PRAMANA
— journal of
physics

Supplement to Vol. 45
October 1995
pp. 209-228

Threshold effects in SUSY and non-SUSY GUTs

M. K. PARIDA
Physics Department, North-Easatern Hill University,
P. O. Box 21, Laitumkhrah, Shillong 793 003.

Abstract. We discuss recent contributions on threshold effects in grand unfiied theories including minimal SUSY SU(5), non-SUSY modifications of the grand desert in SU(5) and SO(10), and SO(10) with single intermediate symmetires. Consequences of theorems on vanishing GUT-scale corrections to $\sin^2\theta_W$ in SO(10) with $SU(2)_L$ X$SU(2)_R$ X$SU(4)_C$ ($g_{2L} = g_{2R}$) intermediate symmetry are discussed and vanishing corrections on the intermediate scale are explicitly demonstrated where predictions are more precise. Threshold and higher dimensional operator effects in SUSY SU(5) recently derived by a number of authors are presented.

1. Introduction

Experimental measurements at the CERN-LEP at the Z-peak and improved estimation of the finestructure constant have rpovided more precise values of $\sin^2\theta_W, \alpha_S$ and α^{-1},

$$\begin{array}{lcccc} \sin^2\theta_W & = & 0.2324 & \pm & 0.0006 \\ \alpha_{s_{-1}} & = & 0.12 & \pm & 0.01 \\ \alpha & = & 127.9 & \pm & 0.2 \end{array} \tag{1}$$

leading to very accurate determination of the standard model gauge couplings,

$$\begin{array}{lcccc} \alpha_1 & = & 0.016887 & \pm & 0.00004 \\ \alpha_2 & = & 0.03322 & \pm & 0.000025 \\ \alpha_3 & = & \alpha_S & & \end{array} \tag{2}$$

at the Z-mass. These have revived interests in grand unified theores [1] resulting in more precise predictions than before[2]. In addition to one- and two-loop contributions to the gauge-symmetry-breaking scales threshold [3] and higher dimensional operator effects [4] have been estimated and their impact on GUT-prediction on $\sin^2\theta_W$,α_S, proton lifetime, and neutrino masses have been calculated. It has also been found in a class of GUTs that in addition tot he vanishing of multiloop corrections at high mass scales, the unknown uncertainties on $\sin^2\theta_W$ and the intermediate scale due to threshold and higher dimensional operator effects are absent leading to more precise predictions[4-6]. The purpose of the present talk is to review these works and other corrections obtained in SUSY and nonSUSY GUTS. In addition we demonstrate explicitly the stability of the G_{224P}-breaking scale under threshold and higher dimensional operator effects leading to a more precise predictions of degenerate and see-saw contributions to neutrino masses in a class of GUTs.

The plan of the talk is organised in the following manner. In Sec.1 we obtain generalised formulas for mass scales and threshold effects. In Sec.2 we discuss in detail the threshold effects in modified grand desert models such as SU(5) and SO(10). In Sec.3 we derive threshold corrections in modified grand desert models [7,8]. In Sec.4 we estimate threshold effects in SO(10) with G_{224p} intermediate symmetry. Threshold effects in other single intermediate scale models of SO(10) are summarized in Sec.5. Section 6 is devoted to discussions on corrections in SUSY SU(5). We provide a brief summary and conclusions in Sec.7.

2. Generalized formulas for mass scales

Although minimal nonSUSY SU(5) is ruled out experimentally, the modified grand desert models based upon SU(5), SO(10) and others with light degrees of freedom are consistent with all the available data. Evidences on neutrino masses would rule out SU(5)-based modifications of the grand desert, but a number of other GUT would survive. In the absence of theoretical formulas for the mass scales or $\sin^2\theta_W$ it might be still possible to study the unification constraints by plotting the coupling constants numerically, but accurate estimations of the model predictions including experimental and theoretical uncertainties analytic formulas are essential. Such formulas can be obtained more easily for single step breaking of any GUT to the standard with or without SUSY[9,10]. We derive here generalised theoretical formulas for mass scales for modified grand desert models and two step breakings in GUTs. These formulas contain explicitly the loop contributions to each order although at present contributions upto two-loops only are calculated. More important is that the formulas have analytic expressions for different types of corrections arising at the intermediate or the GUT-scales[4.7,8,13].

At first we consider the following class of models,

$$(a) \qquad SO(10) \xrightarrow{M_U} G_{213} \xrightarrow{M_Z} G_{13}$$

where $G_{213}(= SU(2)_L X U(1)_Y X SU(3)_C)$ is the standard model (SM), $G_{13} = U(1)_{em} X SU(3)_{C'}$ and the presence of light degrees of freedom corresponding to additional Higgs scalars or fermions has been assumed. We also consider the following class of models based upon $SO(10)$,

$$(b) \qquad SO(10) \xrightarrow{M_U} G_I \xrightarrow{M_I} G_{213} \xrightarrow{M_W} G_{13}$$

where $G = SU(2)_L X U(1)_{I_{3R}} X SU(4)_C (= G_{214})$[12], $SU(2)_L X SU(2)_R X SU(4)_C$ $(= G_{224}, g_{2L} = g_{2R}), SU(2)_L X SU(2)_R X U(1)_{B-L} X SU(3)_C (= G_{2213}, g_{2L} = g_{2R})$[11], $G_{2213P}(= G_{2213} X P, P =$ Parity $=$ Left-right discrete symmetry, $g_{2L} = g_{2R}), G_{224P}$ $(G_{224} X P, g_{2L} = g_{2R})$. It may be noted that SO(10) also permits the left-right discrete symmetry to be broken at a higher scale without breaking $SU(2)_R X U(1)_{B-L}$ or $SU(2)_R X SU(4)_C$ which can survive down to lower scales as in the cases of G_{2213} or G_{224}. In such a situation the parity and the $SU(2)_{2R}$-breakings are decoupled [11].

In models of the type (a) or (b) the renormalisation group equations (RGEs) for the gauge couplings of G_{213} or G_I can be written in the following manner in the two mass ranges,

$M_Z < \mu < M_I$

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_I)} + \frac{a_i}{2\pi} ln \frac{M_I}{M_Z} + \frac{P_i^I}{4\pi} - \frac{L_i^I}{12\pi} \quad (3)$$

In model (a) M_I = degenerate mass of additional degrees of freedom in the desert.

$M_I < \mu < M_U$

$$\frac{1}{\alpha_i(M_I)} = \frac{1}{\alpha_G} + \frac{a_i'}{2\pi} ln \frac{M_U}{M_I} + \frac{P_i^U}{4\pi} - \frac{L_i^U}{12\pi} \quad (4)$$

where α_G is the GUT-coupling constant and the two-loop contributions are represented by terms containing P_i-functions,

$$\begin{aligned} p_i^I &= \textstyle\sum_j B_{ij} ln \frac{\alpha_j(M_I)}{\alpha_j(M_Z)} \quad , \quad B_{ij} = b_{ij}/a_j \\ p_i^U &= \textstyle\sum_j B_{ij}' ln \frac{\alpha_j(M_U)}{\alpha_j(M_I)} \quad , \quad B_{ij}' = b_{ij}'/a_j' \end{aligned} \quad (5)$$

