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How good is the quenched approximation of QCD?

MAARTEN F. L. GOLTERMAN

Department of Physics, Washington University, St. Louis, MO 63130, U S A

Abstract. The quenched approximation for QCD is, at present and in the foreseeable future, unavoidable in lattice calculations with realistic choices of the lattice spacing, volume and quark masses. In this talk, I review an analytic study of the effects of quenching based on chiral perturbation theory. Quenched chiral perturbation theory leads to quantitative insight on the difference between quenched and unquenched QCD, and reveals clearly some of the diseases which are expected to plague quenched QCD.

1. Introduction

The lattice formulation of QCD has proven to be a powerful tool for computing QCD quantities of direct phenomenological interest, such as hadron masses, decay constants, weak matrix elements, the strong coupling constant, *etc.* (For reviews see for instance refs. [1, 2], or the proceedings of Lattice 93 [3].)

In order to perform such computations numerically, one obviously needs to consider a system with a finite number of degrees of freedom, which is accomplished by putting lattice QCD in a finite box. This box is then hopefully large enough to accomodate the physics one is interested in without serious finite size effects. This leads to the requirement that the Compton wavelength of the particles of interest is sufficiently smaller than the linear dimension of the box, *i.e.* the mass has to be large enough for the particles to fit in the box.

In order to have a small enough lattice spacing, small enough masses (in particular for the pion) and a large enough box size, one needs a large number of degrees of freedom in a numerical computation. It turns out that for QCD with realistic choices of the lattice spacing a, volume V and the quark masses (in particular the light quark masses), the presently available computational power is not adequate. The most severe problem comes from the fermion determinant, the logarithm of which is a very nonlocal part of the gluon effective action (specially for light quark masses). This nonlocality slows down the Monte Carlo algorithms dramatically.

In order to circumvent this problem, most numerical computations in lattice QCD have been done in the quenched approximation, in which one simply sets the fermion determinant equal to one [4]. This amounts to ignoring all fermion loops which occur in QCD correlation functions (except those put in by hand through the choice of operators on the external lines). While some handwaving arguments exist as to why this might not be unreasonable, the quenched approximation does introduce an uncontrolled systematic error. Since the effect of a fermion loop is roughly inversely proportional to its mass, this error is expected to be particularly severe for quantities involving light quarks. It appears therefore that chiral perturbation theory ChPT maybe a useful tool for investigating the difference between quenched and unquenched ("full") QCD.

In this talk, I will review a systematic approach to the study of the quenched approximation through ChPT [5, 6, 7, 8]. There are two reasons why ChPT is useful in this context:

• It turns out that ChPT can be systematically adapted to describe the low energy sector of quenched QCD [6]. It will therefore give us nontrivial, quantitative information on the difference between quenched and full QCD.

• ChPT describes the approach to the chiral limit, and can be used for extrapolation of numerical results to small masses and large volumes. If these results come from quenched computations, one will of course need a quenched version of ChPT. (For finite volume ChPT, see refs. [9]. For quenched finite volume results, see refs. [6, 7].)

In this review, I will concentrate on the first point. I will first show how ChPT is developed for the quenched approximation, and then use it for a quantitative comparison between full and quenched QCD. The quantities that I will discuss are f_K/f_π [6, 8] and the octet baryon masses [10].

I will then address a number of theoretical problems that arise as a consequence of quenching. That such problems arise is no surprise, as quenching QCD mutilates the theory quite severely. It is however quite instructive to see what the detailed consequences are.

2. Systematic ChPT for quenched QCD

In this section I will outline the construction of a chiral effective action for the Goldstone boson sector of quenched QCD [6]. I will first introduce the formalism, and then show how it works in some examples. For early ideas on quenched ChPT, see ref. [11, 5].

We will start from a lagrangian definition of euclidean quenched QCD. (We will restrict ourselves entirely to the euclidean theory which can be defined by a pathintegral. Hamiltonian quenched QCD presumably does not exist.) To the usual QCD lagrangian with three flavors of quarks q_a, $a = u, d, s$, we add three ghost quarks $\tilde{q}_a$ with exactly the same quantum numbers and masses m_a, but with opposite, bosonic, statistics [11]:

$$\mathcal{L}_{\text{quarks}} = \sum_a \overline{q}_a(\not{D} + m_a)q_a + \sum_a \overline{\tilde{q}}_a(\not{D} + m_a)\tilde{q}_a, \tag{1}$$

where $\not{D}$ is the covariant derivative coupling the quark and ghost quarks to the gluon field. The gluon effective action produced by integrating over the quark- and ghost quarkfields vanishes, since the fermion determinant of the quark sector is exactly cancelled by that of the ghost sector. Note that the ghost quarks violate the spin-statistics theorem.

We will now assume that mesons are formed as (ghost) quark - (ghost) antiquark pairs just like in ordinary QCD. This is basically equivalent to the notion that it is the dynamics of the gluons which leads to confinement and chiral symmetry breaking. The Goldstone particle spectrum of quenched QCD will then contain not only $q\bar{q}$, but also $\tilde{q}\bar{\tilde{q}}$, $q\bar{\tilde{q}}$ and $\tilde{q}\bar{q}$ bound states. We will denote this 36-plet by

$$\Phi \equiv \begin{pmatrix} \phi & \chi^\dagger \\ \chi & \tilde{\phi} \end{pmatrix} \sim \begin{pmatrix} q\bar{q} & q\bar{\tilde{q}} \\ \tilde{q}\bar{q} & \tilde{q}\bar{\tilde{q}} \end{pmatrix}. \tag{2}$$

Note that the fields χ and $\chi^\dagger$ describe Goldstone fermions.

