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Oscillating neutrinos: theory vs experiment

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Abstract. Possible hints on neutrino masses are reviewed. They come from the deficits in the solar as well as atmospheric neutrinos and from need of a significant amount of hot component in the dark matter of the universe. The role of three generation mixing in simultaneously solving the solar and atmospheric neutrino problem is discussed. All the three hints can be reconciled if three neutrinos are almost degenerate. Models for neutrino masses and mixing implied by the above hints are briefly discussed.

1. Introduction

We discuss in this talk the implications of present experiments on the properties - masses and mixing - of the known neutrinos. So far, one does not have any conclusive evidence from laboratory in favour of neutrino masses. But there exist a number of hints which strongly point to non-vanishing neutrino masses. More importantly, when taken in totality, these hints imply a definite pattern for the neutrino masses. We shall review [1] these hints and try to present general conclusions about the structure of neutrino masses following from them. Then we shall briefly review theoretical schemes which can lead to the desired pattern for neutrino masses and mixing.

2. Hints on neutrino masses

The hints on neutrino masses come at least from three different sources: (1) Solar neutrino deficit (2) Atmospheric neutrino deficit and (3) need for a significant hot component in the dark matter of the universe. Many reviews [2, 3, 4] exist which summarize these hints. We briefly mention salient features of each of these topics.

2.1 Solar neutrino deficit

Four different experiments looking for the electron neutrino of the solar origin have found depletion in the solar ν_e flux compared to theoretical expectations based on the standard solar model (SSM). The recent results are [3]

$$
\begin{aligned}
R_{sol} = \frac{\text{Observedflux}}{\text{SSMflux}} \quad &= 0.318 \pm 0.051 \quad \text{(Homestake)} \\
&= 0.51 \pm 0.04 \pm 0.06 \quad (\text{Kamioka II} + \text{III}) \\
&= 0.58 \pm 0.07 \quad (\text{GALLEX} + \text{SAGE})
\end{aligned} \tag{1}
$$

- The SSM rates are calculated using the flux of Bachall's group.
- Different experiments observed different amount of depletion. Since these experiments are sensitive to different parts of the neutrino energy spectrum, the above results can be understood in terms of some underlying mechanism which can cause the energy dependent suppression. This would occur if the Mikhyev Smirnov Wolfenstein (MSW) [5] conversion or the long wave length vacuum [6] oscillations cause the depletion in the ν_e flux. In contrast, the vacuum oscillations with a wavelength much shorter than the Sun-Earth distance cannot explain the observed energy dependence of depletion.
- It has been argued [7] that various experimental results are indicative of some new physics rather than of uncertainty in our knowledge of the Sun.
- If one assumes neutrino oscillations to be responsible for the depletion of ν_e then the relevant parameter space is restricted to the following in case of the two generation mixing. For the vacuum solution, one has

  $$\delta m^2 \sim (0.55 - 1.1) \times 10^{-10}\text{eV}^2 \quad \sin^2 2\theta \sim 0.75 - 1 \tag{2}$$

  while for the MSW solution, one has

  $$
  \begin{aligned}
  \delta m^2 \sim 6 \times 10^{-6}\text{eV}^2 \quad \sin^2 2\theta \sim 0.007 \quad (\text{Non} - \text{Adiabatic}) \\
  \delta m^2 \sim 9 \times 10^{-6} \quad (\text{eV}^2)\ ; \quad \sin^2 2\theta \sim 0.6(\text{Adiabatic})
  \end{aligned} \tag{3}
  $$

2.2 Atmospheric neutrino deficit

The atmospheric neutrinos are produced in decays of pions, kaons and muons originating from interactions of cosmic rays with the atmosphere. Observations of these neutrinos seem to indicate the depletion in the muon to electron neutrino flux ratio compared to the theoretical expectations. Experimental findings of various groups are as follows [3]:

$$
\begin{aligned}
R &= \frac{(\mu/e)_{OBS}}{(\mu/e)_{MC}} = 0.60^{+0.06}_{-0.05} \pm 0.05 \quad (\text{KAM} \quad \text{Sub GeV}) \\
&= 0.57^{+0.08}_{-0.07} \quad (\text{KAM} \quad \text{Multi GeV}) \\
&= 0.54 \pm 0.05 \pm 0.12, \quad (\text{IMB} - 3) \\
&= 0.69 \pm 0.19 \pm 0.09, \quad (\text{SOUDAN}) \\
&= 0.87 \pm 0.16 \pm 0.08, \quad (\text{FREJUS})
\end{aligned} \tag{4}
$$

where $(\mu/e)_{MC}$ is the ratio generated through the monte carlo simulation using the theoretically calculated fluxes.