In (3)-(5) $a_i(b_{ij})$ and $a_i'(b_{ij}')$ are the one(two)-loop coefficients of the β-function in the two mass ranges. The λ_i-functions in (3) and (4) represent threshold effects due to heavy or superheavy particles whose masses are of the order M_I or M_U. But as these masses are unknown theoretically, the threshold effects due to heavy or superheavy particles whose masses are of the order M_I or M_U. But as these masses are unknown theoretically, the threshold effects contribute to the uncertainties of the model predictions on the mass scales, $\sin^2\theta_W$, and α_S etc. For $\mu \approx M_I$ of M_U, the general expression for $\lambda_i(\mu)$ in nonSUSY theories is[14],

$$\begin{aligned} L_i(\mu) &= Tr(t_{iV}^2) + \lambda_i(\mu) \\ \lambda_i(\mu) &= -21 Tr(t_{iV}^2 ln \frac{M_i^V}{\mu}) + 8 Tr(t_{iF}^2 ln \frac{M_i^F}{\mu}) + Tr(t_{iS}^2 ln \frac{M_i^S}{\mu} \end{aligned} \quad (6)$$

where t_{iV}, t_{iF}, and t_{iS} are the generators in the representations of the heavy or superheavy gauge bosons(V), fermions(F), and scalars(S), respectively. In our notation

$$\lambda_i^I = \lambda_i(\mu = M_I), \quad \lambda_i^U = \lambda_i(\mu = M_U) \quad (7)$$

Physically the first two terms are the vacuum polarisation contribution of the heavy gauge bosons(V) and the next two are similar contributions due to fermions(F) and Higgs scalars(S) in the theory. Using (3) and (4) and suitable linear combinations of the SM gauge couplings we derive and following generalised formulas for mass scales for models(a) and (b),

$$ln \frac{M_U}{M_Z} = [(L_S B_I - L_\theta A_I) + (J_2 B_I - K_2 A_I) + (K_\lambda A_I - J_\lambda B_I)]/D \quad (8)$$

$$ln \frac{M_I}{M_Z} = [(A_U L_\theta - B_U L_S) + (K_2 A_U - J_2 B_U) + (J_\lambda B_U - K_\lambda A_U)]/D \quad (9)$$

where

$$D = A_U B_I - A_I B_U \ . \ L_S = \frac{16\pi}{3}\left(\alpha_S^{-1} - \frac{3}{8}\alpha^{-1}\right)$$

$$L_\theta = \frac{16\pi}{3\alpha}\left(\sin^2\theta_W - \frac{3}{8}\right)$$

In (8) and (9) the first, second, and the thrid terms represent the one-loop, two-loop and the threshold contributions, respectively and the values A_i, B_i, J_i and K_i differ from one model to the other,

MODEL (a)

$$\begin{aligned}
A_I &= a_{2L} + \tfrac{5}{3}a_Y - \tfrac{8}{3}a_{3C} - \left(a'_{2L} + \tfrac{5}{3}a'_Y - \tfrac{8}{3}a'_{3C}\right) \\
B_I &= \tfrac{5}{3}(a_Y - a_{2L} - a'_Y + a'_{2L} \\
A_U &= a'_{2L} + \tfrac{5}{3}a'_Y - \tfrac{8}{3}a'_{3C} \\
B_U &= \tfrac{5}{3}(a'_Y - a'_{2L}) \\
J_2 &= \tfrac{1}{2}[(P^I_{2L} + \tfrac{5}{3}P^I_Y - \tfrac{8}{3}P^I_{3C}) + (P^U_{2L} + \tfrac{5}{3}P^U_Y - \tfrac{8}{3}P^U_{3C})] \\
K_2 &= \tfrac{5}{6}(P^I_Y - P^I_{2L}) + \tfrac{5}{6}(P^U_Y - P^U_{2L}) \\
J_\lambda &= \tfrac{1}{6}(L^U_{2L} + \tfrac{5}{3}L^U_Y - \tfrac{8}{3}L^U_{3C}) \\
K_\lambda &= \tfrac{5}{18}(L^U_Y - L^U_{2L})
\end{aligned} \qquad (10)$$

MODEL (b)

(i) $G_I = G_{224}$ or G_{224P}

$$\begin{aligned}
A_I &= \tfrac{8}{3}a_{3C} - a_{2L} - \tfrac{5}{3}a_Y - 2a'_{4C} + a'_{2L} + a'_{2R} \\
B_I &= \tfrac{5}{3}(a_{2L} - a_Y) + a'_{2R} + \tfrac{2}{3}a'_{4C} - \tfrac{5}{3}a'_{2L} \\
A_U &= 2a'_{4C} - a'_{2L} - a'_{2R} \\
B_U &= \tfrac{5}{3}a'_{2L} - a'_{2R} - \tfrac{2}{3}a'_{4C} \\
J_2 &= \tfrac{1}{2}(P^I_{2L} + \tfrac{5}{3}P^I_Y - \tfrac{8}{3}P^I_{3C}) + \tfrac{1}{2}(P^U_{2L} + P^U_{2R} - 2P^U_{4C}) \\
K_2 &= \tfrac{5}{6}(P^I_Y - P^I_{2L}) + \tfrac{1}{2}(P^U_{2R} + \tfrac{2}{3}P^U_{4C} - \tfrac{5}{3}P^U_{2L}) \\
J_\lambda &= \tfrac{1}{6}(L^U_{2L} + L^U_{2R} - 2L^U_{4C}) + \tfrac{1}{6}(L^I_{2L} + \tfrac{5}{3}L^I_Y - \tfrac{8}{3}L^I_{3C}) \\
K_\lambda &= \tfrac{1}{6}(L^U_{2R} + \tfrac{2}{3}L^U_{4C} - \tfrac{5}{3}L^U_{2L}) + \tfrac{5}{18}(L^I_Y - L^I_{2L})
\end{aligned} \qquad (11)$$

(ii) $G_I = G_{2213}$ or G_{2213P}

Generalised formulas for mass scales are obtained from (11) of model (b) (i) by the replacements,

$$2a'_{4C} \longrightarrow \tfrac{8}{3}a'_{3C} - \tfrac{2}{3}a'_{BL} \;,\; \tfrac{2}{3}a'_{4C} \longrightarrow \tfrac{2}{3}a'_{BL}$$
$$2L^U_{4C} \longrightarrow \tfrac{8}{3}L^U_{3C} - \tfrac{2}{3}L^U_{BL} \;,\; \tfrac{2}{3}L^U_{4C} \longrightarrow \tfrac{2}{3}L^U_{BL} \qquad (12)$$
$$2P^U_{4C} \longrightarrow \tfrac{8}{3}P^U_{3C} - \tfrac{2}{3}P^U_{BL} \;,\; \tfrac{2}{3}P^U_{4C} \longrightarrow \tfrac{2}{3}P^U_{BL}$$

(iii) $G_I = G_{214} = SU(2)_L X U(1)_{I_{3R}} X SU(4)_C$

Generalised formulas for mass scales are obtained from (7), (8) and (10) by replacing

$$a'_{2R} \longrightarrow a'_{1R} = a'_{I_{3R}}$$
$$P^U_{2R} \longrightarrow P^U_{1R} \;,\; L^U_{2R} \longrightarrow L^U_{1R} \qquad (13)$$

(iv) $G_I = G_{2113} = SU(2)_L X U(1)_{I_{3R}} X U(1)_{B-L} X SU(3)_C$

The generalised formulas in this case are obtained by combining (12) and (13).

Before closing this section it is worth pointing out that other corrections due to Yukawa couplings, top quark mass, higher dimensional operators etc. can be derived following the same procedure as threshold effects with appropriate replacements of the λ-functions.

