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Polarised structure functions

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Abstract. This talk discusses the current understanding in some aspects of polarised lepton–nucleon deep inelastic scattering. The first section focuses on the non-singlet part of available spin dependent structure function data. The results, though consistent with theoretical predictions in this sector, are extremely sensitive to extrapolation of the data to the $x \to 1$ region and seem to indicate that the quarks *do* carry a very small fraction of the nucleon spin. We then review the formalism of DIS processes with emphasis on the gluon sector.

In this talk we focus on some aspects of polarisation phenomena in deep inelastic scattering (DIS) of high energy leptons (e, μ) off a nucleon target through virtual photon exchange. The energy involved in the process is such that the target completely breaks up. The debris remains unobserved; only the lepton is detected after the collision (its energy and angle of scattering). If both the initial particles are polarised (here, longitudinally, along the beam axis), then the lepton probes the internal spin structure of the nucleon. Hence such experiments can shed light on the distribution of the nucleon spin in terms of that of its component partons (quarks and gluons).

Section 1 discusses the current data and the phenomenology of polarised DIS with emphasis on the nonsinglet sector. While the data turn out to be consistent with theoretical expectations in this sector, the total contribution of quarks to the spin of the proton is small. There is thus a possibly significant gluon contribution to the proton spin. Section 2 therefore addresses the question of the gluonic contribution to the polarised DIS cross section. This section is basically a pedagogical review of the formalism of such processes. We establish that the gluon contribution to the spin of the proton is well defined, though it *cannot* be extracted from data in polarised DIS.

1. Polarised Deep Inelastic Scattering

1.1 The Formalism

The European Muon Collaboration (EMC) measured the asymmetry with respect to the polarised μp Deep Inelastic Scattering (DIS) cross sections when the muon and proton spins were parallel and antiparallel respectively. Theoretically, this asymmetry is given in terms of the spin dependent and spin independent proton structure functions, g_1^p and F_1^p, as,

$$A_1^p(x) = \frac{g_1^p(x)}{F_1^p(x)} .$$

Here, x, as usual, is the Bjorken scaling variable, $x = Q^2/(2p \cdot q)$, where $Q^2 = -q^2$, p and q are the 4-momenta of the virtual photon and the proton in the DIS process and $0 < x \leq 1$. The SMC and E142 measured the asymmetries,

$$A_1^D(x) = \frac{g_1^D(x)}{F_1^D(x)} , \qquad A_1^n(x) = \frac{g_1^n(x)}{F_1^n(x)} ,$$

where the quantities are correspondingly defined for the deuteron and neutron. The spin dependent structure functions are then extracted as the denominators are known. These structure functions are a measure of the distribution of the nucleon spin in terms of that of the partons.

There are other structure functions occurring not only in polarised DIS experiments but also in other processes such as Drell Yan in polarised pp collisions. These have recently been the subject of intense study. For want of adequate time, however, we do not discuss them here [1].

The structure functions of interest can be expressed in terms of parton densities (to leading order) as

$$F_1^p(x) = \frac{1}{2}\sum_f e_f^2 q(x) \qquad g_1^p(x) = \frac{1}{2}\sum_f e_f^2 \widetilde{q}_f(x) ,$$

where the summation runs over both quark as well as antiquark flavours and q and $\widetilde{q}$ are the spin summed and spin dependent quark densities. x is then interpreted as the fraction of nucleon momentum carried by that parton. The Bjorken sum rule relates the moments of the proton and neutron spin dependent structure functions. To $\mathcal{O}(\alpha_s)$, we have

$$\int_0^1 (g_1^p(x) - g_1^n(x))\,\mathrm{d}x = \frac{g_A}{6}\left(1 - \frac{\alpha_s}{\pi}\right) .$$

Note that higher order (through $\mathcal{O}(\alpha_s)^3$) corrections have been calculated and are less than 2% at the scale of interest, $Q^2 = 5$ GeV^2.

1.2 The data

The five year old data on the spin dependent proton structure function [2] has recently been augmented by data from the Spin Muon Collaboration (SMC) [3] and the E142 Collaboration [4] on spin dependent deuteron and neutron structure functions respectively.

The new information gained from these experiments is in the non-singlet sector since these experiments probe three different combinations of the two (u and d) valence densities in a nucleon. In fact, the problem is over specified. Hence we can get stringent bounds not only on the Bjorken Sum Rule [5] (which involves moments of non-singlet densities) but also on the x-dependence of these non-singlet densities.

However, in order to extract the g_1 structure functions from the measured asymmetries in these experiments, we need to know the denominators in the asymmetry equations, i.e., the x-dependent unpolarised structure functions, F_1.

Recent data in the unpolarised sector from both NMC [6] and HERA [7] have radically changed our understanding of the small-x behaviour of spin independent structure functions [8]. This will affect the extraction of the spin dependent functions from the asymmetry data. The EMC used the old EMC F_2 data and obtained F_1 by setting the ratio, R (of longitudinal to transverse photoproduction cross sections), to zero while the E142 collaboration used their older F_2 and R data. (Specifically, $F_1 = F_2/(2x(1+R))$.) The NMC have used their own current F_2 data as well as corrected for non-zero R. Furthermore, all the experiments are at different average Q^2 values. Table 1 shows the results as announced by the various groups, in comparison with theoretical predictions.

Table 1. The results of various polarised DIS experiments: the moment of g_1 vs the Ellis Jaffe [9] prediction and the total quark contribution ($2\Delta\Sigma \equiv \Delta u + \Delta d + \Delta s$) to the proton spin are listed. Also shown are the Bjorken Sum Rule [5] computations using two data sets at a time.

