frac{1}{m_b}\left[O_0 + \frac{1}{2m_b}O_1 + \frac{1}{4m_b^2}O_2 + \ldots\right] \tag{17}$$

$$\begin{aligned} O_0 &= \bar{\mathrm{b}}\,\Gamma\,\mathrm{b}\,, \\ O_1 &= \bar{b}_v\,\Gamma\,\mathrm{i}\mathrm{D}_\mu b_v\,, \\ O_2 &= \bar{b}_v\,\Gamma\,\mathrm{i}\mathrm{D}_\mu \mathrm{i}\mathrm{D}_\nu b_v \end{aligned}$$

As before, at leading order, we have

$$\langle B|\,\bar{\mathrm{b}}\gamma^\mu \mathrm{b}\,|B\rangle = 2P_B^\mu \tag{18}$$

Now consider the first order mass suppressed terms.

$$\langle M|\,O_1\,|M\rangle = \langle M|\,\bar{h}\,\Gamma\,\mathrm{i}\mathrm{D}_\mu h\,|M\rangle = \langle M|\,\bar{h}\,\Gamma v_\mu v\cdot \mathrm{i}\mathrm{D}h\,|M\rangle \tag{19}$$

This vanishes by the equation of motion! Notice this critically depends on our phase choice in defining heavy quark field.

An arbitrary counterterm

$$\delta m \overline{b_v} b_v \tag{20}$$

was taken to vanish. With this choice of quark mass, all linear corrections to the free quark result vanish!

At higher order in $1/m$, one needs the two matrix elements

$$\begin{aligned} K_b &\equiv -\langle H_b(v,s)|\,\bar{b}_v \frac{(iD)^2}{2m_b^2}\, b_v\,|H_b(v,s)\rangle \\ G_b &\equiv Z_b\,\langle H_b(v,s)|\,\bar{b}_v\,\frac{gG_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b^2}\, b_v\,|H_b(v,s)\rangle\,, \end{aligned} \tag{21}$$

The second matrix element is known because it is related to the known spin dependent mass splittings. The first is not known, and is a parameter to be determined.

So, to summarize, the leading order result for the rate and spectrum is the free quark result. The first order corrections vanish. At second order, there is one unknown coefficient and one which has been determined. This analysis has assumed we are far from endpoint, where the OPE is valid. Had we applied this near the endpoint, higher dimension operators would not be suppressed and it would be necessary to average the spectrum over a range of energies.

One can compare this result to the predictions of models. It is interesting to see how models reflect and differ from these general QCD predictions. Consider for example the ACCMM model. Here I review work of Csaba Csaki and myself [8], but other interesting references are that by Grant Baillie [9] and Ref. [10]. One finds that one can always *define* a quark mass so that $1/m$ corrections vanish. However, at $1/m^2$, the model would differ from the QCD result.

In the ACCMM model, one models a B meson decay as *disintegration*. There is a spectator quark with mass m_{sp} and momentum distribution $\phi(|p|)$.

$$\phi(|p|) = \frac{4}{\sqrt{\pi} p_f^3} \exp\left(-\frac{|p|^2}{p_f^2}\right) \tag{22}$$

The b quark momentum is determined by kinematic constraints. The lepton spectrum is determined from the decaying off shell b quark. Define

$$E_W = M_B - \sqrt{p^2 + m_{sp}^2} \tag{23}$$

The invariant mass of the b quark will then be

$$W^2 = m_B^2 + m_{sp}^2 - 2m_B\sqrt{p^2 + m_{sp}^2} \tag{24}$$

Define

$$\begin{aligned} x &= \frac{2E_e}{m} \\ \epsilon &= \frac{m_f^2}{m_b^2} \\ x_m &= 1 - \epsilon \end{aligned}$$

Recall the formula for free quark decay.

$$\frac{d\Gamma(m_b, E)}{dx} = \frac{G_F^2 m^5}{96\pi^3} \frac{x^2(x_m - x)^2}{(1-x)^3} \left[(1-x)(3-2x) + (1-x_m)(3-x)\right] \tag{25}$$

Then the lepton spectrum in the ACCMM model is

$$\begin{aligned} \frac{d\Gamma_B}{dE} &= \int_0^{p_{max}} dp p^2 \phi(|p|) \int \frac{1}{\gamma} \frac{d^2\Gamma(W, E')}{dE' d\cos\theta} dE' \times \\ & \qquad d\cos\theta \int \frac{d\cos\theta_p}{2} \delta(E - \gamma E' - \gamma\beta E' \cos\theta_p) \end{aligned}$$

$$\frac{d\Gamma_B}{dE} = \int dpp^2\phi(|p|)\frac{1}{2\beta\gamma^2}\int\frac{d^2\Gamma}{dE'd\cos\theta}d\cos\theta'\frac{dE'}{E'} \tag{26}$$

Using $\beta\gamma^2 = pE_W/W(p)^2$ yields

$$\frac{d\Gamma_B}{dE} = \int dpp^2\phi(|p|)\frac{W^2}{2pE_W}\int_{E_-}^{E_{max}}\frac{dE'}{E'}\frac{d\Gamma(W,E')}{dE'} \tag{27}$$

where

$$E^{\mp} = \frac{EW}{E_W \pm p} \tag{28}$$

$$E_{max} = min\{E_+, \frac{W}{2}(1-x_m)\} \tag{29}$$

$$p_{max} = \frac{m_B}{2} - \frac{m_f^2}{m_B^2} \tag{30}$$

Now do a heavy meson mass expansion.

$$\frac{d\Gamma_B}{dE} = \int_0^{p_{max}}\phi(|p|)p^2\frac{W^2}{2pE_W}\frac{G_F^2W^4}{48\pi^3}\int_{E_-}^{E_+}\frac{dE'}{E'}\left(\frac{2E}{W}\right)^2\left(3-\frac{4E'}{W}\right) \tag{31}$$