The following remarks are in order:

- The absolute flux of ν_e and ν_μ have about 30% uncertainty but their ratio is much less uncertain ($\sim$ 5%). Thus the experimental indication in the depletion in R can be pointing to genuine effects like oscillations.
- The deficit is seen [8] by Kamioka in two sets of data. One corresponding to the contained events with energy $\leq \mathcal{O}$(GeV) approximately. The other set consists of contained and partially contained events in the multi GeV range.
- IMB group has also looked for the upward going muons. Their numbers in this case are consistent with no oscillation hypothesis.
- The Kamioka group finds [8] strong dependence on the zenith angle in the multi GeV data. The down coming neutrinos do not show much depletion compared to the upgoing ones which are depleted by about 50%. Since, the latter travels much larger distance compared to the down going neutrinos, this zenith angle dependence is strongly suggestive of the oscillation phenomena with typical wavelength of order few tens of km. In fact, the best fit values of parameters quoted by Kamioka assuming the oscillations to be the cause of depletion are

$$\Delta m^2 \simeq 1.6 \times 10^{-2}\ eV^2 \ , \ \sin^2 2\theta \simeq 1 \ . \tag{5}$$

2.3 Dark matter

Combinations of experimental evidences and strong theoretical arguments lead one to believe that our universe is mostly made up of non baryonic dark matter [9, 10].

Neutrinos are the only known particles among various candidates for the dark matter that have been proposed. But it is likely that neutrinos by themselves may not be responsible for all the non baryonic dark matter. This follows from the arguments based on the structure formation [10] in the early universe. Because of relatively small masses, neutrinos are relativistic till quite late in the evolution of the universe. Free streaming of these neutrinos cannot allow for the growth of small structures which could have evolved to the galactic sizes at present. The structures at smaller scales arise more naturally if the dark matter is cold, for example, heavy weakly interacting particle like the lightest supersymmetric state. But this by itself is not successful in explaining the large scale structure. This has given rise [11] to suggestion of the mixed dark matter scenario with simultaneous presence of the hot and cold component in the dark matter. Assuming the hot component to be in the form of neutrinos one finds that roughly 30% contribution by them in addition to cold matter simultaneously fit the COBE and IRAS observations. Taken at the face value this suggests that the sum of the neutrino masses [11] should be

$$\Sigma m_{\nu_i} \sim 7\text{eV} \tag{6}$$

3. Pattern of neutrino masses

It is likely that neutrino masses may not be responsible for one or more of the phenomena discussed in the last section or that the experimental findings themselves may need some revision. But if one takes all these hints seriously than one could draw strong theoretical conclusions about the pattern of neutrino masses. First, we discuss the possible pattern of neutrino masses and mixing necessary in order to simultaneously solve the solar and atmospheric neutrino problems. If one also wants neutrinos to provide the hot component in the dark matter then the structure of neutrino masses get further constrained. We discuss this in the subsequent subsection.

3.1 Solar and atmospheric neutrino deficits

The restrictions imposed by the solar and atmospheric neutrino deficits on masses and mixing have already been displayed in eqs.(2,3) and eq. (5). One can draw the following obvious conclusions from these equations:

(*i*) At least two of the three masses and mixing angles among neutrinos must be non-zero.

(*ii*) At least one of the mixing angles must be large and

(*iii*) two independent (mass)^2 differences between neutrinos must be hierarchical. Such hierarchy can result if the non-zero masses themselves are hierarchical or if two are nearly degenerate with the common (mass)^2 around 10^{-2} (eV)^2 and (mass)^2 splitting around $10^{-5} - 10^{-6}$ (eV)^2.