Collab. & Target(Q^2)	$\int g_1 dx$ measured/EJSR	$2\,\Delta\Sigma$
EMC p(10GeV^2)	$0.126 \pm 0.01 \pm 0.015/$ 0.189 ± 0.005	$0.12 \pm 0.09 \pm 0.13$
SMC D(5GeV^2)	$0.023 \pm 0.02 \pm 0.015/$ 0.094 ± 0.005	$0.06 \pm 0.2 \pm 0.15$
E142 n(2GeV^2)	$-0.022 \pm 0.011/$ -0.021 ± 0.018	0.57 ± 0.11
	B.S.R. (Measured/Predicted)	
EMC + SMC pD(5GeV^2)	$0.20 \pm 0.05 \pm 0.04/0.189 \pm 0.005$	
SMC + E142 Dn(2GeV^2)	$0.146 \pm 0.021/0.183 \pm 0.007$	

While the combined EMC/SMC data are essentially in agreement with the Bjorken sum rule, the EMC/E142 data were found to differ by nearly two standard deviations. Various reanalyses have been done, taking into consideration the fact that these data are at different Q^2 values and cannot be combined per se and also by including higher twist effects. Recently, the SMC [10] has done a combined analysis of all these data. They claim that there is no conflict between the data; furthermore, that the Bjorken Sum Rule is confirmed to within 17% uncertainty using these data. Their procedure involved a reanalysis of the *neutron* spin dependent structure function data, using the EMC/SMC data, which have smaller errors, to determine the neutron spin dependent structure function, $g_1^n(x)$, at small-x values. Hence, there exist widely differing viewpoints [11], [12], [13] on whether the Bjorken sum rule is violated or not.

1.3 The phenomenology

In what follows, we shall study, in depth, the behaviour of *both* the asymmetries and the structure functions at small- and large-x respectively and the impact on the moments and Bjorken sum rule of extrapolation to these experimentally inaccessible kinematic regimes.

Before we begin the actual analysis, the following comments are in order:

1. The spin independent as well as spin dependent densities can be constructed from the helicity densities, $q_f^\uparrow(x)$ and $q_f^\downarrow(x)$, of f-flavour quarks with momentum fraction x and spins parallel and opposed to that of the nucleon. Specifically, $\widetilde{q} = q^\uparrow - q^\downarrow$ and $q = q^\uparrow + q^\downarrow$. For a large-x behaviour, $q^{\uparrow,\downarrow} \sim (1-x)^{\alpha_\uparrow,\alpha_\downarrow}$, *both* $\widetilde{q}$ and q have a behaviour given by $q, \widetilde{q} \sim (1-x)^\alpha$, where α is the smaller of $(\alpha_\uparrow, \alpha_\downarrow)$. Hence, they vanish at large-x at the same rate.

2. $q^\uparrow$, $q^\downarrow$ are positive definite densities; hence, $|\widetilde{q}| \leq |q|$ for all x. Hence the partonic asymmetries, $A_q \equiv \widetilde{q}/q$, are bounded by ± 1 . In fact, unless $\alpha_\uparrow = \alpha_\downarrow$ (which is unlikely), $A_q \to \pm 1$ as $x \to 1$. Furthermore, it is experimentally established that the unpolarised u-quark density dominates over the d and s at large-x; hence, the proton, neutron and deuteron asymmetries **must** approach each other as $x \to 1$.

Our observations now follow automatically. We use the parametrisation of $F_2^{p,d}$ as given by the NMC [6] (with $F_2^n = 2F_2^D - F_2^p$) and use $R = \sigma_L/\sigma_T$ as given by the SLAC group [14]. This way, there is no need to specify the individual parton densities. We assume all the A_1 to be Q^2 independent, as observed, and analyse all data at $Q^2 = 5$ GeV^2 .

1.4 The unconstrained fits

The corresponding g_1 from the three experiments are shown in fig. 1; the solid curve is the best fit to the data for a parametrisation, $g_1(x) = Nx^\alpha(1+bx)(1-x)^\beta$, with $b = 0$ for the proton case and a large-x dependence, $(1-x^2)^\beta$, specially for the neutron case due to the very fast fall-off of g_1^n at large-x. (The errors in the deuteron data preclude us from specifying the exact crossing point, x_0 $(= -1/b)$, where the structure function (or asymmetry) changes sign; the two curves shown indicate the maximum extent of variation of b). These fits to the structure functions result in fits to the asymmetry shown by the dotted lines in fig. 2. It is observed that the intermediate-x asymmetry is distinctly different from zero only for the proton.

The corresponding moments are shown in table 2. We assume our errors to be the same as the experimentally quoted ones as the errors due to the fit are negligible in all but the deuteron case. The range of variation in the deuteron case due to the allowed range, $0.04 \leq x_0 \leq 0.06$, may be considered as an error in the moment coming from the fit. The moments are not all in agreement (to within 1σ) with the predictions of the Bjorken sum rule. Of course, the proton and deuteron data are also not in accordance with the Ellis Jaffe prediction [9], although we do not discuss this further.

Table 2. The nonsinglet moments, $\int(g_1^p - g_1^n)dx$, $\int 2(g_1^p - g_1^d)dx$ and $\int 2(g_1^d - g_1^n)dx$ are shown for $Q^2 = 5$ GeV^2.