3. Threshold effects in modified grand desert models

It has been found that the SU(5) model with additional light degrees of freedom corresponding to fermions or Higgs scalars is consistent with the CERN-LEP data and proton lifetime for the $p \longrightarrow e^+\pi^0$ mode [8,15-160,

$$(\tau_p)_{expt.} > 3 \times 10^{32}\ yrs. \qquad (14)$$

The added presence of new degrees of freedom such as the Higgs scalars significantly below M_U needs additional finetuning of parameters which is not natural. In order to keep this unnatural act to a minimum we have successfully implemented new minimal modification of the grand desert which needs only one additional SM irreducible real scalar representation transforming as ζ(3,0,8) and needs only one additional finetuning of parameters [7]. Moreover the model has been implemented in SU(5) and the popular GUT like SO(10) thereby accounting for the CERN-LEP data, the proton lifetime and small neutrino masses necessary to explain the solar neutrino flux and the dark matter of the Universe in the latter case. The additional Higgs scalar ζ(3,0,8) C **75** of SU(5) and **210** of SO(10). Thus the implementation needs the Higgs representations **24**, 5 and **75** in SU(5) and **45, 126, 10** and **210** in SO(10). Thus keeping $M_Z < M_I = M_\zeta < M_U$ and evaluating the coefficients in (7)-(10) yields the following analytic expressions for the mass scales.

$$\begin{aligned} ln\frac{M_U}{M_Z} &= \frac{16\pi}{187\alpha}\left(\frac{7}{8} - \frac{10\alpha}{3\alpha_S} + \sin^2\theta_W\right) + \frac{5}{187}\left[\frac{8}{3}(P^U_3 + P^\zeta_3)\right. \\ &- \left.\frac{3}{2}(P^U_2 + P^\zeta_2) - \frac{7}{6}(P^U_1 + P^\zeta_1)\right] + \frac{5}{3366}(7\lambda_1 \\ &+ 9\lambda_2 - 16\lambda_3) + \frac{25}{561} \end{aligned} \qquad (15)$$

$$
\begin{aligned}
ln\frac{M_\zeta}{M_Z} &= \tfrac{4\pi}{187\alpha}(15-\tfrac{23\alpha}{\alpha_S}+63\sin^2\theta_W)+\tfrac{3}{187}[16(P_2^U+P_2^\zeta) \\
&- \tfrac{23}{3}(P_3^U+P_3^\zeta)-\tfrac{25}{3}(P_1^U+P_1^\zeta)]+\tfrac{1}{561}(25\lambda_1-48\lambda_2 \\
&+ 23\lambda_3)+\tfrac{9}{187}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\frac{1}{\alpha_G} &= \tfrac{3}{8\alpha}+\tfrac{1}{187\alpha}(\tfrac{347}{8}+\tfrac{466\alpha}{3\alpha_S}-271\sin^2\theta_W)-\tfrac{1}{4488\pi}[932(P_3^U+P_3^\zeta) \\
&- 945(P_2^U+P_2^\zeta)+1135(P_1^U+P_1^\zeta)]+\tfrac{1}{13464\pi}(1135\lambda_1-945\lambda_2 \\
&+ 932\lambda_3)+\tfrac{196}{561\pi}
\end{aligned} \tag{17}
$$

where the invrse of the GUT-coupling constant α_G^{-1} has been derived from the finestructure constant relation,

$$\frac{1}{\alpha(M_Z)} = \frac{5}{3\alpha_1(M_Z)}+\frac{1}{\alpha_2(M_Z)} \tag{18}$$

and the RGEs for the gauge couplings. Clearly the threshold effects are,

$$
\begin{aligned}
\Delta_U &= \Delta ln\,\frac{M_U}{M_Z} = \tfrac{5}{3366}(7\lambda_1+9\lambda_2-16\lambda_3) \\
\Delta_\zeta &= \Delta ln\,\frac{M_\zeta}{M_Z} = \tfrac{1}{561}(25\lambda_1-48\lambda_2-23\lambda_3) \\
\Delta_G &= \Delta\alpha^{-1}G = \tfrac{1}{13464\pi}(1135\lambda_1-945\lambda_2-932\lambda_3)
\end{aligned} \tag{19}
$$

where $\lambda_i = \lambda_i^U$ which are evaluated using the decomposition of the superheavy components of the Higgs representations under G_{213}.

3.1 The SU(5) model

$$
\begin{aligned}
\underline{24} &\supset D_1(3,0,1)+D_2(1,0,8) \\
\underline{5} &\supset C(1,-\tfrac{2}{3},3) \\
\underline{75} &\supset E_1(1,\tfrac{10}{3},3)+E_2(2,\tfrac{5}{3},3)+E_3(1,-\tfrac{10}{3},\bar{3})+E_4(2,-\tfrac{5}{3},\bar{3}) \\
&+ E_5(2,-\tfrac{5}{3},\bar{6})+E_6(2,\tfrac{5}{3},6)+E_7(1,0,8)
\end{aligned}
$$

Defining $\eta_i = ln\frac{M_i}{M_U}$, extremisation of Δ_U requires

$$
\begin{aligned}
\eta_{D_1} &= \eta_{E_1} = \eta_{E_2} = \eta_{E_3} = \eta_{E_4} = \eta^{(+)} = \pm\, ln\beta \\
\eta_C &= \eta_{D_1} = \eta_{E_2} = \eta_{E_6} = \eta_{E_7} = \eta^{(-)} = \mp\, ln\beta
\end{aligned}
$$

leading to

$$\Delta_U = \pm 0.38\, ln\beta\ ,\ \Delta_\zeta = \pm 1.0\, ln\beta\ ,\ \Delta_G = \pm\frac{3795}{6232\pi}ln\beta \tag{20}$$

Here the significance of β is that the superheavy scalar components can have a mass $\beta M_U(M_U/\beta), \beta = 1-10$ in analogy with the standard model Higgs boson. Without threshold effects the solutions are,

$$M_\zeta^0 = 10^{10.2\pm0.6}\ ,\ M_U^0 = 10^{15\pm0.2}GeV\ ,\ \alpha_G^{0^{-1}} = 39.05\pm0.7 \tag{21}$$

The coupling constant trajectories have been plotted in ref.[7]. The trajectories of $\alpha^{-1}i(\mu)$ are prevented to cross below the unification mass because of the presence of ζ at $M_I^0 = M_\zeta^0 = 10^{10.2\pm0.3}GeV$ which changes the slopes of $\alpha_{2L}^{-1}(\mu)$ and $\alpha_{3C}^{-1}(\mu)$ for $\mu > M_\zeta^0$. The unification mass has been increased by almost an order of magnitude increasing the proton lifetime consistent with the experimental limit. Inlcuding threshold effects the values of mass sclaes and coupling constants are given in Table 1. Clearly the model predictions with $\beta = 10$ saturates the Suprkamiokande limit for $p \longrightarrow e^+ + \pi^0$.

Table 1. Threshold effects on mass scales, GUT coupling and proton lifetime predictions in the degenerate(D) and nondegenerate(ND) cases of superheavy scalars in nonSUSY SU(5) with modified grand desert.