The quenched QCD lagrangian (1) for vanishing quark masses has a much larger symmetry group than the usual $U(3)_L \times U(3)_R$ flavor group; it is invariant under the graded group $U(3|3)_L \times U(3|3)_R$ [6], where $U(3|3)$ is a graded version of $U(6)$ since it mixes the fermion and boson fields q and $\tilde{q}$. Writing an element U of $U(3|3)$ in block form as

$$U = \begin{pmatrix} A & C \\ D & B \end{pmatrix}, \tag{3}$$

the 3×3 matrices A and B consist of commuting numbers, while the 3×3 matrices C and D consist of anticommuting numbers.

We can now construct a low energy effective action for the Goldstone modes along the usual lines. We introduce the unitary field

$$\Sigma = exp(2i\Phi/f), \tag{4}$$

which transforms as $\Sigma \to U_L \Sigma U_R^\dagger$ with U_L and U_R elements of $U(3|3)$. Because we are dealing here with a graded group, in order to build invariants, we need to use the supertrace *str* and the superdeterminant *sdet* instead of the normal trace and determinant, with [12]

$$\begin{aligned} str(U) &= tr(A) - tr(B), \\ sdet(U) &= exp(str\log(U)) = det(A - CB^{-1}D)/det(B). \quad (strsdet) \end{aligned} \tag{5}$$

To lowest order in the derivative expansion, and to lowest order in the quark masses, the chiral effective lagrangian consistent with our graded symmetry group is

$$\mathcal{L}_0 = \frac{f^2}{8} str(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger) - v\; str(\mathcal{M}\Sigma + \mathcal{M}\Sigma^\dagger), \tag{6}$$

where $\mathcal{M}$ is the quark mass matrix

$$\mathcal{M} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}, \qquad M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}. \tag{7}$$

f and v are bare coupling constants which are not yet determined at this stage.

The symmetry group is broken by the anomaly to the smaller group $[SU(3|3)_L \times SU(3|3)_R] \circledS U(1)$ (the semidirect product arises as a consequence of the graded nature of the groups involved; the details are irrelevant for this talk). $SU(3|3)$ consists of all elements $U \in U(3|3)$ with $sdet(U) = 1$. The anomalous field is $\Phi_0 = (\eta' - \tilde{\eta}')/\sqrt{2}$, where the relative minus sign comes from the fact that in order to get a nonvanishing triangle diagram, one needs to choose opposite explicit signs for the quark and ghost quark loops, due to the different statistics of these fields. η' is the field describing the normal η' particle, while $\tilde{\eta}'$ is the ghost η' consisting of ghost quarks and ghost antiquarks. We will call the field Φ_0 the super-η' field. The field $\Phi_0 \propto str\log\Sigma = \log sdet\,\Sigma$ is invariant under the smaller symmetry group, and we should include arbitrary functions of this field in our effective lagrangian. Following ref. [13], the correct chiral effective lagrangian is

$$\begin{aligned}\mathcal{L} = \; & V_1(\Phi_0)str(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger) - V_2(\Phi_0)str(\mathcal{M}\Sigma + \mathcal{M}\Sigma^\dagger) \\ & + V_0(\Phi_0) + V_5(\Phi_0)(\partial_\mu\Phi_0)^2, \end{aligned} \tag{8}$$

where the function multiplying $i\, str(\mathcal{M}\Sigma - \mathcal{M}\Sigma^\dagger)$ can be chosen equal to zero after a field redefinition. This lagrangian describes quenched ChPT systematically, as we will show now with a few examples.

For our first example, let us isolate just the quadratic terms for the fields η' and $\tilde{\eta}'$, choosing degenerate quark masses for simplicity. We expand

$$\begin{aligned} V_1(\Phi_0) &= \tfrac{f^2}{8} + \ldots, \\ V_2(\Phi_0) &= v + \ldots, \\ V_0(\Phi_0) &= \text{constant} + \mu^2\Phi_0^2 + \ldots, \\ V_5(\Phi_0) &= \alpha + \ldots, \end{aligned} \tag{9}$$

and obtain

$$\begin{aligned}\mathcal{L}(\eta', \tilde{\eta}') = \; & \tfrac{1}{2}(\partial_\mu\eta')^2 - \tfrac{1}{2}(\partial_\mu\tilde{\eta}')^2 + \tfrac{1}{2}\alpha(\partial_\mu\eta' - \partial_\mu\tilde{\eta}')^2 \\ & + \tfrac{1}{2}m_\pi^2(\eta')^2 - \tfrac{1}{2}m_\pi^2(\tilde{\eta}')^2 + \tfrac{1}{2}\mu^2(\eta' - \tilde{\eta}')^2 + \ldots, \end{aligned} \tag{10}$$

where $m_\pi^2 = 8mv/f^2$. The relative minus signs between the η' and $\tilde{\eta}'$ terms in (10) come from the supertraces in (8), and are related to the graded nature of the chiral symmetry group of quenched QCD.