Target	$\int g_1\, dx\ (Q^2)$ (expt)	(fit to g_1)	$\int g_1 dx$ (5 GeV^2) from fit to A_1.
p	$0.126 \pm 0.01 \pm 0.015(10)$	0.125(10) 0.124(5)	0.131
n	-0.022 ± 0.011 (2)	−0.030(2) −0.036(5)	−0.035
d	$0.023 \pm 0.02 \pm 0.015(5)$	0.0176(5)	0.052
d′	"	0.0135(5)	0.042

We now move on to the x dependence of these structure functions, which is really the focus of the talk. We first express every quark density as a sum of valence and sea parts, i.e.,

$$q^f(x) = q_V^f(x) + q_s^f(x)\ ; \qquad \overline{q}^f(x) = q_s^f(x).$$

We ignore (small) corrections due to deviations from the Gottfried sum rule, i.e., we assume $\overline{u} = \overline{d}$. The three measurements that we have in the spin dependent sector are *not* sufficient to extract any of the unknown densities individually; we can only extract the non-singlet combination $(\widetilde{u}(x) - \widetilde{d}(x))$. This is because all the experiments measure quantities whose singlet parts are identical. Then the combinations, $(g_1^p - g_1^n)$, $2(g_1^p - g_1^d)$ and $2(g_1^d - g_1^n)$ all give information on the non-singlet quantity, $(\widetilde{u} - \widetilde{d})$. We use the parametrised fits to the various structure functions at $Q^2 = 5$ GeV^2 to plot these differences which we denote as *pn*, *pd* and *dn*, as functions of x in fig. 3. We have chosen the deuteron parametrisation, d, corresponding to $x_0 = 0.06$. Similar results are obtained on using the d', i.e., the $x_0 = 0.04$ fit. The non-singlet moments of all combinations are shown in table 3. The *dn* combination is seen to deviate substantially from the Bjorken sum rule prediction.

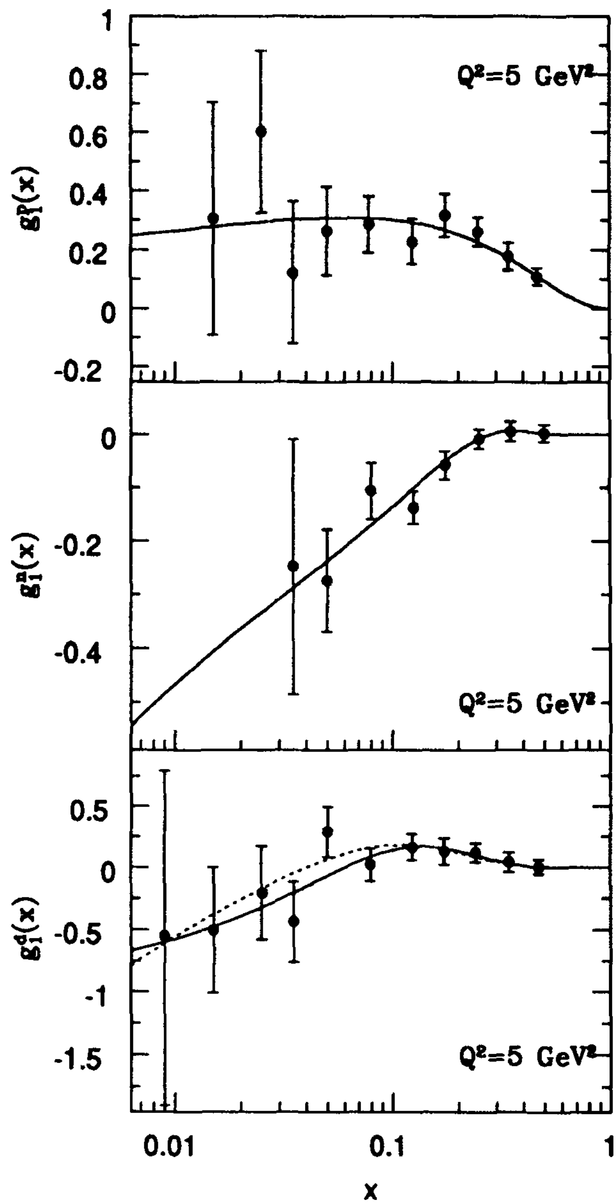

Figure 1. The structure function data for the proton, neutron and deuteron are shown as a function of x. The solid curves are our fits to the data.