We can explicitly evaluate the E' integral, to get

$$\frac{d\Gamma_B}{dE} = \frac{G_F^2E^2}{24\pi^3}\int_0^{p_{max}}\frac{dp}{E_W}\left(6E_WW^2 - 8EE_W^2 - \frac{8}{3}p^2E\right)p^2\phi(|p|) \tag{32}$$

Take p_f small.

$$\frac{d\Gamma_B}{dE} = \frac{d\Gamma_B^0}{dE} + \frac{G_F^2E^2M_B^2}{12\pi^3\sqrt{\pi}}\left(8\frac{E}{m_B} - 12\right)\frac{p_f}{m_B} + P(\frac{p_F^2}{m_B})^2\frac{d\Gamma_q}{dE} \tag{33}$$

It looks like there are nonvanishing linear corrections, but that is because we expanded in the meson mass m_B, rather than the quark mass m_b. To do a heavy quark expansion, one needs to *define* a quark mass. This can be done so that the linear correction vanishes.

$$\frac{d\Gamma_B}{dE} = \frac{G_F^2E^2}{24\pi^3}\int_0^{p_{max}}\frac{dp}{E_W}\left(6E_WW^2 - 8EE_W^2 - \frac{8}{3}p^2E\right)p^2\phi(|p|) \tag{34}$$

If we define

$$m_b = \langle W(p)\rangle \tag{35}$$

Up to quadratic terms in p,we have

$$\langle W\rangle = \langle E_W\rangle = m_b = m_B - \langle p\rangle \tag{36}$$

$$\langle f(W,E_W)\rangle = f(m_B,m_B) - \langle p\rangle f'(m_B,m_B) + O(p^2) = f(m_b,m_b) + O(p^2) \tag{37}$$

We see that it was possible to define a b quark mass so that linear corrections vanished because the p dependence was all through E_W. So we can eliminate linear corrections through a proper choice of m_b. It is easy to extend this to nonzero spectator mass.

$$\langle E_W\rangle = m_B - \langle\sqrt{m_{sp}^2 + p^2}\rangle \tag{38}$$

Now consider the total rate and the full spectrum.

$$\int \frac{d\Gamma_B}{dE} = \int dpp^2 \int \frac{d\cos\theta_p}{2} \int dE' \frac{d\Gamma(W,E')}{E'} \delta(E - \gamma E' - \gamma\beta E' \cos\theta_p) \tag{39}$$

However, the spectra themselves are very different near the endpoint.

$$\begin{aligned} \frac{d\Gamma_B}{dE} &= \int_0^{p_1} dpp^2 \phi(|p|) \frac{W}{2p} \int_{E_-}^{E_+} \frac{d\Gamma(W,E')}{E'} \\ &+ \int_{p_1}^{p_{max}} dpp^2 \phi(|p|) \frac{W}{2p} \int_{E_-}^{E_1} \frac{d\Gamma(W,E')}{E'} \end{aligned}$$

where $E_1 = W(1-\epsilon)/2$, $p_1 = m_B/2(1 - \frac{m_f^2}{m_B^2} - E$ for $m_{sp} = 0$. This spectrum extends up to $E_{max} = \frac{m_B - m_{sp}}{2}\left(1 - \frac{m_f^2}{(m_B - m_{sp})^2}\right)$

We see the ACCMM spectrum and the free quark spectrum deviate due to two effects near the endpoint. The first term is not integrated up to $p \approx \infty$ and the second term integrates over a rapidly falling spectrum so it is no longer a good approximation to replace $\langle f(W) \rangle$ by $f(\langle W \rangle)$. The spectrum extends beyond naive quark mass endpoint.

Since we have a specific model, we can also investigate the question of how large an energy interval must be averaged over to get good agreement between the model and the free quark prediction. In ref. [8] we used the averaging function

$$\frac{d\Gamma}{dE}(E_0) = \int \frac{1}{\sqrt{\pi}\Delta E} e^{-\left(\frac{(E-E_0)^2}{\Delta E^2}\right)} \frac{d\Gamma(E)}{dE}(E) \tag{40}$$

We expect to require an average of approximately $2p_f$ to get $(p_f/m)^2$ agreement. This was appproximately correct. We also checked that the best fit quark mass is generally very close to the exact quark mass, so the deviation of the model from the free quark prediction is probably not very important for practical purposes.

At higher order, the predictions of this model will not agree with general QCD predictions. This is because gauge invariance is not incorporated. There are corrections to the free quark result due to the fact that terms proportional to $\langle p_0^2 \rangle$ and $\langle \vec{p}^2 \rangle$ both yield $1/m^2$ corrections. Recall that in the heavy quark theory the operator $\langle D_0^2 \rangle$ contributions only at higher order in the heavy quark mass expansion by the equation of motion. In the ACCMM model, both contribute at the same order. Obviously, the ACCMM model also doesn't give you spin dependent gluon operator. We conclude that although the model is probably adequate in practice (as would be free quark decay) it does not properly incorporate QCD dynamics.

We conclude that there has been much advancement in our understanding of the lepton decay spectrum. It is unclear at what level this will be tested. Most of the large deviations from the free quark prediction are in the endpoint region, where one must smear to get reliable predictions. Furthermore, the higher order correction involves an unknown parameter and requires very accurate measurements. However, from a practical point of view, when using the spectrum to extract b quark couplings, we see that the free quark decay models b quark decay very well. It might be better to fit the spectrum to a free quark spectrum that that of a model, such as the ACCMM model. The vanishing of Λ_{QCD}/m corrections is very important to this conclusion.

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