The inverse propagator in momentum space,

$$(p^2 + m_\pi^2)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (\mu^2 + \alpha p^2)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \tag{11}$$

clearly cannot be diagonalized in a p independent way, which is quite different from what one would expect from a normal field theory! Treating the $\mu^2 + \alpha p^2$ term as a twopoint vertex, one can easily show that the repetition of this vertex on one meson line vanishes, due to the fact that the propagator matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ multiplied on both sides by the vertex matrix $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ gives zero. This result coincides exactly with what one would expect from the quark flow picture for η' propagation, as depicted in fig. 1. The straight-through and double hairpin contributions do not contain any virtual quark loops, and are therefore present in the quenched approximation. All other contributions should vanish because they do contain virtual quark loops, and this is exactly what happens as a consequence of the (admittedly strange) Feynman rules for the propagator in the η'–$\tilde{\eta}'$ sector! This propagator is given by the inverse of (11) and reads

$$\frac{1}{p^2 + m_\pi^2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{\mu^2 + \alpha p^2}{(p^2 + m_\pi^2)^2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \tag{12}$$

in which the two terms correspond to the two first diagrams in fig. 1.

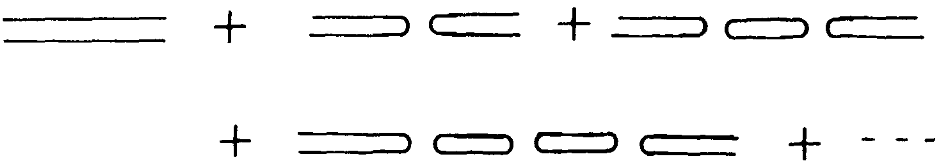

Figure 1. Quark flow diagrams for the η' propagator in full QCD

From (12) we learn several things. First, because μ^2, which in full ChPT would correspond to the singlet part of the η' mass, appears in the numerator, we need to keep the η' (and its ghost partner) in quenched ChPT: it cannot be decoupled by taking μ^2 large. Second, this "propagator" is definitely sick, due to the double pole term. It should be stressed here that this double pole term is an unescapable consequence of quenched QCD, and does not result from our way of setting up chiral perturbation theory. In the case of nondegenerate quark masses, this double pole also shows up in the π^0 and η propagators, due to mixing with the η'. I will return to these strange properties of quenched QCD in section 4.

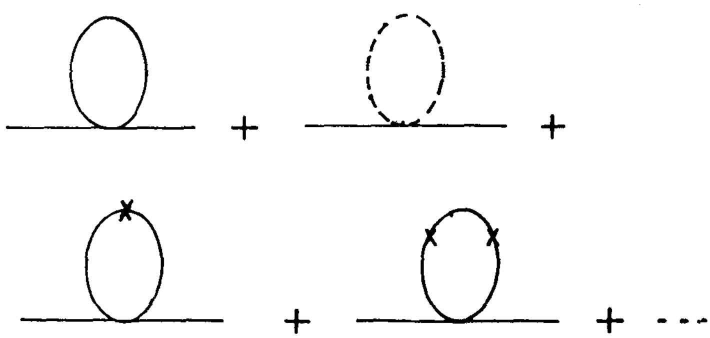

Figure 2. One loop pion selfenergy in quenched ChPT.

As a second example, we will consider the (charged) pion selfenergy at one loop, again with degenerate quark masses. I will set $\alpha = 0$ for simplicity. At one loop, the pion selfenergy only contains tadpoles, with either ϕ or χ lines (*cf.* (2)) on the loop. Also, on the ϕ loop, one can have an arbitrary number of insertions of the vertex μ^2 if the internal ϕ line is an $SU(3)$ singlet. These various contributions are drawn in fig. 2, where a solid line denotes a ϕ line, a dashed line denotes a χ line, and a cross denotes a μ^2 vertex. One finds that the diagrams with the ϕ and χ lines on the loop without any crosses cancel, and then, of course, that the diagrams with more than one cross vanish, using our earlier result for the η'–$\tilde{\eta}'$ propagator. We are left with only one term, and the result is

$$\Sigma_\pi(p) = \frac{2m_\pi^2}{3f^2}\mu^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+m_\pi^2)^2}. \tag{13}$$

The pion selfenergy is logarithmically divergent, but the origin of this divergence is completely different from those that arise in the unquenched theory, as it is proportional to μ^2. One can easily convince oneself that the diagrams in fig. 2 which cancel or vanish correspond to diagrams with virtual quark or ghost quark loops in the quark flow picture. (For early discussions of the quenched pion selfenergy in the quark flow picture, see refs. [11, 5].)

Before I go on to look at some quantitative results, I would like to discuss one aspect of the chiral expansion in quenched ChPT. The chiral expansion is basically an expansion in the pion mass (see *e.g.* ref. [14]). However, as we have argued above, in quenched ChPT there is unavoidably another mass scale, namely the singlet part of the η' mass, μ^2. For our expansion to be systematic as an expansion in the pion mass, we would have to sum up all orders in μ^2, at a fixed order in the pion mass. This is clearly a formidable task. In order to avoid this complication in a systematic way, we can think of $\mu^2/3$ (which turns out to be the natural parameter as it appears in the chiral expansion) as an independent small parameter. To check whether this makes any sense, one may note that from the experimental value of the η' mass one obtains a value $\mu^2/3 \approx (500\ MeV)^2$, which is roughly equal to the kaon mass squared, m_K^2. Of course, for quenched QCD the parameter μ^2 need not have the same value, after all quenched QCD is a different theory. A lattice computation of this parameter [15] gives $\mu^2_{\rm quenched}/\mu^2_{\rm full} \approx 0.75$. ($\alpha$ can be estimated from η–η' mixing, and is very small.) Finally, one may also note that both μ^2 and α are of order $1/N_c$, where N_c is the number of colors [16]. I will return to this point in section 4.