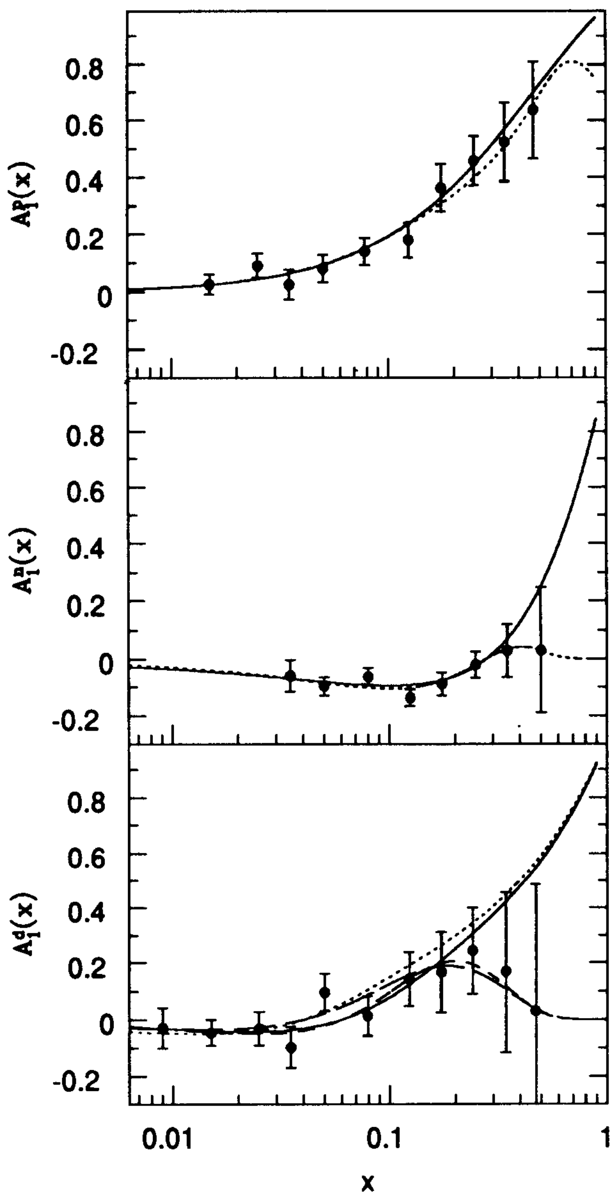

Figure 2. The proton, neutron and deuteron asymmetry data are shown as a function of x. The solid curves are our constrained fits to the data ($A_1(1) = 1$), while the dotted ones come from fits to the structure functions as shown in fig. 1.

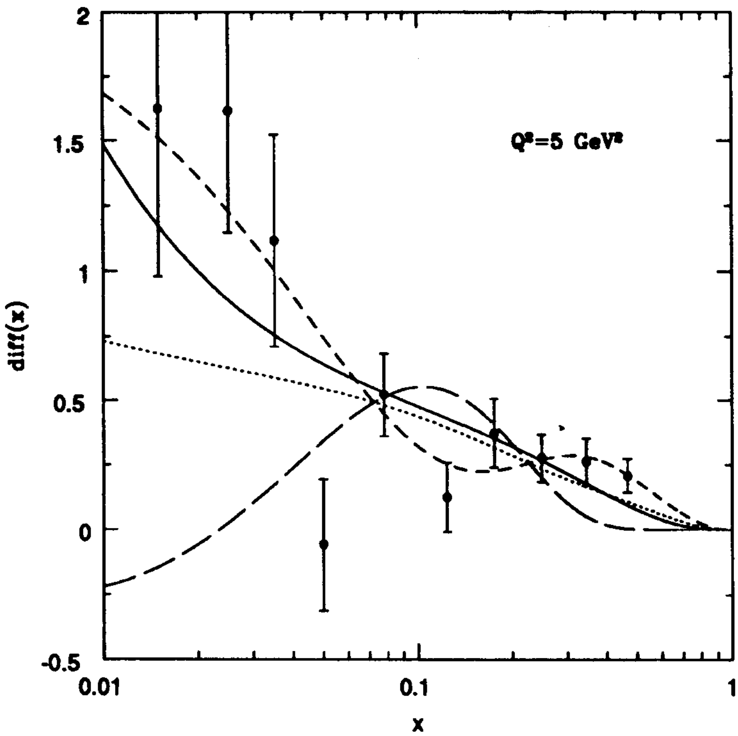

Figure 3. The non-singlet combinations, $(g_1^p - g_1^n)$ (dotted curves), $2(g_1^p - g_1^d)$ (dashed curves) and $2(g_1^d - g_1^n)$ (long dashed curves) are plotted as a function of x. The solid line corresponds to a prediction in agreement with the Bjorken sum rule.

Table 3. The nonsinglet moments, $\int(g_1^p - g_1^n)\mathrm{d}x$, $\int 2(g_1^p - g_1^d)\mathrm{d}x$ and $\int 2(g_1^d - g_1^n)\mathrm{d}x$ are shown for $Q^2 = 5$ GeV^2.

Set	Moments of non-singlet structure functions from fits to	
	$g_1(x)$	$A_1(x)$
$p-n$	0.161 ± 0.015	0.166 ± 0.015
$2(p-d)$	(0.215–0.223) ± 0.032	(0.160–0.179) ± 0.032
$2(d-n)$	(0.099–0.107) ± 0.033	(0.154–0.173) ± 0.033
Average value:		0.166 ± 0.013
Bjorken Sum Rule prediction:		0.189 ± 0.005

To proceed further, we need some more input. For this, we parametrise the *spin dependent* valence densities [15] using the Carlitz Kaur model [16]. Specifically, we have

$$\begin{aligned}\widetilde{u_V}(x) &= \left(u_V(x) - \tfrac{2}{3}\,d_V(x)\right)\cos 2\phi(x)\ ,\\ \widetilde{d_V}(x) &= -\tfrac{1}{3}\,d_V(x)\cos 2\phi(x)\ ,\end{aligned}$$

where $\cos 2\phi(x) = 1/(1+\gamma\sqrt{1-x}/x^{0.1})$ is a one-parameter spin dilution function. We use the GRV parametrisations [17] for the spin independent densities, though results are *not* sensitive to choice of parametrisation. Since u_V and d_V are known, γ fixes $\widetilde{u_V}$ and $\widetilde{d_V}$ individually.