The (*i*) above imply that all three neutrinos would be involved in the oscillations and conventional analysis of the neutrino oscillations in terms of two generations may be inadequate in general. More importantly, in many cases the observed atmospheric neutrino deficit can influence the region of parameter space allowed by the solar neutrino experiments and vice versa.

A priori, the analysis of neutrino experiments in terms of three neutrino generations seems complicated since in general two independent (mass)^2 differences and three mixing angles are involved. But the fact that (mass)^2 differences are hierarchical greatly simplifies the analysis of oscillations both in vacuum as well as in the presence of the matter. Thus it becomes possible to simultaneously analyze the combined implications of the solar and atmospheric neutrino data.

Consider first the vacuum oscillations and the simplest mass hierarchy $m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$ with $\Delta_{32} \sim 10^{-2}$ (eV)^2, $\Delta_{12} \sim 10^{-5}$ (eV)^2. The length scale associated with Δ_{12} is $\sim 10^5(\frac{E}{100\,MeV})$ km. The effects of such oscillations cannot be felt in the laboratory or in the atmospheric neutrino experiments. Hence, one can take $m_1 = m_2$ in analyzing these phenomena and all relevant conversion probabilities

[12] depend only on Δ_{32}.

$$\begin{array}{lclrcl} P_{\nu_e\,\nu_e} & = & 1-4s_{13}^2c_{13}^2S; & P_{\nu_e\,\nu_\mu} & = & s_{13}^2c_{13}^2s_{23}^2S \\ P_{\nu_\mu\,\nu_\mu} & = & 1-4s_{23}^2c_{13}^2(1-c_{13}^2s_{23}^2)S; & P_{\nu_e\,\nu_\tau} & = & 4s_{13}^2c_{13}^2s_{23}^2S \\ P_{\nu_\tau\,\nu_\tau} & = & 1-c_{23}^2c_{13}^2(1-c_{13}^2c_{23}^2)S; & P_{\nu_\mu\,\nu_\tau} & = & 4s_{23}^2c_{13}^4c_{23}^2S \end{array} \tag{7}$$

where $S = \sin^2 \frac{\Delta_{23}L}{4E}$. Note that only two of the three mixing angles appear in all the probabilities. Thus one has effectively one more parameter compared to the conventional analysis in terms of only two generations. The detailed restrictions on the angles θ_{13} , θ_{23} and on Δ_{23} following from the laboratory search of the oscillations can be found for example in ref.[12].

Consider now the matter effects inside the Sun. The matter can enhance the conversion of ν_e if the latter go through the MSW resonance inside the Sun. As is well known, this requires the relevant (mass)^2 differences around 10^{-5} (eV)^2. The oscillations characterized by $\Delta_{32} \sim 10^{-2}$ (eV)^2 do not go through a resonance and the matter effects also get simplified in spite of the presence of three generations [12, 13, 14]. More specifically, the effective matter dependent (mass)^2 matrix can be expressed as

$$M_A^2 = \begin{pmatrix} s_{12}^2\Delta_{21} + Ac_{13}^2 & s_{12}c_{12}\Delta_{21} & 0 \\ s_{12}c_{12}\Delta_{21} & c_{12}^2\Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix} + O(\frac{\sin 2\theta_{13}A}{2\Delta_{32}}) \tag{8}$$

where $A = 2\sqrt{2}G_F n_e E$. The above equation is formally the same as in case of two generations. There are two important differences however. The resonance condition following from eq.(8) now becomes

$$\Delta_{21}\cos 2\theta_{12} = c_{13}^2 A \tag{9}$$

Secondly, the survival probability for the electron neutrino now takes the form

$$P_{\nu_e\,\nu_e} = c_{13}^4 P_2(\theta_{12}, \Delta_{12}) + s_{13}^4 \tag{10}$$

where $P_2(\theta_{12}, \Delta_{12})$ is the standard survival probability in case of the two generations. Both eqs (9) and (10) differ from the corresponding two genration case because of the presence of one more mixing angle namely, θ_{13}.