3. Quantitative comparison of quenched and full ChPT

Let us first consider the quenched result for the ratio of the kaon and pion decay constants f_K and f_π [6, 8]. I will set $\alpha = 0$ and take $m_u = m_d \equiv m$:

$$\left(\frac{f_K}{f_\pi}\right)^{1-\rm loop}_{\rm quenched} = 1+\frac{\mu^2/3}{16\pi^2 f_\pi^2}\left[\frac{m_K^2}{2(m_K^2-m_\pi^2)}\log\left(\frac{2m_K^2}{m_\pi^2}-1\right)-1\right]+(m_s-m)L. \tag{14}$$

L is a certain combination of "low energy constants" [13]. Since this constant is a bare parameter of the quenched chiral lagrangian, the result (14) is not directly comparable to the equivalent result for the full theory. In other words, in order to compare quenched and full QCD, we have to consider quantities which are independent of the bare parameters of the effective action. (Alternatively, we would need to extract the values of the bare parameters from some independent measurement or lattice computation, in this case, we would need independent determinations of L in both quenched and full QCD.) In the full theory, $(f_\eta f_\pi^{1/3})/f_K^{4/3}$ is such a quantity [13], but in the quenched theory this quantity is not well defined, due to the double poles which occur in the propagators of neutral mesons.

We will therefore choose to consider a slightly different theory, in which sufficiently many decay constants referring only to charged (*i.e.* off-diagonal) mesons are present [8]. This theory is a theory with two light quarks $m_u = m_d = m$ and two heavy quarks $m_s = m_{s'} = m'$. This theory contains a $u\bar{d}$ pion π, an $s'\bar{s}$ pion π' and a $u\bar{s}$ kaon K, with mass relation

$$m_K^2 = \frac{1}{2}(m_\pi^2 + m_{\pi'}^2). \tag{15}$$

One can show that the ratio $f_K/\sqrt{f_\pi f_{\pi'}}$ is independent of the low energy constants L. For the quenched theory we find

$$\left(\frac{f_K}{\sqrt{f_\pi f_{\pi'}}}\right)^{1-\text{loop}}_{\text{quenched}} = 1 + \frac{\mu^2/3}{16\pi^2 f_\pi^2}\left[\frac{m_\pi^2 + m_{\pi'}^2}{2(m_{\pi'}^2 - m_\pi^2)}\log\left(\frac{m_{\pi'}^2}{m_\pi^2}\right) - 1\right], \tag{16}$$

whereas in the full theory

$$\left(\frac{f_K}{\sqrt{f_\pi f_{\pi'}}}\right)^{1-\text{loop}}_{\text{full}} = 1 - \frac{1}{64\pi^2 f_\pi^2}\left[m_\pi^2\log\left(\frac{m_K^2}{m_\pi^2}\right) + m_{\pi'}^2\log\left(\frac{m_K^2}{m_{\pi'}^2}\right)\right]. \tag{17}$$

Note again that the logarithms in the quenched and unquenched expressions are completely different in origin.

We may now compare these two expressions using "real world" data, where we'll determine the value of the π' mass from the mass relation (15). With $m_\pi = 140\ MeV$, $m_K = 494\ MeV$ and $\mu^2/3 = 0.75 \times (500\ MeV)^2$ we find

$$\begin{aligned}\left(\frac{f_K}{\sqrt{f_\pi f_{\pi'}}}\right)^{1-\text{loop}}_{\text{quenched}} &= 1.049,\\ \left(\frac{f_K}{\sqrt{f_\pi f_{\pi'}}}\right)^{1-\text{loop}}_{\text{full}} &= 1.023,\end{aligned} \tag{18}$$

a difference of 3%. If we choose $\mu^2/3 = (500\ MeV)^2$, we find a difference of about 4%. This difference is small. Note however, that this is due to the fact that for this particular ratio, ChPT seems to work very well, both for the full and the quenched theories. If one only considers the size of the one loop corrections (the numbers behind the decimal point), the quenched and full results are very different. It is also possible, and in fact not unlikely, that part of the difference between the full and quenched theory gets "washed out" by the fact that we are considering a "ratio of ratios". It follows that the relative difference is a lower bound on the difference between the quenched and full values of the decay constants. For another quantity for which the difference between quenched and full ChPT has been calculated, see ref. [8].