The input that we use is the Bjorken sum rule prediction itself! We *choose* γ so that the combination, $(\widetilde{u_V}(x) - \widetilde{d_V}(x))/6$ integrates to g_A, so the Bjorken sum rule is satisfied. The resulting $\mathcal{O}(\alpha_s)$ corrected non-singlet combination, [18] $(\widetilde{u_V}(x) - \widetilde{d_V}(x))(1-\alpha_s/\pi)/6$, is plotted as the solid curve in fig. 3, for comparison with the data differences. We see that the *pd* and *pn* combinations are quite close to the Carlitz-Kaur fit while *dn* deviates severely from it, especially at small-x. Furthermore, at large-x ($x \geq 0.2$), the combination *dn* underestimates the fit while *pd* overestimates it. We have also plotted the actual data points for the *pd* combination (made possible as EMC and SMC have data in the same x-bins) to give an idea of the errors involved.

Therefore, deviations from the expected behaviour (where by 'expected' we mean an x-dependence of the non-singlet density compatible with the Bjorken sum rule) occur at both large as well as small values of x. Hence deviations in the moments of these differences also arise from both these kinematically distinct regions. Is there any way to explain these deviations?

1.5 The constrained fits

Note that the errors on the data, especially at $x \geq 0.2$, are fairly large. Also recall the phenomenological bias that *all* the asymmetries should approach each other at large-x. Furthermore, the parton model expects this value to be ± 1 at $x = 1$. We now *reparametrise* the data with this bias or constraint. Our motivation for such a procedure is that the proton asymmetry is *clearly distinct from zero* at large-x values. Hence, the deuteron and neutron asymmetries (which are not so clear, due to large errors at large-x, as can be seen seen from fig. 2) *must* also follow this trend. So we fit the asymmetries to the form, $A_1^{p,d,n}(x) = N x^{\alpha}\,\{1 - \exp\left[a\,(x - x_0)\right]\}$, where the constraint $A_1(1) = 1$ determines N from $N\,\{1 - \exp\left[a\,(1 - x_0)\right]\} = 1$, and $x_0 = 0$ for the proton. These fits are shown as solid lines in fig. 2. The differences between the two fits are strongly marked at larger x values for the neutron and deuteron. Surprisingly, the χ^2 values are not very different for these new fits as compared to the case when the data were fitted without any theoretical bias (see table 4). This is obvious for the proton case as the data are clearly compatible with the assumption that $A_1^p \to 1$ as $x \to 1$. For the deuteron and neutron data, this happens because the data in the intermediate-x region are weighted by large errors.

Table 4. The χ^2 values of the fits (at $Q^2 = 5$ GeV^2) to the data in figs. 1 and 2 are shown. d (d') is the deuteron fit corresponding to $x_0 = 0.06$ (0.04).

Target	No. of data points	No. of parameters in fit	χ^2/d.o.f from fits to $g_1(x)$	$A_1(x)$
p	10	3	4/7	4/8
n	8	4	4/4	5/5
d	11	4	5/7	6/8
d′	11	4	5/7	8/8

These new fits to the asymmetry yield corresponding spin dependent structure functions as shown in fig. 4.
While the small-x fits are good, the large-x data are systematically overestimated (while remaining within 1σ) for the neutron and deuteron. The next logical step is to find the moments of *these* fits; these are shown in the last column of table 2. The moments of the proton and neutron do not change much, as expected; however, the deuteron moment goes up remarkably. The deuteron result is very sensitive to the value of x_0. Earlier, the (negative) integral in the small-x region was being more or less cancelled by the positive contribution from the remaining interval. By increasing the large-x contribution, the moment is now clearly positive.

We show the x-dependence of these new (theoretically constrained) non-singlet combinations, pn, pd and dn, as defined earlier, in fig. 5. The small-x fits do not change much; what is dramatic, however, is the behaviour of all the combinations at large-x, which now precisely satisfy the theoretical requirements. The agreement of *all* the three combinations with the predicted non-singlet density is visible from $x = 0.2$ onwards. (Recall that the earlier parametrisations had yielded fits which either overestimated or underestimated the non-singlet density in this region). Furthermore, the corresponding moments of all these non-singlet distributions are now very much in accordance with the prediction of the Bjorken sum rule, as seen from table 3. The errors are those from the data; errors in the fit are only accounted for in the deuteron case, leading to corresponding range of values for the pd and dn moments. We emphasise that these fits, though tailored to theoretical considerations, are definitely consistent with the data.

To sum up, while the results are sensitive to large-x extrapolation, the non-singlet data from all the experiments are compatible, within errors (1σ), with theoretical expectations in the large-x region and the Bjorken sum rule. The small-x non-singlet data still deviate considerably from the naïve parton model picture. This sector is already controversial and poorly understood. There has been a great deal of literature on the subject; we do not comment on it further.

Since we do not have three independent measurements, we cannot specify the individual quark contributions (Δu, Δd and Δs) without a further assumption We choose this to be the constraint coming from the non-singlet SU(3) symmetric combination:

$$\Delta u + \Delta d - 2\Delta s = g_8 \ ,$$

where $g_8 = 0.584 \pm 0.018$ based on nucleon and Σ^- decay data.