β	M_ζ/M_ζ^o	M_U/M_U^o	$\Delta\alpha_G^{-1}$	τ_p/τ_p^o
10(D)	$10^{\substack{-1.0\\+1.0}}$	$10^{\substack{+.08\\-.02}}$	$\substack{+2.9\\-1.9}$	$10^{\substack{+.30\\-.08}}$
5(ND)	$10^{\substack{-1.7\\+2.2}}$	$10^{\pm.29}$	$\substack{-.3\\+.4}$	$10^{\pm.92}$
10(ND)	$10^{\substack{-2.6\\+3.0}}$	$10^{\pm.38}$	$\substack{-.5\\+.7}$	$10^{\pm1.52}$

3.2 The SO(10) model

One important advantage of SO(10) over SU(5) is the generation of Majorana neutrino masses over a wide range of values. The real scalar $\zeta(3,0,8) \subset$ **210** of SO(1). The implementation of the modified grand desert model in SO(10) requires the Higgs representation **10**, **45**, **126**, and **210** although the model also works by replacing **45** by another **210**. The superhevy components are identified in each representation. Then following the procedure explained for SU(5) we obtain[7],

$$\Delta_U = [\substack{+.44\\-.64}]ln\beta$$
$$\Delta_I = [\substack{-2.37\\+2.65}]ln\beta \tag{22}$$

Numerical values on threshold effects on mass scales, GUT coupling contants and proton lifetimes are presented in Table 2 in the degenerate (D) and nondegenerate cases for $\beta = 5 - 10$ where $\beta = M_i/M_U.M.$ being the superheavy Higgs scalar masses. It is clear that for superheavy masses differing by a factor 10(1/10) from M_U the increase in the proton lifetime could be as large as 50 times the uncorrected value and the model can not be rules out by the planned experiments in near future. The Majorna netrino masses in the jodel can arise in various ways. For example the induced contributions[7] may be made to dominate giving rise to,

$$m_{\nu_i} = \lambda h_i < \phi^0 <^2 < \Delta_R^0 > / M_{\Delta_L}^2 \ , \ i = e, \mu, \tau \tag{23}$$

with

$$\begin{aligned} m_{\nu_e} &= (2.5 \times 10^{-6} \quad -- \quad 2.5 \times 10^{-4}) eV \\ m_{\nu_\mu} &= (7.5 \times 10^{-4} \quad -- \quad 2.5 \times 10^{-2}) eV \\ m_{\nu_\tau} &= (7 \times 10^{-2} \quad -- \quad 7) eV \end{aligned} \tag{24}$$

where Δ_R^0 is the neutral comonent in $\Delta_R(1, 3, \bar{10})$ carrying $B{-}L = 2$ and $\Delta_L(3, 1, 10)$ is the corresponding left-handed triplet. Both are contained in **126** of SO(10). Here ϕ^0 is the netural comonent of the standard doublet $\subset$ **10** of SO(10) and $\lambda(h_i)$ is the Higgs quartic (Yukawa) coupling between **126** and **10**. Here we have used $h_i < \phi^0 >= m_{q_i}$. It is evident that while m_{ν_e} and m_{ni_μ} are compatible with the solar neutrino flux n_{ν_τ} value can be made to be consistent with the dard matter of the universe. The model can be also modified slightly to produce degenerate and see-saw contributions[17-18].

Table 2. Same as Table 1 but for nonSUSY SO(10).

β	M_ζ / M_ζ^o	M_U / M_U^o	$\Delta\alpha_G^{-1}$	τ_p / τ_p^o
$10(D)$	$10^{\substack{+.85 \\ -.00}}$	$10^{\substack{-.02 \\ +.00}}$	$\substack{-12.0 \\ +0.0}$	$10^{\substack{-.24 \\ +.00}}$
$5(ND)$	$10^{\substack{+2.4 \\ -2.6}}$	$10^{\substack{-.46 \\ +.30}}$	$\substack{+4.5 \\ -7.2}$	$10^{\substack{-1.7 \\ +0.8}}$
$10(ND)$	$10^{\substack{+3.6 \\ -3.7}}$	$10^{-6.8}$	$\substack{+6.8 \\ -10.0}$	$10^{\substack{-.57 \\ +1.61}}$

Although corrections to the intermediate mass M_ζ is quite significant that on M_U and hence on τ_p are small. Such results have been also obtained[8] in computing threshold effects in SU(5) with split multiplets[16]. We observe from these analyses that the presence of G_{213} below the GUT-scale reduces uncertainty on the proton lifetime prediction.

4. Threshold effects in SO(10) with intermediate scale and theorems on vanishing corrections

Here we discuss threshold effects in SO(10) with G_{224P} intermediate symmetry in detail and breifly mention the results with other intermediate symmetries.

(a) G_{224P} Intermediate Symmetry

With the choice of minimal Higgs representations **54**, **126**, and **10**, we have

$$a_i'^T = (11/3, 11/3, -14/3)\ ,\ A_U = -50/3\ ,\ B_U = 50/9$$

$$A_I = -17/3\ ,\ B_I = -53/3$$

$$B'_{ij} = \begin{bmatrix} 584/3 & 3 & 765/2 \\ 3 & 584/3 & 765/2 \\ 153/2 & 153/2 & 1759/6 \end{bmatrix} \tag{25}$$

Using these in (8)-(10) we obtain,

$$\begin{aligned} \Delta_U &= -\tfrac{3}{4400}(-9\lambda_{2R}'^U + \tfrac{88}{3}\lambda_{4C}'^U - \tfrac{61}{3}\lambda_{2L}'^U \\ &\quad -15\lambda_Y'^I - \tfrac{61}{3}\lambda_{2L}'^I + \tfrac{106}{3}\lambda_{3C}'^I) \end{aligned} \tag{26}$$

$$\Delta_I = -\frac{1}{88}(\lambda_{2L}'^U - \lambda_{2R}'^U + \lambda_{2L}'^I + \frac{2}{3}\lambda_{3C}'^I - \frac{5}{3}\lambda_Y'^I) \tag{27}$$

It is important to note that the presence of L-R symmetry for $\mu > M_I$ demands $\lambda_{2L}'^U = \lambda_{2R}'^U$ which leads to vanishing GUT-threshold corrections on Δ_I or equivalently on M_I. This is a manifestation of theorems on vanishing corrections on $\sin^2\theta_W$ to be discussed a little later.

We make the simplifying assumption that all the superheavy components belinging to a single GUT representation have the same mass. Also we use $M_{L_i} = M_L$ and $M_{R_i} = M_R$ leading to $\eta_{L_i} = \eta_L, \eta_{R_i} = \eta_R$,

$$\begin{aligned} \Delta_U &= \tfrac{1}{1100}(11\eta_{54} - 88\eta_{126} - 44\eta_{10} + 11\eta_\phi - 131\eta_R + 189\eta_L) \\ \Delta_I &= \tfrac{1}{22}(9\eta_R - 10\eta_L) \end{aligned} \tag{28}$$

(a.1) Theorem on vanishing corrections on $\sin^2\theta_W$

It is important to note that the GUT threshold contributions such as η_{10}, η_{54} and η_{126} are absent in Δ_I. Also the contributions of two loop functions P_{2L}^U, P_{2R}^U and P_{4C}^U in Δ_I cancel out. These are due to theorems proved recently in SO(10) with G_{224P} intermediate symmetry occuring at the highest intermeidate scale[5,6].

Theorem 1 In all GUTs where the G_{224P} symmetry breaks at the highest intermediate scale (M_I), the GUT - threshold contribution to $\sin^2\theta_W$ vanishes[5].

Theorem 2 In all GUTs where the G_{224P} symmetry breaks at the highest intermediate scale (M_I), the gauge boson renormalization to every m - loop order ($m \geq 2$) in the mass range $M_I - M_U$ has vanishing contribution to $\sin^2\theta_W$.