Next, I will review some very recent work on baryons in quenched ChPT by Labrenz and Sharpe [10]. They calculated the one loop corrections to the octet baryon mass coming from the cloud of Goldstone mesons. They employed an effective lagrangian for quenched heavy baryon ChPT, constructed using the same techniques as described in section 2. In the case of degenerate quark masses, the result for the nucleon mass is

$$\begin{aligned}m_N = \quad &\overline{m} - \tfrac{3\pi}{2}(D-3F)^2\tfrac{\mu^2/3}{8\pi^2 f_\pi^2}m_\pi + 2(b_D - 3b_F)m_\pi^2\\ &+ \left[\tfrac{2}{3}(D-3F)(2D+3\gamma) + \tfrac{5}{6}\alpha(D-3F)^2\right]\tfrac{m_\pi^3}{8\pi f_\pi^2}.\end{aligned} \tag{19}$$

In this equation, $\overline{m}$, D, F, b_D, b_F and γ are bare parameters which occur in the baryon effective action. $\overline{m}$ is the bare "average" octet mass, D and F are the well known baryon-meson couplings, b_D and b_F are low energy constant which arise as a consequence of renormalization (see for instance refs. [18, 19]). γ is a new coupling which occurs because of the unavoidable presence of the super-η' in the quenched approximation. The term proportional to μ^2 comes from a diagram with a cross on the ϕ internal line, *i.e.* an insertion of the μ^2 twopoint vertex. Note that in this case there are also one loop corrections not involving μ^2 which survive the quenched approximation, in contrast to the pion selfenergy, (13), or f_K/f_π, (14). The authors of ref. [10] then calculated the coefficients using full QCD values for the various parameters (from ref. [19]). With $\alpha = 0$ and $\gamma = 0$ ($\gamma = 0$ is consistent with available information, which however is limited [17]), they obtained

$$m_N = 0.97 - 0.5\frac{\delta}{0.2}m_\pi + 3.4m_\pi^2 - 1.5m_\pi^3, \tag{20}$$

with $\delta \equiv \mu^2/(24\pi^2 f_\pi^2)$ and $\delta \approx 0.2$ for the full theory.

In ref. [10], (19) was also compared to recent numerical results from ref. [20]. These data are presented in fig. 3, where the scale $a^{-1} = 1.63\ GeV$ is set by f_π [10]. If one calculates the coefficients in (19) by "fitting" the four data points, one finds

$$m_N = 0.96 - 1.0m_\pi + 3.6m_\pi^2 - 2.0m_\pi^3. \tag{21}$$

This is only four data points for four parameters, and the "fit" is quite sensitive to for instance an additional m_π^4 term. From the agreement between (20) and (21) it appears that it is reasonable to apply ChPT to the results of ref. [20]. Note that the individual terms in (20) are quite large for the two higher pion masses in fig. 3 (this is not unlike the case of unquenched ChPT). From fig. 3 it is also clear that $(\overline{m}/f_\pi)_{\rm quenched} \neq (\overline{m}/f_\pi)_{\rm full}$ because of the term linear in (19), which is absent in full ChPT.

Labrenz and Sharpe then went on to consider octet mass splittings. In order to remove effects which can be accomodated by a change of scale, they calculated the ratios

$$R_{ij} = \frac{m_i}{m_j}, \quad i,j = N,\Lambda,\Sigma,\Xi \tag{22}$$

in quenched ChPT, and compared these with similar ratios obtained from ref. [19]. They assumed that all bare parameters in the equations for the octet masses (for explicit expressions, see their paper) are equal in the full and quenched theory, and then calculated the ratios

$$r_{ij} = \frac{R_{ij}^{\rm quenched}}{R_{ij}^{\rm full}}. \tag{23}$$

With the assumption that the bare parameters of the quenched and full theories are equal, b_D and b_F drop out of the ratios, and with $\gamma = 0$, $\alpha = 0$ and D and F equal to their full QCD values, they obtain

$$\begin{aligned} r_{\Sigma N} &= 1 + 0.19(\delta/0.2) + 0.13 = 1.31[1.27] \ \text{ for } \delta = 0.2[0.15], \\ r_{\Xi N} &= 1 - 0.46(\delta/0.2) + 0.43 = 0.97[1.09] \ \text{ for } \delta = 0.2[0.15], \\ r_{\Lambda N} &= 1 - 0.39(\delta/0.2) + 0.26 = 0.87[0.97] \ \text{ for } \delta = 0.2[0.15]. \end{aligned} \tag{24}$$

(The choice $\delta = 0.15$ corresponds to the value reported in ref. [15].)

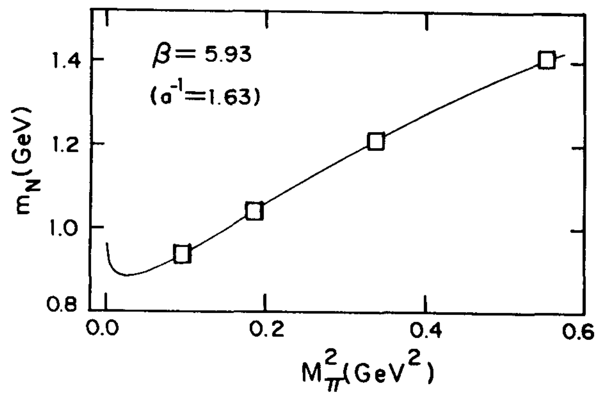

Figure 3. The nucleon mass from the lattice [20] (copied from ref. [10]). The curve is from a fit to the form $m_N = \overline{m} + am_\pi + bm_\pi^2 + cm_\pi^3$.

From this, one would conclude that one can expect errors from quenching of at least 20% in the octet splittings. These differences between the quenched and full theories cannot be compensated for by a change in scale between quenched and full QCD.