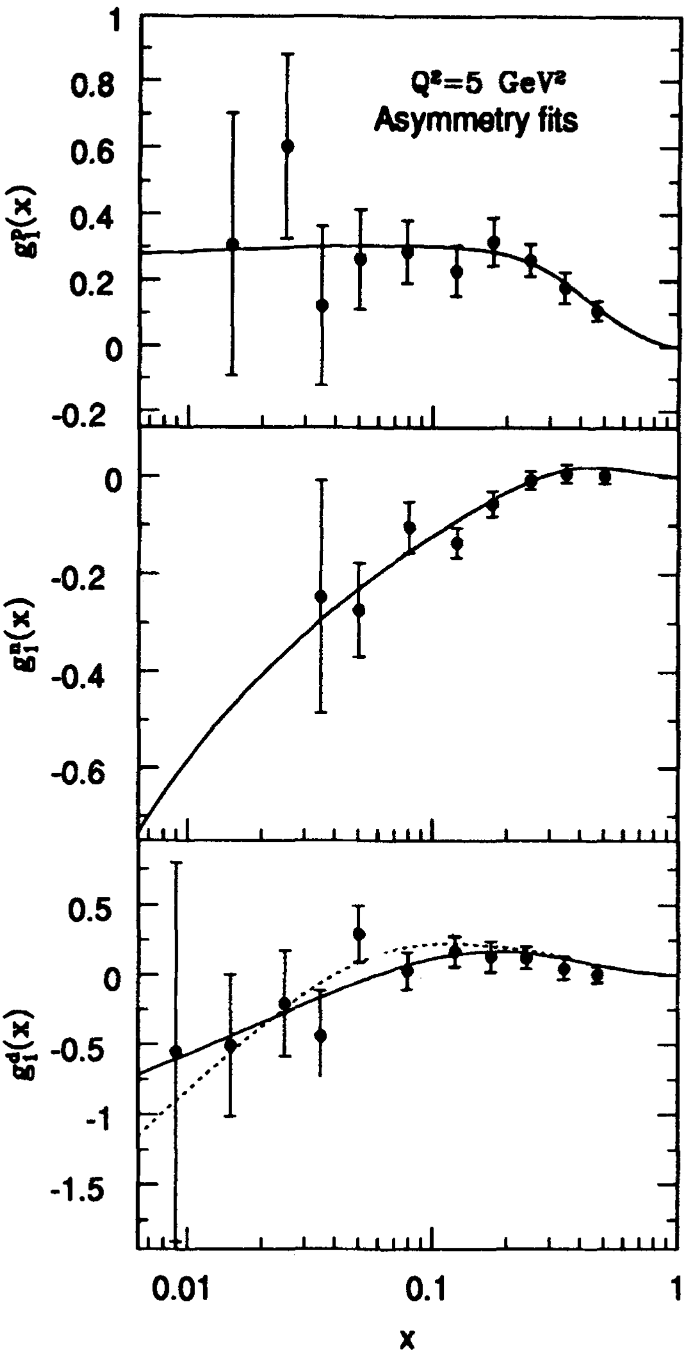

Figure 4. The same as fig. 1, but with the solid curves corresponding to constrained fits to the asymmetry data.

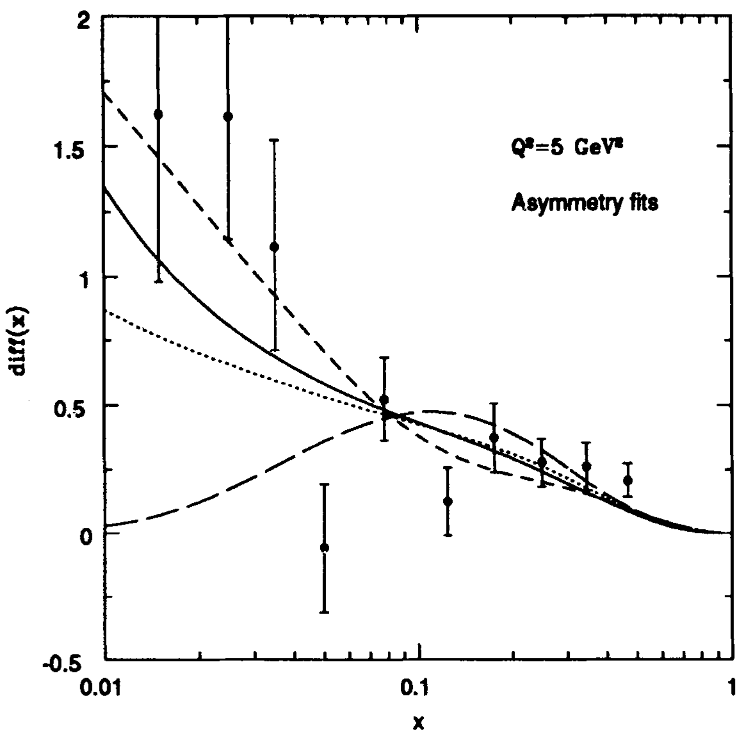

Figure 5. The same as fig. 3, with the structure functions evaluated from constrained fits to the asymmetry data.

We then use the moment of the proton g_1 structure function and $(\Delta u - \Delta d) = 1.097 \pm 0.083$ obtained from tables 2 and 3 respectively to get

$$\Delta u = 0.701 \pm 0.037, \quad \Delta d = -0.396 \pm 0.091, \quad \Delta s = -0.140 \pm 0.050 ,$$

so that

$$\Sigma \equiv \frac{1}{2}(\Delta u + \Delta d + \Delta s) = 0.083 \pm 0.055 .$$

This is still small, just as the nearly zero result first obtained by the EMC (though with large errors). In some sense then, the new data not only indicate that the Bjorken sum rule is valid, but also that the original controversy on the nature of the nucleon spin and the smallness of the quark contribution remains.

We emphasise the need for more data, particularly in the intermediate and large-x region. Only when the data improve to a point where the large-x p, d and n asymmetries clearly differ from each other can we say that the old proton "singlet spin puzzle" has been joined by a new "non-singlet spin puzzle!" In short, we believe that current data are not in conflict with the Bjorken sum rule.