One could distinguish between two physically different possibilities:

(a) The solar neutrino deficit arises due to ν_e ν_μ oscillations while the ν_μ ν_τ oscillations solve the atmospheric neutrino problem [12].

(b) Alternatively, the solar neutrino problem can be solved by the ν_e ν_τ oscillations and atmospheric neutrino by the ν_e ν_μ [14]. The survival probability is given in this case by

$$P_{\nu_e \nu_e} = c_{12}^4 P_2(\theta_{13}, \Delta_{13}) + s_{12}^4 \tag{11}$$

Note that in case (b), the additional angle entering the survival probabilities for the solar ν_e is required to be large from the atmospheric neutrino anomaly in contrast to the case (a) where the angle θ_{13} could be small. Thus, in case (b), one *cannot* treat the solar neutrino deficitė as purely two generation problem and the allowed region in parameter space changes. The region allowed by the solar neutrino data including the energy distribution observed by the Kamioka is determined for the case (b) in [14]. The allowed region is considerably larger in comparision to the two generation scenario.

Note that the cases (b) above requires a mass hierarchy different from the conventional one namely $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$. But such inverted hierarchies are not uncommon and occur for example in one of the most popular models for the neutrino masses namely, the Zee model [15]. The detailed mixing pattern predicted in this model is however not favoured [16] by the data in case of scenario (b).

3.2 Combined solution

While it is possible to solve the atmospheric and solar neutrino problems in variety of ways, combined solution of all three problems imply stringent constraints on the neutrino mass spectrum. The requirement that neutrinos provide the hot component in the dark matter demands that at least one of the neutrino mass should be in the eV range. But if there are only three neutrinos then one cannot generate two $(\text{mass})^2$ differences both of which are much smaller then O(eV) unless all the neutrinos are degenerate to a high degree. Alternatively, one needs to introduce additional light sterile [4] neutrino. We shall consider the minimal scenario without any light sterile neutrinos in the following.

4. Models for neutrino masses

The mass terms for three light neutrinos can be written in general as follows:

$$-\mathcal{L}_{mass} = \frac{1}{2}\nu_L^T \, m_{eff} \, \nu_L \tag{12}$$

where m_{eff} is a matrix in generation space transforming under $SU(2)_L$ as a triplet. m_{eff} breaks the lepton number but it can preserve some linear combinations of the family lepton numbers. m_{eff} can be generated in one of the following ways [4]:

- Add an $SU(2)$ triplet scalar field T and assign non-zero vacuum expectation value (vev) to its neutral component. This way of generating mass term gives rise to a triplet -Goldstone boson if the lepton number is not explicitly broken in the Higgs potential and is ruled out by the measured value of the invisible width of Z.

- Generate an effective triplet term by radiative corrections. A proto-type of this way of mass generation is provided by the Zee model [15].
- Couple the ν_L to one or more heavy neutrinos. This results in the effective couplings of the above type. The prime example of this is the well known seesaw mechanism which is characterized by the following mass matrix between ν_L and ν_R

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \tag{13}$$

leading to the effective mass matrix

$$m_{eff} = -m_D M_R^{-1} m_D^T \tag{14}$$

Any one or more of above can be used to generate masses for the known left-handed neutrinos.

The seesaw models naturally arise in grand unified theories like $SO(10)$. The simplest of such theory also relates the neutrino Dirac masses to the mass matrix for the charged 2/3 quarks. As a result, the neutrino masses display strong generation dependence as long as M_R is generation blind. Typically,

$$m_{\nu_e} \sim \frac{m_u^2}{M_R} \;; m_{\nu_\mu} \sim \frac{m_c^2}{M_R} \;; m_{\nu_\tau} \sim \frac{m_t^2}{M_R} \tag{15}$$

The above equations imply

$$\frac{\Delta_{32}}{\Delta_{21}} \approx (\frac{m_t}{m_c})^4 \sim 10^8 \tag{16}$$

Hence it is not possible to solve both the atmospheric and solar neutrino problems simultaneously in the seesaw model based on the simplest $SO(10)$. It should however be remembered that the above results, many times taken as predictions of the seesaw scheme, are strongly model dependent. If M_R shows some generation dependence then the predictions could change drastically. For example, it is possible [17] to construct models where all the three neutrinos have masses in the eV range and two of them have mass splitting which could account for the solar neutrino deficit through the MSW mechanism. Moreover, such structure follows from natural assumptions namely m_D coinciding with the up quark masses and all the nonzero elements of M_R having the same order of magnitudes. Such models are interesting from the point of view that they display completely different pattern compared to the standard seesaw prediction. But, one cannot still obtain almost degenerate spectrum needed for the simultaneous solution of all the three problems discussed above.