At this point I would like to comment on the above mentioned assumption that was used in order to obtain (24). Let us consider in particular the parameters b_D and b_F. They correspond to higher derivative terms in the baryon-meson effective action, and are needed in order to absorb the UV divergences which arise at one loop in ChPT. Since the size of these divergences is in principle different between the full and quenched theories, one expects that $b_{D,\text{quenched}}$ and $b_{F,\text{quenched}}$ can be different from $b_{D,\text{full}}$ and $b_{F,\text{full}}$. If we want to proceed without assuming that the quenched and full b's are equal, we have to consider ratios of quantities independent of the parameters b_D and b_F. The situation is essentially the same as in the case of f_K/f_π. From the available results [10], only one ratio independent of b_D and b_F can be formed:

$$X = \frac{m_\Sigma m_\Lambda^3}{m_N^2 m_\Xi^2}. \tag{25}$$

If we expand X in the Goldstone meson masses using ChPT, $X-1$ measures the deviation from the Gell-Mann–Okubo formula (*cf.* ref. [19] for the full theory).

Setting $m_\pi = 0$ keeping only m_K as in ref. [10], one finds

$$\begin{aligned} X_{\text{quenched}} &= 1 - 1.1046\tfrac{m_K^3}{8\pi\overline{m}f_\pi^2}\left(D^2-3F^2\right) + 1.3333\delta\tfrac{\pi m_K}{\sqrt{2}\overline{m}}\left(D^2-3F^2\right), \\ X_{\text{full}} &= 1 - 0.4125\tfrac{m_K^3}{8\pi\overline{m}f_\pi^2}\left(D^2-3F^2\right). \end{aligned} \tag{26}$$

(The parameter γ drops out of this particular combination and we again take $\alpha = 0$.) These quantities still depend on the other bare parameters, D, F and $\overline{m}$. Again, they could be different in the quenched and full theories, and I will leave the quenched values as free parameters. Substituting $m_K = 495\ MeV$, $f_\pi = 132\ MeV$, $D_{\text{full}} = 0.75$, $F_{\text{full}} = 0.5$ and $\overline{m}_{\text{full}} = 1\ GeV$ [19] finally gives

$$\frac{X_{\text{quenched}}}{X_{\text{full}}} = 1 - 0.0214 + \left[0.293\frac{\delta}{0.2} - 0.306\right]\frac{\left(D^2-3F^2\right)_{\text{quenched}}}{\overline{m}_{\text{quenched}}/1\ GeV}. \tag{27}$$

For any reasonable values of $\overline{m}$ and δ, and for $\left(D^2-3F^2\right)_{\text{quenched}}$ not too far from its full theory value of -0.1875, the difference between the quenched and full theories as measured by the ratio $X_{\text{quenched}}/X_{\text{full}}$ is not more than a few percent. Of course the same comment that applied in the case of f_K/f_π applies here, that part of the difference may have been washed out by taking "ratios of ratios". Summarizing, the conclusion of this analysis seems to be that the error from quenching for octet baryon masses is at least a few percent, and could be as much as 20%.

4. A sickness of quenched QCD

Let us again consider the quenched result for f_K/f_π, (14), as a function of the quark masses (using treelevel relations between meson masses and quark masses),

$$\left(\frac{f_K}{f_\pi}\right)^{1-\text{loop}}_{\text{quenched}} = 1 + \frac{\mu^3/3}{16\pi^2 f_\pi^2}\left[\frac{m_u+m_s}{2(m_s-m_u)}\log\frac{m_s}{m_u} - 1\right] + (\text{L}-\text{terms}).$$

From this expression it is clear that we cannot take $m_u \to 0$ keeping m_s fixed, or, to put it differently, that if we take both m_u and m_s to zero keeping the ratio fixed, the limit depends on this ratio, and is not equal to one! This is quite unlike the case of full ChPT, where one can take any quark mass to zero uniformly, and deviations from $SU(3)$ symmetry due to this quark mass vanish in this limit. Technically, the reason for this strange behavior is that there is another mass μ, which, as we argued before, cannot be avoided in quenched ChPT. This mass is related to the singlet part of the η' mass, and is not a free parameter of the theory. Even if we do not consider any Green's functions with η' external lines, this mass shows up through the double pole term in 12 on internal lines. Because of the double pole, such contributions can lead to new infrared divergences in the $m_\pi \to 0$ limit. This problem with the chiral limit of quenched ChPT shows up in other quantities, such as meson masses and $\langle\overline{\psi}\psi\rangle$ [6, 7, 21].