(1.2) Vanishing corrections on $\Delta ln(M_I/M_Z)$ *and precise ptediction on neutrino masses*

In ref[5] it was shown that all perturbative and non-pertubative corrections including gravity-induced higher dimensional operator effects occuring at the GUT sclae also have vanishing contributions on $\sin^2\theta_W$ making the intermediate scale M_I quite stable. Here we provide a proof on the stability of M_I against all such uncertainities. Since left-right symmetry is restored in the presence of G_{224P}, we have

$$a'_{2L} = a'_{2R}\ ,\ \lambda_{2L}'^U = \lambda_{2R}'^U\ ,\ C_{2L}^U = C_{2R}^U \tag{29}$$

where C_{2L}^U, C_{2R}^U and C_{4C}^U are all other corrections including multiloop effects and contributions due to higher-dimensional operators. The corrections to the coupling constants at $\mu = M_U$ can be written in the form

$$\frac{1}{\alpha_i(M_U)} = \frac{1}{\alpha_G} - \frac{(\lambda_i^U + C_i^U)}{12} \;, \; i = 2L, 2R, 4C$$

Then all the formulas derived so far hold hold if we replace $\lambda_i'^U \longrightarrow \lambda_i'^U + C_i^U$. In eq.(9),

$$\begin{aligned}
A_U &= 2(a'_{4C} - a'_{2L}) \,, \\
B_U &= \tfrac{2}{3}(a'_{2L} - a'_{4C}) \,, \\
A_I &= \tfrac{8}{3}a_{3C} - a_{2L} - \tfrac{5}{3}a_Y - 2(a'_{4C} - a'_{2L}) \,, \\
B_I &= \tfrac{5}{3}(a_{2L} - a_Y) - \tfrac{2}{3}(a'_{2L} - a'_{4C}) \,, \\
J_\lambda^U &= \tfrac{1}{3}(\lambda_{2L}^U + C_{2L}^U - \lambda_{4C}^U - C_{4C}^U) \,, \\
K_\lambda^U &= \tfrac{1}{9}(\lambda_{4C}^U + C_{4C}^U - \lambda_{2L}^U - C_{2L}^U) \,.
\end{aligned} \tag{30}$$

which by virtue of (29) lead to

$$\Delta_I^U = 0 \tag{31}$$

This can also be verified through eqs.(17) and (29). It is to be noted that (31) holds irrespective of the nature of the correction arising at the GUT scale and it is indeoendent of the presence or absence of supersymmetry. The cancellation occurs for any possible nonperturbative contributions also. Thus the G_{224P} -breaking scale is not affected due to any corrections emerging from the GUT-sclae although the GUT-scale itself is affected by the corrections. In the absence of threshold corrections we obtain

$$\begin{aligned}
M_U^0 &= 10^{15 \pm 0.25} GeV \\
M_I^0 &= 10^{13.6 \pm 0.2} GeV \\
\alpha_G^{-1} &= 40.6 \pm 0.2
\end{aligned} \tag{32}$$

where the uncertainties are due to those in the input parameters.

The threshold effects are computed by extremising Δ_U or Δ_I while taking into account the parity restoration constraint for $\mu \geq M_U$ with

$$\eta_L \leq 0 \;, \; \eta_R \leq 0 \;, \; \eta_\phi \leq 0 \,.$$

Numerical values of corrections are given in Table 2 for $\beta = 5 - 10$. Predictions for proton lifetime for the $p \longrightarrow e^+\pi^0$ is amde using[21]

$$\tau_p^0 = \frac{5}{8}\left[\frac{\alpha_G^{SU(5)}}{\alpha_G^{SO(10)}}\right]^2 \times 4.5 \times 10^{29 \pm .7}\left(\frac{M_U}{2 \times 10^{14} GeV}\right)^4 Yrs. \tag{33}$$

For $\beta = 10$ the model predicts

$$\tau_p = 1.44 \times 10^{32.1 \pm .7 \pm 1.0 \pm 1.9} Yrs. \tag{34}$$

where the first. second and third uncertainties are due to matrix element, input parametrs, and threshold effects. It is clear that the maximum value for τ_p exceeds the current Super-kamiokande limit by almost one order. Thus, as against the conclusions of ref[19], this model can not be ruled out by any improvement on proton-lifetime measurement.

Table 3. Threshold effects in SO(10) with G_{224P} intermediate symmetry.

β	M_I/M_I^o	M_U/M_U^o	$\Delta\alpha_G^{-1}$	τ_p/τ_p^o
$5(ND)$	$10^{+.26}_{-.20}$	$10^{+.27}_{-.30}$	$^{-1.4}_{+0.2}$	$10^{\mp 1.2}$
$10(ND)$	$10^{+.40}_{-.30}$	$10^{+.40}_{-.44}$	$^{-2.0}_{+0.2}$	$10^{\mp 1.7}$

Threshold effect on M_I shown in Table 3 is due to that at the intermediate-scale boundary only.

$$\log_{10}(M_I/M_I^0) = \pm 0.4$$

The higher-dimensional operators scaled by the Planck mass do not modify the intermediate scale. Also imposition of a horizontal symmetry does not change the scale unless additional light scalars are introduced below M_I. Even if the left-handed triplet is not given a VEV explicitly[28], it can be induced[29,30],

$$\begin{aligned} m_\nu &= \gamma\frac{M_Z^2}{M_I} = (2-200)\times 10^{\pm 0.2\pm 0.4} f\lambda eV \\ &= (0.5-800) f\lambda eV\,, \end{aligned} \tag{35}$$

where we have evaluated[30]

$$\gamma \approx 10 f\lambda/\beta^2\ ,\ \beta = 0.1-1\,,$$

$f(\lambda)$ being a Yukawa(quartic) coupling. The see-saw contributions are,

$$\begin{aligned} m_{\nu_e} &= 5\times 10^{-10.6\pm .2\pm .4} eV\,, \\ m_{\nu_\mu} &= 1.6\times 10^{-3.6\pm .2\pm .4} eV\,, \\ m_{\nu_\tau} &= 0.5\times 10^{-.6\pm .2\pm .4} eV\,. \end{aligned} \tag{36}$$

These neutrino masses are capable of explaining the solar and the atmospheric neutrino oscullations while offering neutrinos as strong candidates for the dark matter of the Universe. In addition the degenerate neutrino mass for ν_e could be confirmed by the neutrinoless double-beta decay experiments.

5. Threshold effects with other single intermediate symmetries in SO(10)

Threshold effects in SO(10) with $G_I = G_{224}$ or G_{2213} without parity ($g_{2L} \neq g_{2R}$) have been extensively discussed in ref.[3]. Although larger undertainty was noted earlier[22] allowing for parity restoration and hence complete SO(10) symmetry at $\mu > (10-30)M_U$, it has been found in ref[3] that the imposition of the parity restoration constraint at the GUT-scale can reduce the uncertainties significantly. Among the two the models, the one with $G_I = G_{224}$ appear to be more favoured for explaining the solar neutrino puzzle.

For $G_I = G_{2213P'}$ the threshold effects have been computed in ref.[20] and for $G_I = G_{214}$ they have been computed in ref.[12]. The computations in all cases can be made following the methods outlined in Secs.2-4. We summarise the results without going into details in Table 4. The predictions on the proton lifetime in different cases with $\beta = 10$ are given below for extremised $\Delta_{U'}$

Table 4. Threshold effects in SO(10) with single intermediate symmetries: G_{224} (model A), G_{2213} (model B), G_{2213P} (model C), and G_{214} (model D) for $\beta = 10$ in the nondegenerate case.