We briefly comment on possible higher twist effects. These affect the large-x data. The aymmetries are seen to be largely Q^2 independent. So all the Q^2 dependence of $g_1^{p,d,n}$ *must* come from that of F_2 and R. This has already been accounted for. In fact, the NMC parametrisation of F_2 includes both logarithmic and higher twist corrections. Hence, it is unlikely that higher twist terms play any major role in the process of lining up the data with the Bjorken sum rule. Certainly we have shown that agreement can be brought about without the use of such extra terms.

Finally, it seems that non-singlet densities are interesting and deserve to be studied in their own right. In this context, we recall a proposal [19] for studying spin dependent valence densities through semi-inclusive hadroproduction in polarised DIS, which can be immediately performed without difficulty. In particular, the asymmetry for K meson production, (defined to be the usual spin asymmetry, but for the difference in production of K^+ and K^- mesons) is

$$A^K(x) \equiv \frac{d\sigma^{K^+}(\uparrow\downarrow - \uparrow\uparrow) - d\sigma^{K^-}(\uparrow\downarrow - \uparrow\uparrow)}{d\sigma^{K^+}(\uparrow\downarrow + \uparrow\uparrow) - d\sigma^{K^-}(\uparrow\downarrow + \uparrow\uparrow)} = \frac{\widetilde{u_V}(x)}{u_V(x)} ,$$

i.e., it measures $\widetilde{u_V}$. The asymmetry for π production involves both $\widetilde{u_V}$ and $\widetilde{d_V}$ although it uses the currently controversial [20] assumption of isospin invariance (the former does not). Certainly these two asymmetries should well be able to supplement results from inclusive DIS as both are expected to be large and positive over all x.

2. The spin dependent gluon densities

Finally, we briefly address the question of extraction of the spin aependent gluon densities. Since the nucleon spin is not saturated by the quark contribution, the gluons are likely to contribute substantially, although they occur at $\mathcal{O}(\alpha_s)$ in the expression for the spin dependent structure function. This has been the subject of a great deal of controversy; in fact, it was even argued that the magnitude of the gluonic contribution to the spin of the proton may well turn out to be renormalisation scheme dependent! We outline why this is not the case, and pinpoint the source of ambiguity in this sector. To do this, we go back to the parton interpretation of $g_1(x)$. This section is mainly a review of already existent formalism [21, 22, 23, 24] addressing the question, "Is there a sizable singlet quark/gluon contribution to the first moment of g_1?"

Look at the leading Q^2 dependence of the moments of the parton densities (or spin contributions to that of the nucleon):

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left(\alpha_s(Q^2)\Delta g(Q^2)\right) &= 0 \\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\Delta\Sigma(Q^2)\right) &= 0\,, \end{aligned}$$

where $t = \ln(Q^2/\mu^2)$ and μ is the renormalisation scale.

Since α_s vanishes logarithmically as $Q^2 \to \infty$, Δg *grows* with Q^2 at the same rate. Hence Δg may be large even at finite Q^2. Since quarks and gluons (as also their relative orbital angular momentum) contribute to the nucleon spin, there is *apriori* no reason to leave this contribution out of the g_1 calculation. This means we need to go beyond the leading order. (Contrast this with the spin independent case where the momentum, $xg(x) \to 0$ as $\alpha_s \to 0$).

2.1 The parton model calculation

Parton densities are not uniquely defined beyond the leading order. Furthermore, the gluon contribution to g_1 suffers from both infrared and collinear divergences. In other words, the result depends on the mode of regularisation of the contributing box diagram (Fig 6).

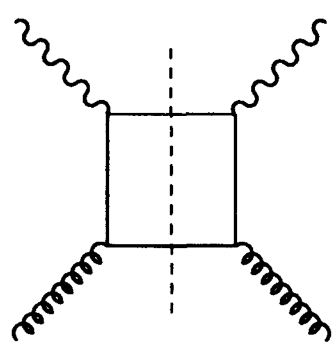

Figure 6. The gluon contribution to the g_1 structure function in the parton model.

2.2 The factorisation approach

Here, there is a clean separation of the 'hard' and 'soft' components of the total cross section. The structure function symbolically evaluates to

$$g_1^{\gamma^* N} = \hat{g}_1^q \otimes f_{\frac{\Delta q + \Delta \bar{q}}{N}} + \hat{g}_1^g \otimes f_{\frac{\Delta g}{N}} \ . \qquad (1)$$

The two terms represent the quark and gluon contributions to the structure function, respectively. $f_{p+/N}(x)$ ($f_{p-/N}(x)$) is the probability of locating a parton ($p = q, \bar{q}, g$) in a nucleon, N, with spin parallel (antiparallel) to that of the parent nucleon and $f_{\Delta p/N}$ is the difference of these two probabilities and was denoted as $\tilde{p}$ in the previous section for simplicity. The symbol $\otimes$ stands for the convolution,

$$a(x) \otimes b(x) \equiv \int_x^1 \frac{dy}{y} a(x/y) b(y) \ .$$

Each term is therefore a convolution of the soft (nonperturbative, and therefore incomputable) parton densities and the hard (perturbative and computable) scattering coefficients. These hard scattering coefficients, $\hat{g}_1^{q,g}$ are target independent. The simplest way to calculate them is to replace the nucleon target N by a known (point) target, say, gluon. $f_{\Delta q/g}$ and $f_{\Delta g/g}$ are now computable. So is $g_1^{\gamma^* g}$. Hence the only unknowns in eq. (1) are the hard scattering coefficients. However, since $f_{\Delta q/g}$ is of order $\mathcal{O}(\alpha_s)$ we need to compute $\hat{g}_1^q$ only to leading order, $\mathcal{O}(1)$. This is already known ($= \delta(1-x)/2$). This leaves $\hat{g}_1^g$ as the only unknown which can thus be found.