A question one might ask is whether this problem is an artifact of one loop quenched ChPT [8]. For instance, if we would sum all contributions to the η'

propagator, maybe the double pole term would become softer in the $p \to 0$ limit, improving the infrared behavior of diagrams in which the double pole terms appear. Let us address this question in the chiral limit, $m_a = 0$, where the problem is most severe. In the full theory, we can write the fully dressed η' propagator as

$$\frac{Z(p)}{p^2 + \Sigma(p)}, \tag{28}$$

and define $\mu_F^2(p) = \Sigma(p)$, which onshell is the η' mass in the chiral limit. Likewise, in the quenched theory we can write the dressed η', $\tilde{\eta}'$ propagator as

$$Z_Q(p)\left[\frac{1}{p^2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} - \frac{\mu_Q^2(p)}{(p^2)^2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right], \tag{29}$$

which defines $\mu_Q^2(p)$. To leading order in $1/N_c$, these two definitions of $\mu^2(p)$ should lead to the same result:

$$\mu_Q^2(p) = \mu_F^2(p)\left(1 + O\left(\frac{1}{N_c}\right)\right). \tag{30}$$

We also believe that $\mu_F^2(p = 0)$ is not equal to zero, since we expect the η' to remain a well-behaved, massive particle in the chiral limit. This implies, in sofar as we can rely on the large N_c expansion, that $\mu_Q^2(0) \neq 0$, and that the double pole in (12) is a true feature of the theory.

While this argument is not very rigorous, I believe that the foregoing discussion implies that the chiral limit of quenched QCD really does not exist. This believe is futhermore supported by the following remarks:

• Sharpe [7] has summed a class of diagrams in the case of degenerate quark masses for a very simple quantity (the pion mass), and found a result that is actually more divergent than the one loop result.

• With nondegenerate quark masses there are many more diagrams that are infrared divergent in the chiral limit, and it is even less probable that resummation will improve the situation.

• Any mechanism improving the infrared behavior would have to work for each divergent quantity. One expects that such a mechanism would be related to the double pole term in the η' propagator, which created the problem in the first place. But this seems unlikely in view of the arguments given above.

• The bare quark mass parameter appearing in the chiral effective action is not the same as that appearing in the (unrenormalized) QCD lagrangian. But one can argue that the two bare quark masses should be analytically related, and the infrared problem is not just a problem of quenched ChPT, but of quenched QCD.

5. Conclusion

The quenched approximation leads to an unknown systematic error in all lattice calculations that employ this approximation. It would of course be very nice to have a parameter that interpolates between full and quenched QCD, and in principle the quark masses could play such a role, since one expects that quenched QCD corresponds to full QCD with very heavy quarks. One would have to distinguish here between valence and sea quark masses, and it is the sea quark mass that would play the role of such a parameter. This distinction can indeed be made by considering so-called partially quenched theories [22], but no practical scheme to implement this idea is known.

Quenched QCD can be defined from a euclidean pathintegral as rigorously as full QCD. In this talk I have explained that euclidean quenched ChPT can be used as a tool for a systematic investigation of quenched QCD. Quenched ChPT does not quite accomplish a task equivalent to that of an interpolating parameter. Since the bare parameters appearing in the quenched and full chiral effective actions are not the same (as explained in section 3) one cannot directly compare quantities calculated in full and in quenched ChPT. However, one can calculate combinations of physical quantities which do not depend on the bare parameters, and in that case a direct comparison between quenched and full QCD is possible, as we demonstrated with an example involving meson decay constants. This makes it possible to estimate lower bounds on the differences which come from quenching; these estimates are dependent on the values of the meson masses, which can be taken to be the (known) independent parameters of the theory. For realistic values of these masses, such differences turn out to be of the order of a few percent for ratios of decay constants and for baryon octet splittings.

The disadvantage of this more conservative approach is that part of the difference maybe hidden, because these specific combinations of physical quantities maybe less sensitive to the effects of quenching than other quantities of interest. This is particularly clear in the example of baryon octet masses. In this case, a comparison based on the assumption that the bare parameters of the full and quenched effective theories are the same, lead to differences of up to 20% and more. Of course, it is not known to what extend this assumption is valid.

The differences between the quenched and full theories become markedly larger for decreasing quark masses. This is due to the fact that new infrared divergences occur in quenched QCD, which do not have a counterpart in full QCD. These divergences lead to the nonexistence of the chiral limit for quenched ChPT (as discussed in section 4). The origin of this phenomenon can be traced to the special role of the η' in the quenched approximation. In the quenched approximation, the η' is a Goldstone boson (it develops massless poles in the chiral limit), but an additional double pole term arises in its propagator, rendering it a "sick" particle. For nondegenerate quark masses this problem is also inherited by the π^0 and the η. In section 4 I argued that the nonexistence of the chiral limit is a fundamental feature of quenched QCD.

In principle therefore, the chiral expansion breaks down for quenched QCD. For very small quark masses, at fixed values of the singlet part of the η' mass μ^2, the expansion becomes unreliable. In order to make progress, one may take the expansion to be an expansion in $\mu^2/3$ (which was shown to be roughly equal to m_K^2 phenomenologically), with coefficients which are functions of the quark mass. These functions then can be expanded in terms of the quark masses, sometimes leading to divergent behavior of the leading term (*e.g.* the one loop correction to f_K/f_π). If such divergences occur, the expansion is only valid for a range of quark masses which are neither too small, nor too large. It would be interesting to see whether this point of view can be made solid.

It is in principle interesting to study any quantity which is being computed in quenched lattice QCD in ChPT, for those quantities for which ChPT is applicable (meson masses, decay constants, condensates and the kaon B parameter have been calculated [6, 5, 8]). As discussed, this includes not only Goldstone meson physics *per se*, but also chiral corrections to baryon masses [10], and for the same reason, to mesons containing heavy quarks.