	G_{224} (A)	G_{2213} (B)	G_{2213P} (C)	G_{214} (D)
M_I/M_I^o	$10^{-0.07}{}^{+2.60}$	$10^{-.1}{}^{+.3}$	$10^{\pm .1}$	$10^{+.45}{}^{-.15}$
M_U/M_U^o	$10^{-1.24}{}^{+0.13}$	$10^{-.5}{}^{+.2}$	$10^{\pm .43}$	$10^{-.20}{}^{+.66}$
τ_p/τ_p^o	$10^{-5.0}{}^{+0.5}$	$10^{-2.0}{}^{+0.8}$	$10^{\pm 1.7}$	$10^{-0.8}{}^{+2.6}$

(A) $G_I = G_{224}$

$$\tau_p = 1.44 \times 10^{37.4 \pm .7 \pm 1.0^{+0.5}_{-5.0}} Yrs.$$

(B) $G_I = G_{2213}$

$$\tau_p = 1.44 \times 10^{37.7 \pm .7 \pm .9^{+0.7}_{-2.0}} Yrs.$$

(C) $G_I = G_{2213P}$

$$\tau_p = 1.44 \times 10^{34.2 \pm .7 \pm .8^{\pm 1.7}} Yrs.$$

(D) $G_I = G_{214}$

$$\tau_p = 1.44 \times 10^{29.8 \pm .7 \pm .8 \pm^{2.6}_{0.8}} Yrs.$$

Some comments and conclusions are in order. Including threshold effects we find that even if $\beta = 7$ the model (D) is consistent with the available experimental limit on proton lifetime. As against the conclusion of ref.[19] the model is not rules out. If β is permitted to be sufficiently large ($\beta = 30$) it is extremely difficult to rule out the model in near future on the basis of the proton lifetime measurments.

6. Threshold effects in minimal SUSY SU(5)

Two different appraoches have been adopted in the minimal SUSY SU(5) model (MSSM). In one approach the superpartner scale has been introduced at M_S which is expected not to exceed few TeVs. Below M_S global SUSY is assumed to have broken down but the effects of superpartner masses is taken into account by threshold correction at that scale,

(i) $SU(5) \times SUSY \xrightarrow{M_U} G_{213} \times SUSY \xrightarrow{M_S} G_{213} \xrightarrow{M_Z} G_{13}$ In the other case it is assumed that SUSY survives down to M_Z and the effects of superpartner masses are taken into account through threshold effects at M_Z.

(ii) $SU(5) \times SUSY \xrightarrow{M_U} G_{213} \times SUSY \xrightarrow{M_Z} G_{13}$

Threshold effects due to superheavy masses are estimated at M_U and the corrections due to the top quark is computed as in the SM at M_Z in both cases.

6.1 Threshold effects on M_S

With the chain(i) the generalized form of eqs.(3)-(5) and (8) - (9) apply with $M_I = M_S$ but different numerical values of the coefficients for $i, j = 1, 2, 3$

$$a'_i = \begin{vmatrix} 33/5 \\ 1 \\ -3 \end{vmatrix} \qquad b'_{ij} = \begin{vmatrix} 7.96. & 5.4 & 17.6 \\ 1.8 & 25 & 24 \\ 2.2. & 9 & 14 \end{vmatrix}$$

and with

$$L_i^U/12\pi \longrightarrow \Delta_i^U \;\cdot\; L_i^I/12\pi \longrightarrow \Delta_i^S$$

where

$$\Delta_i^U = \Delta_i^{CNV} + \Delta_i^{SH} \tag{37}$$

The first term in the RHS of (37) represents conversion from DR to $\overline{MS}$ scheme and the second term includes the superheavy particle effects near M_U. In order to find the latter, it is noted that the scalar multiplets have the components,

$$\begin{aligned}
\underline{24} &= M_\sigma(1,0,8) + M_\sigma(3,0,1) + M_V(2,5/6,3) + M_V(2,-5/6.\bar{3}) + M_0(1,0,1)\,, \\
\underline{5} &= M_{H_C}(1,1/3,3) + M_{H^0}(2,1/2.1)\,, \\
\underline{5} &= M_{H_C}(1,-1/3,\bar{3}) + M_{H_2}(2.-1/2.1)\,.
\end{aligned}$$

At first evaluating only the GUT-threshold corrections to the gauge couplings by using the superheavy-particle - one-loop β-functions yields

$$\begin{aligned}
\alpha_3^{-1} &= \alpha_G^{-1} + \tfrac{1}{2\pi}[-4ln\tfrac{M_S}{M_Z} + ln\tfrac{M_V}{M_Z} - 3ln\tfrac{M_\sigma}{M_Z} - ln\tfrac{M_{H_C}}{M_Z}] + \tfrac{1}{4\pi}[P_3 + P'_3] \\
&\quad + \tfrac{1}{4\pi} - \Delta_3^S \\
\alpha_2^{-1} &= \alpha_G^{-1} + \tfrac{1}{2\pi}[-\tfrac{25}{6}ln\tfrac{M_S}{M_Z} + 3ln\tfrac{M_V}{M_Z} - 2ln\tfrac{M_\sigma}{M_Z}] + \tfrac{1}{4\pi}[P_2 + P'_2] \\
&\quad + \tfrac{1}{6\pi} - \Delta_3^S \\
\alpha_1^{-1} &= \alpha_G^{-1} + \tfrac{1}{2\pi}[-\tfrac{5}{2}ln\tfrac{M_S}{M_Z} + 7ln\tfrac{M_V}{M_Z} - \tfrac{2}{5}ln\tfrac{M_{H_C}}{M_Z}] + \tfrac{1}{4\pi}[P_1 + P'_1] \\
&\quad - \Delta_1^S
\end{aligned}$$

The superheavy-mass contributions to the corrections are given in Table 5. Using (38) and the standard procedure gives

$$\begin{aligned}
ln\frac{M_U}{M_Z} &= \frac{\pi}{19\alpha}\left(\frac{1}{2}+7s^2-\frac{25}{\alpha_s}\right)+\frac{5}{76}\left[\frac{1}{6}(P_1'+P_1)+\frac{3}{2}(P_2'+P_2)\right. \\
&\quad \left.-\frac{5}{3}(P_3'+P_3)\right]+\delta_U \\
ln\frac{M_S}{M_Z} &= \frac{12\pi}{19\alpha}\left(1-5s^2+\frac{7\alpha}{3\alpha_s}\right)-\frac{1}{19}[5(P_1'+P_1)-12(P_2'+P_2) \\
&\quad +7(P_3'+P_3)]+\delta_S \\
\alpha_G^{-1} &= \frac{3}{8\alpha}+\frac{3}{76\alpha}\left(\frac{47}{2}+\frac{250\alpha}{\alpha_S}-146s^2\right)[45(P_1'+P_1) \\
&\quad -51(P_2'+P_2)]+25(P_3'+P_3)]+\delta_S
\end{aligned} \tag{39}$$

Table 5. Threshold and 5-dim. operator contributions to the matching functions [9].