A possible mechanism to obtain (almost) degenerate spectrum and the required departures from degeneracy was proposed in ref.[18, 19]. The mechanism presented there is quite natural in the context of the left right symmetric theories with manifest parity invariance. The majorana masses for the right handed neutrinos come from the $SU(2)_R$ triplet. But then the manifest left right symmetry requires the presence of a left handed triplet field with non-zero vev resulting in the following structure instead of eq.(13)

$$\mathcal{M}_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} \tag{17}$$

where m_L is the majorana mass for the left handed neutrinos. This generates the following effective mass

$$m_{eff} = m_L - m_D M_R^{-1} m_D^T \tag{18}$$

If there exists some horizontal symmetry [22] which makes the majorana mass matrices m_L,M_R proportional to identity matrix then the following pattern emerges for the masses

$$m_{\nu_e} \sim m_0 - \frac{m_u^2}{M_R} \tag{19}$$

$$m_{\nu_\mu} \sim m_0 - \frac{m_c^2}{M_R} \tag{20}$$

$$m_{\nu_\tau} \sim m_0 - \frac{m_t^2}{M_R} \tag{21}$$

(22)

This leads to the following prediction [19]

$$\frac{\Delta_{32}}{\Delta_{12}} \sim (\frac{m_t}{m_c})^2 \sim 10^4 \tag{23}$$

Thus the hierarchy needed to solve the solar and atmospheric problems simultaneously is automatically reproduced in this picture. Moreover, the common degenerate mass m_0 can be writen as

$$m_0 \sim (2.5eV) \left(\frac{\Delta_{21}}{10^{-6}eV^2}\right) \left(\frac{M_R}{10^{16}\, GeV}\right) \tag{24}$$

Hence if the Δ_{12} is in the range appropriate for a solution to the solar neutrino problem through the MSW mechanism and M_R around the GUT scale then the degenerate mass m_0 is automatically fixed in the eV range needed to provide the hot component in the dark matter.

One possible problem in the above scheme is difficulty in generating a large mixing needed to solve the atmospheric neutrino problem. The mixing in the lepton sector is correlated to mixing in the quark sector in $SO(10)$ type models. In case of the ordinary seesaw model, one could destroy this correlation by making the M_R [23] to be generation dependent. This does not happen in the present case since m_L and hence M_R is required to be proportional to the identity matrix. This can be avoided if one introduce extra singlet [20] fermions or makes the Higgs structure complex enough to destroy the interesting relations between femion masses existing in the simple $SO(10)$ scenario.

The above scheme predicts a non-zero effective mass of the O(2 eV) for the neutrinoless double beta decay amplitude. This is to be contrasted with the latest [24] limit of about 0.68 eV. Thus even after allowing for a factor of two uncertainty in the nuclear matrix elements, the simplest model seems to be ruled out. This however does not exclude the theoretically interesting possibility of degenerate neutrino spectrum since in principal, there could be cancellations among different neutrino contributions. Constructing a realistic model displaying this cancellation is however quite challanging.

5. Conclusions

We have reviewed the hints on neutrino masses following from present experiments and have discussed their implications. All the hints can be simultaneously satisfied within the minimal scenario only if all the three neutrinos are almost degenerate. This seemingly different pattern compared to other fermion masses can be naturally understood in the seesaw picture with additional discrete symmetry. The present models for degenerate masses do not seem to be favoured by the neutrinoless double beta decay results if the degenerate mass is to provide the hot dark matter. This may be a pointer to the existence of more than three light neutrinos.

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