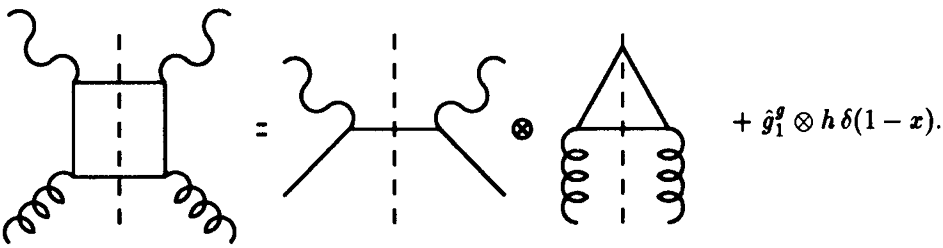

Figure 7. The gluon–virtual photon scattering cross section in the factorisation approach. The LHS corresponds to fig. 6. The terms in the RHS are the quark and gluon contributions respectively.

Specifically, with gluon replacing the nucleon target in eq. (1), the equation can be diagrammatically represented as shown in fig. 7. The LHS of fig. 7 is known (it is the parton model result) and has both IR and collinear divergences. However, these are exactly cancelled by corresponding divergences on the RHS. However, the RHS is renormalisation scheme dependent, while the LHS is not. The result is that $\hat{g}_1^g$ is hard, prescription independent, and scheme dependent. Due to this, the gluonic contribution to the first moment of the nucleon structure function ($\int g_1(x) dx$) vanishes or does not (equals $-\alpha_s/2\pi$) depending on whether the computation is performed in the $\overline{\rm MS}$ or MOM scheme.

This means that meaningful results can be obtained only if all quantities (F_2, g_1, etc) are defined upto next-to-leading order in the same scheme. Then only will the experimentally measurable cross sections be scheme invariant. However, we emphasise that the spin dependent gluon density and its moment Δg are well defined always and therefore can be unambiguously measured. In fact, the advantage of the factorisation approach is that it yields [25] an *operator definition* of the densities (both spin independent and spin dependent). It is the coefficient of the Δg term in the cross section that is scheme dependent.

A simple way of seeing this is to write the spin dependent DIS cross section for a nucleon target of mass, M:

$$\sigma_N(x,Q^2,M) = \hat{\sigma}_q(x,Q^2,\mu) \otimes \tilde{q}(x,M,\mu) + \hat{\sigma}_g(x,Q^2,\mu) \otimes \tilde{g}(x,M,\mu) \,, \quad (2)$$

where $\tilde{q} = f_{\Delta q/N}$ and $\tilde{g} = f_{\Delta g/N}$ as before. Replacing N by a gluon target, factorisation yields

$$\begin{aligned} \sigma_g(x,Q^2,M) &= \hat{\sigma}_q(x,Q^2,\mu) \otimes f_{\Delta q/g}(x,M,\mu) \\ &\quad + \hat{\sigma}_g(x,Q^2,\mu) \otimes f_{\Delta g/g}(x,M,\mu) \,. \end{aligned} \quad (3)$$

Here $\hat{\sigma}_{q,g}$ are target independent while $f_{\Delta g/g} \sim \delta(1-x)$. Substituting for $\hat{\sigma}_g$ from eq. (3) in eq. (2), we have

$$\sigma_N = \hat{\sigma}_q \otimes \tilde{q} + \sigma_g \otimes \tilde{g} - \hat{\sigma}_q \otimes f_{\Delta q/g} \otimes \tilde{g} \,.$$

The last term can be thought of as either a correction to $\tilde{q}$ or a correction to σ_g. Hence we are free to redefine

$$\begin{aligned} \tilde{q}' &= \tilde{q} - \delta f_{\Delta q/g} \otimes \tilde{g} \\ \text{OR} \quad \hat{\sigma}_{g'} &= \sigma_g - \hat{\sigma}_q \otimes \delta f_{\Delta q/g} \,. \end{aligned}$$

That is, there is ambiguity in q and/or $\hat{\sigma}_g$ to order $\mathcal{O}(\alpha_s)$, *but none in* $\tilde{g}$ or in $g_1(x)$. Hence, although g_1 itself may not be able to distinguish between models with large and small Δg, it will be possible (say, in direct photon production in polarised pp collisions) to establish the gluonic contribution to the spin of the nucleon.

3. Conclusions

We have established that current polarised DIS data on spin dependent nucleon structure functions are consistent with predictions in the nonsinglet sector (Bjorken Sum Rule). The quark contribution to the spin of the proton remains small (about 17%). It is therefore likely that gluons may contribute substantially. In view of this, and other controversies in the field, we have reviewed the formalism of DIS and the contribution of the gluonic term to the structure functions, in the factorisation approach. We emphasise that, although DIS cannot measure the gluon contribution to the proton spin, such a quantity is well-defined and can be measured elsewhere. In fact, several new structure functions [25] have recently been defined, occurring in both DIS and pp collision processes in longitudinal as well as transversely polarised collisions, that may throw more light on the issue of the proton spin.

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