	Δ_i^{CNV}	Δ_i^L	Δ_i^{SH}	Δ_i^{5-dim}
Δ_1	0	$\frac{5}{4\pi}ln\frac{M_1}{M_Z}$	$-\frac{5}{\pi}ln\frac{M_V}{M_U}+\frac{1}{5\pi}ln\frac{M_{Hc}}{M_U}$	$-.015\eta$
Δ_2	$-\frac{1}{6\pi}$	$\frac{25}{12\pi}ln\frac{M_2}{M_Z}$	$-\frac{3}{\pi}ln\frac{M_V}{M_U}+\frac{1}{\pi}ln\frac{M_{Hc}}{M_U}$	$-.042\eta$
Δ_3	$-\frac{1}{4\pi}$	$\frac{2}{\pi}ln\frac{M_3}{M_Z}$	$-\frac{2}{\pi}ln\frac{M_V}{M_U}+\frac{3}{2\pi}ln\frac{M_\Sigma}{M_U}$	
			$+\frac{1}{2\pi}ln\frac{M_{Hc}}{M_U}$	$-.028\eta$

$$\begin{aligned}
\delta_U &= \frac{5\pi}{19}\left[\frac{1}{6}(\Delta_1^U+\Delta_1^S)+\frac{3}{2}(\Delta_2^U+\Delta_2^S)-\frac{5}{3}(\Delta_3^U+\Delta_3^S)\right] \\
\delta_S &= \frac{4\pi}{19}[5(\Delta_1^U+\Delta_1^S)-12(\Delta_2^U+\Delta_2^S)+7(\Delta_3^U+\Delta_3^S)]
\end{aligned} \tag{40}$$

$$\delta_G = \frac{1}{19}[45(\Delta_1^U+\Delta_1^S)-5(\Delta_2^U+\Delta_2^S)+25(\Delta_3^U+\Delta_3^S)] \tag{41}$$

Ignoring all threshold corrections except Δ_1^{CNV} and using the input parameters from eq.(1) in (38)-(40) gives

$$M_U^0 = 10^{16.1\pm.49}GeV,\ M_S^0 = 10^{2.44\pm1.85}GeV,\ \alpha_G^{-1}=26.5\pm.3 \tag{42}$$

Two loop contributions which were ignored from M_S^0 estimation in [24] is found to be comparable to the one-loop contribution in $ln(M_S^0/M_Z)$,

$$ln(M_S^0/M_Z) = -2.53+3.66\pm4.26 \tag{43}$$

where the first, second and the third entries on the RHS of (43) are the one-loop, two-loop, and the input-parameter-uncertainty contributions, respectively. The

GUT-threshold corrections are

$$\begin{aligned} \delta_U &= \tfrac{4}{19} ln\tfrac{M_{H_c}}{M_Z} &&- \tfrac{5}{19} ln\tfrac{M_\Sigma}{M_Z} \\ \delta_S &= \tfrac{18}{19} ln\tfrac{M_\Sigma}{M_U} &&- \tfrac{6}{19} ln\tfrac{M_\Sigma}{M_U} \\ \delta_G &= \tfrac{1}{48\pi}(43 ln\tfrac{M_{H_c}}{M_U} &&- 27 ln\tfrac{M_\Sigma}{M_U} \end{aligned} \qquad (44)$$

The GUT-threshold effects are estimated in Table 6 in the degenerate(D), and nondegenerate(ND) cases assuming the superheavy masses to vary between M_U/β and βM_U with $\beta = 5 - 10$ where, as per the conclusion of ref.[24], the uncertainty in M_S is of the order $10^{\pm 1.2}$ for $\beta = 10$.

Table 6. GUT-threshold effects on the unification mass and superpartner scale in MSSM in the degenerate(D) and nondegenerate(ND) cases.

β	M_U/M_U^o	M_S/M_S^o
$5(D)$	$10^{\mp .1}$	$10^{\pm .44}$
$10(D)$	$10^{\mp .14}$	$10^{\pm .66}$
$5(ND)$	$10^{\pm .33}$	$10^{\pm .66}$
$10(D)$	$10^{\pm .47}$	$10^{\pm 1.26}$

6.2 Constraint on superheavy masses from the CERN-LEP data

Hisano, Murayama and Yanagida [25] have observed that the combination $p/\alpha_1 + q/\alpha_2 + r/\alpha_3$ is dependent only on the Higgs colour-triplet mass(M_{H_C}) if $(p, q, r) = (-1, 3, -2)$. Similarly the other combination $(5, -3, -2)$ depends on M_V and M_Σ leading to,

$$\begin{aligned} (-2\alpha_3^{-1} + 3\alpha_2^{-1} - \alpha_1^{-1}) &= \frac{1}{2\pi}\left[\frac{12}{5} ln(M_{H_c}/M_Z) - 2 ln\frac{M_S}{M_Z}\right] \\ (5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}) &= \frac{1}{2\pi}\left[12 ln\frac{M_V^2 M_\Sigma}{M_Z^3} + 8 ln\frac{M_S}{M_Z}\right] \end{aligned} \qquad (45)$$

Incorporating Δ_i^S effects leads to the replacements of second terms in the RHS of (45) by the logarithmic functions of super partner and the second Higgs-doublet masses [25]. Restricting the model to the minimal supergravity SU(5), the particle spectrum and the CERN-LEP measurements impose the following constraints on the colour troplet mass and the effective unification scale $(M_V^2 M_\Sigma)^{1/3}$,

$$2.2\times10^{13}GeV < M_{H_c} < 2.3\times10^{17}GeV$$
$$0.95\times10^{16}GeV < (M_V^2 M_\Sigma)^{1/3} < 3.3\times10^{16}GeV \quad (46)$$

for the gluino mass 100 GeV - 1TeV.

6.3 Higher-dimensional operator effects

The effects of the 5-dim. operator,

$$-\frac{\eta}{2M_G}Tr[F_{\mu\nu}\Sigma_{(24)}F^{\mu\nu}] \quad (47)$$

on the MSSM predictions have been analysed by Langacker and Polonsky [9] and Hall and Sarid [26]. The analytic expressions for the strong interaction coupling and M_{H_c} are [26],

$$\alpha_s = .132\{1-.024\sigma-.02ln[\mu^{4/5}M_{H_2}^{1/5}/M_Z]+.025ln[\frac{M_{H_c}}{9\times10^{16}GeV}]$$
$$-.025\eta(1-.1\alpha)(m_{1/2}/M_Z)^{-2/9\lambda_{24}-1/2}\} \quad (48)$$
$$M_{H_c} = (3\times10^{16}GeV)\lambda_5(1-.1\sigma)\lambda_{24}^{-1/3}(m_{1/2}/M_Z)^{-2/9}$$

where $M_G=(8\pi G_N)^{-1/2}=2.4\times10^{18}GeV, M_{H2}$ = second Higgs doublet mass $m_{1/2}$ = universal gaugino mass, μ = two-doublets mixing parameter, $\sigma=(s^2-.2326)/.008$, and λ_5 and λ_{24} are the trilinear couplings in the superpotential,

$$W = \lambda_5\bar{H}_5\Sigma H_5 + \frac{1}{3}\lambda_{24}tr(\Sigma^3) + \ldots$$

It is clear from (48) that α_S increases with logarithmic increment of the colour triplet mass when $\eta=0$. Due to gravitational smearing (e.g., $\eta=-1$ to $+1$) the allowed region in the α_s vs. M_{Hc} plot is found to be enhanced and blurred.