Recently, also attempts have been made to compute pion and nucleon scattering lengths [23, 24] from quenched lattice QCD. If one tries to calculate the $I = 0$ pion scattering amplitude in quenched ChPT, one actually finds that the imaginary part is divergent at threshold, even for nonvanishing pion mass [25]! Again, this can be related to double pole terms in the η' propagator. An additional reason is that apparently euclidean quenched correlation functions in general cannot be analytically continued to Minkowski space-time. (The euclidean four pion correlation functions are well defined.) Further work is needed on pion scattering lengths.

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References

[1] S.R. Sharpe, in *The Fermilab Meeting*, 7th meeting of the DPF, eds. C.H. Albright, P.H. Kasper, R. Raja and J. Yoh, World Scientific, 1993.

[2] P.B. Mackenzie, preprint FERMILAB-CONF-93-343-T, to be published in the proceedings of the 16th International Symposium on Lepton and Photon Interactions, 1993, hep-ph/9311242; A.S. Kronfeld and P.B. Mackenzie, Ann. Rev. Nucl. Part. Phys. **43** (1993) 793.

[3] Lattice'93, proceedings of the International Symposium on Lattice Field Theory, Dallas, Texas, 1993.

[4] H. Hamber and G. Parisi, Phys. Rev. Lett. **47** (1981) 1792; E. Marinari, G. Parisi and C. Rebbi, Phys. Rev. Lett. **47** (1981) 1795; D.H. Weingarten, Phys. Lett. **109B** (1982) 57.

[5] S.R. Sharpe, Phys. Rev. **D41** (1990) 3233; Nucl. Phys. **B** (Proc.Suppl.) **17** (1990) 146; G. Kilcup *et al.*, Phys. Rev. Lett. **64** (1990) 25; S.R. Sharpe, DOE/ER/40614-5, to be published in *Standard Model, Hadron Phenomenology and Weak Decays on the Lattice*, ed. G. Martinelli, World Scientific.

[6] C.W. Bernard and M.F.L. Golterman, Phys. Rev. **D46** (1992) 853; Nucl. Phys. **B** (Proc.Suppl.) **26** (1992) 360.

[7] S.R. Sharpe, Phys. Rev. **D46** (1992) 3146; Nucl. Phys. **B**(Proc.Suppl.) **30** (1993) 213.

[8] C.W. Bernard and M.F.L. Golterman, Nucl. Phys. **B**(Proc.Suppl.) **30** (1993) 217.

[9] J. Gasser and H. Leutwyler, Phys. Lett. **184B** (1987) 83,Phys. Lett. **188B** (1987) 477, and Nucl. Phys. **B307** (1988) 763; H. Leutwyler, Nucl. Phys. **B** (Proc.Suppl.) **4** (1988) 248 and Phys. Lett. **189B** (1987) 197; H. Neuberger, Nucl. Phys. **B300** (1988) 180; P. Hasenfratz and H. Leutwyler, Nucl. Phys. **B343** (1990) 241.

[10] J.N. Labrenz and S.R. Sharpe, preprint UW-PT-93-07, to be published in the proceedings of the International Symposium on Lattice Field Theory, Dallas, Texas, 1993, hep-lat/9312067.

[11] A. Morel, J. Physique **48** (1987) 111.

[12] For a description of the properties of graded groups, see for example, B. DeWitt, *Supermanifolds*, Cambridge, 1984.

[13] J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 465.

[14] S. Weinberg, Physica **96A** (1979) 327.

[15] Y. Kuramashi, M. Fukugita, H. Mino, M. Okawa and A. Ukawa, preprint KEK-CP-13, 1994; preprint KEK-CP-010, to be published in the proceedings of the International Symposium on Lattice Field Theory, Dallas, Texas, 1993, hep-lat/9312016.

[16] E. Witten, Nucl. Phys. **B156** (1979) 269; G. Veneziano, Nucl. Phys. **B159** (1979) 213.

[17] T. Hatsuda, Nucl. Phys. **B329** (1990) 376; S.R. Sharpe, private communication.

[18] E. Jenkins and A. Manohar, Phys. Lett. **255B** (1991) 558; E. Jenkins, Nucl. Phys. **B368** (1992) 190.

[19] V. Bernard, N. Kaiser and U. Meissner, Z. Phys. **C60** (1993) 111.

[20] F. Butler, H. Chen, J. Sexton, A. Vaccarino and D. Weingarten, Phys. Rev. Lett. **70** (1993) 2849; Nucl. Phys. **B** (Proc.Suppl) **30** (1993) 377.

[21] M.F.L. Golterman and N.D. Hari Dass, in preparation.

[22] C.W. Bernard and M.F.L. Golterman, Phys. Rev. **D49** (1994) 486; preprint WASH-U-HEP-93-61, to be published in the proceedings of the International Symposium on Lattice Field Theory, Dallas, Texas, 1993, hep-lat/9311070.

[23] R. Gupta, A. Patel and S.R. Sharpe, Phys. Rev. **D48** (1993) 388.

[24] Y. Kuramashi, M. Fukugita, H. Mino, M. Okawa and A. Ukawa, Phys. Rev. Lett. **71** (1993) 2387.

[25] C.W. Bernard, M.F.L. Golterman, J.N. Labrenz, S.R. Sharpe and A. Ukawa, preprint Wash. U. HEP/93-62, to be published in the proceedings of the International Symposium on Lattice Field Theory, Dallas, Texas, 1993.