6.4 Threshold effects through effective mass parameters

Using the symmetry breaking pattern(i) Langacker and Polonsky[9] have computed SUSY particle threshold effects near the boundary M_Z, in addition to the GUT-boundary corrections[9] by parametrising these corrections in terms two separate sets of effective masses. The RGEs for the three gauge couplings can be written as,

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_G}+\frac{a_i}{2\pi}ln\frac{M_u}{M_z} + \frac{1}{4\pi}P_i'-\Delta_i \quad (49)$$

The correction term Δ_i iccludes several contributions,

$$\Delta_i = \Delta_i^U + \Delta_i^{ho} + \Delta_i^t + \Delta_i^Y + \Delta_i^L$$

where Δ_i^U is the same as (37), Δ_i^L is the anlogue of Δ_i^S but evaluated at the boundary M_Z, and the others are the Higgs(ho), the top quartk(t), and the Yukawa coupling(Y) contributions evaluated at the boundary M_Z. Using the RGEs and following the standard procedure it is straightforward to derive the following analytic expressions which are independent of the GUT symmetry but only dependent upon the grand desert hypothesis. We furnish analytic expressions on threshold effects for one set of predictions,

$$\begin{aligned}
ln\frac{M_U}{M_Z} &= 2\pi(3\alpha^{-1} - 8\alpha_S^{-1})/F - (5P_1' + 3P_2' - 8P_3')/(2F) + \delta_U \\
S^2(M_Z) &= \sin^2\theta_w(M_Z) = [3(a_1' - a_3') + (a_1' - a_2')\alpha/\alpha_s]/ - 5\alpha[(a_2' - a_3')P_1' \\
&\quad +(a_3' - a_1')P_2' + (a_1' - a_2')P_3']/(4\pi F) + \delta_{s^2} \\
\alpha_G^{-1} &= [-3a_3'\alpha^{-1} + (5a_1' + 3a_2')\alpha_s^{-1}]/F + [(5P_1' + 3P_s')a_3' \\
&\quad -(5a_1' + 3a_2')P_3']/(4\pi F) + \delta_G
\end{aligned} \tag{50}$$

The threshold corrections are given below which can be computed numerically from the Tables given in ref.[9],

$$\begin{aligned}
\delta_U &= \Delta ln\frac{M_U}{M_Z} = 2\pi(5\Delta_1 + 3\Delta_2 + 8\Delta_3)/F \\
\delta_{s^2} &= \Delta\sin^2\theta_w(M_Z) = 5\alpha[(a_2' - a_3')\Delta_1 + (a_3' - a_1')\Delta_2 + (a_1' - a_2')\Delta_3]/F \\
\delta_G &= [(5a_1' + 3a_2')\Delta_3 - (5\Delta_1 + 3\Delta_2)a_3']/F \\
F &= 5a_1' + 3a_2' - 8a_3'
\end{aligned} \tag{51}$$

Using $\alpha^{-1}(M_Z) = 127.9, \alpha_s(M_Z) = 0.12$ gives

$$M_U = 10^{15.9}GeV, \alpha_G^{-1} = 23.5, \sin^2\theta_w = 0.2335\,.$$

Table 7. The top quark and the Yukawa coupling corrections. Here m_t is in GeV and h_t is the Higgs-top quark Yukawa coupling [9].

	Top quark corrections corrections	Yukawa coupling
Δ_1	$-.15 + .13ln(m_t/138) + .15(m_t/138)^2$	$.17h_t^2$
Δ_2	$-.25 + .065ln(m_t/138) + .025(m_t/138)^2$	$.20h_t^2$
Δ_3	$.04 + .105ln(m_t/138)$	$.13h_t^2$

The top quark and Yukawa coupling corrections which are the same as the Standard Model are given in Table 7[9]. The corrections at M_Z-boundary due to the SUSY particle masses have been parametrized in terms of three effective mass parameters $(M_{1'} M_{2'}, M_{3'})$.

$$\Sigma_\zeta \frac{a_i^\zeta}{2\pi} ln\left(\frac{M_\zeta}{M_z}\right) = \frac{a_i^{SUSY} - a_i^{SM}}{2\pi} ln\left(\frac{M_i}{M_z}\right) \; ; \; i = 1,2,3 \qquad (52)$$

where $a_i^{SUSY} = a_i', a_i^\zeta$ is the analogue of a_i but for particle ζ only, and M_ζ is the mass of ζ. Using the SUSY particle spectrum [27] the effective mass parameters have been evaluated : for example a possible solution is

$$m_t = 160Gev\ ,\ M_1 = 261Gev\ ,\ M_2 = 207Gev\ ,\ M_3 = 352Gev$$

For evaluation of threshold effects at M_U a parametrisation similar to (52) has also been suggested,

$$\Sigma_\zeta \frac{a_i^\zeta}{2\pi} ln \frac{M_\zeta}{M_Z} = \frac{a_i^{matter}}{M_U} \; ; \; i = 1,2,3$$

where ζ stands for superheavy particles near the GUT-scale. The prediction for $\sin^2\theta_W(M_Z)$ turns out to be [9],

$$\sin^2\theta_W(M_Z) = 0.2334 \pm .0025 \pm .0014 \pm .0006 \,{}^{+.0013}_{-.0005} \pm .0016 \qquad (53)$$

To predict (53) one starts with the input parameters $\alpha_S(M_Z)$ and $\alpha(M_Z)$. One can start with the parameters $\sin^2\theta_W(M_Z)$ and $\alpha(M_Z)$ to predict $\alpha_S(M_Z), M_U$ and α_G[9],

$$\alpha_S(M_Z) = .125 \pm .001 \pm .005 \pm .002 \,{}^{+.005}_{-.002} \pm .006 \qquad (54)$$

The first number on the RHS of (53) or (54) is for $m_t = 138GeV$. The second entry in (53) ((54)) is due to the uncertainty in the input parameter $\alpha_S(\sin^2\theta_W)$. The third, fouth, fifth, and sixth entries are due to SUSY threshold. m_t and m_{h_o}, GUT-threshold and 5-dimensional operator effects. Predicted values of M_U and α_G with uncertainties are given in ref.[9].

7. Summary and conclusions

We have discussed threshold effects in modified nonSUSY grand desert models with SU(5) and SO(10) GUTs, single intermediate scale models of SO(10), and in the minimal SUSY SU(5). We have also estimated neutrino masses predicted in nonSUSY SO(10) with G_{224P} intermediate symmetry where vanishing corrections on the intermediate scale has been explicitly demonstrated. We conlcude that

1. A minimal extension of the grand desert by the introduction of a single real scalar $\zeta(3,0,8)$ makes SU(5) consistent with the CERN-LEP data and the proton lifetime measurment. In addition SO(10) can account for small neutrino masses needed for solar neutrino oscillation and the dark matter of the Universe. Also the model has the potentiality to provide degenerate neutrino masses.

2. When the GUT symmetry breaks down to the SM gauge group in one step threshold uncertainty appears to be smaller on the proton lifetime prediction.

3. Certain models appearing to be ruled out by two-loop calculations could survive when threshold and 5-dim. operator effects are taken into account. It is essential to include such effects while questioning their survival against the CERN-LEP data and the proton lifetime.

4. Including threshold effects alone the SO(10) model with G_{214} intermediate symmetry is consistent with the CERN-LEP data, proton lifetime and small neutrino masses.

5. In a number of GUTs with G_{224P} intermediate symmetry we prove explicitly due to high-sclae-loop effects, GUT-threshold and higher dimensional operator effects with

$$M_I = 10^{13.6\pm0.2\pm0.4} GeV$$

leading to more precise values of degenerate and see-saw values of neutrino masses needed to explain the solar and atmospheric neutrino oscillations and the dark matter of the universe. The degenerate mass predictions, $m_{\nu_e} = 0(1)eV$, could be confirmed by the neutrinoless double beta-decay experiments. This prediction is universal to a large class of grand unified theories such as SO(10), SO(18), SU(16), $E_{6'}SU(8)_L \times SU(8)_R$ and all $SO(2N)$ with $N \geq 5$.

6. As shown by Hisano et.al. [25] the Higgs-colour-triplet mass and the effective unification mass in MSSM can be constrained by the CERN-LEP data.

7. There could be gravitational smearing effects in MSSM on α_s prediction due to 5-dim. operator effects[26].

8. There threshold uncertainties on MSSM predictions can also be parametrised in terms of effective mass parameters[9].

Acknowledgement

The author is thankful to Professors R.Ramachandran, G.Rajasekaran, Partha Ghose, and S.Uma Sankar for hospitality during the workshop and acknowledges support in the research project SP/S2/K-09/91 from the Department of Science and Technology, New Delhi.

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