PRAMANA
— journal of
physics

© Printed in India

Supplement to Vol. 45
October 1995
pp. 113-116

Family symmetry

PAUL H. FRAMPTON
Institute of Field Physics, Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599-3255, USA

Abstract. In this presentation, I discuss an extension of the standard model which is motivated by the desire to allow non-trivial accommodation of three families and is testable by the production of new particles (especially dileptons).

The present talk includes (1) a description of the model (331 model); (2) the cubic see-saw for neutrino masses; (3) brief comments on phenomenology of the 331 model for hadronic and leptonic colliders.

(1) Desciption of the Model

Family symmetry is usually taken to mean a horizontal symmetry, either global or gauged, under which the three families transform under some nontrivial representation. The family symmetry is *broken* in order to avoid unobserved mass degeneracies. In this meaning of family symmetry, there is no explanation of *why* there are three families which is an input. Rather the hope is that the postulated family symmetry may explain the observed hierarchies of the fermion masses and the quark flavor mixing angles.

In the present model, the aim is indeed to attempt to address such unexplained hierarchies *and* to explain why there are three families. This may be a necessary first step to understanding the hierarchies themselves.

To introduce the 331 model the following are motivating factors:

(1) Consistency (unitarity, renormalizability) of a gauge theory requires that chiral anomalies be cancelled. This requirement alone is able to fix all electric charges in the standard model, and to explain charge quantization, e.g. the electric neutrality of the hydrogen atom, without the necessity for grand unification.

(2) This does not explain why there are three families but is sufficiently impressive to suggest that $N_g = 3$ may be explicable by anomaly cancellation in an extension of the standard model. This requires *both* that each extended family have a non-vanishing anomaly *and* that the three families are not all treated similarly.

(3) A striking feature of the mass spectrum of the standard model is the top quark mass suggesting that the third family be treated differently from the first two and that the anomaly cancellation be proportional to the combination $+1+1-2=0$ for the three families.

(4) There is a factor -2 (suggested by (3) above) already occurring in the theory, namely the ratio of the electric charges of the up and down quarks. this will underly the anomaly cancellation in the 331 model.

(5) The electroweak gauge group extension from $SU(2)$ to $SU(3)$ adds five gauge bosons. The adjoint of $SU(3)$ breaks down to $3 + (2 + 2) + 1$. The 1 is a Z' and the two doublets are readily identified as dileptonic gauge bosons of the type first encountered in $SU(15)$ grand unification.

Several interesting contributions to $SU(15)$ have been made since 1991 (the WHEPP2 meeting in Calcutta) by B. Brahmachari of the Physical Research Laboratory in Ahmedabad. These include (with Mann, Steele and Sarker) the influence of Higgs on unification for different patterns of symmetry breaking - published in Phys. Rev. D in 1991; (with Sarkar) the increase in the GUT scake for supersymmetrized SU(15) - published in Phys. Lett. B in 1992; and the extension to SU(16), including a right-handed neutrino, and the symmetry breaking patterns and phenomenology - published in Phys. Rev. D in 1993. There have also been significant contributions to $SU(15)$ by Deshpande, Lavowa, and others.

The two principal shortcomings of $SU(15)$ grand unification are (1) The group is so large that the ratio of unknown particles to known ones is unacceptably high; and (2) the anomaly cancellation is inelegant, being accomplished by mirror fermions.

One feature of $SU(15)$ which survives in the 331 model is the presence of an $SU(3)$ electroweak gauge group. But it is not a subgroup of $SU(15)$ as can be seen by the fact that the dileptonic gauge bosons do not couple to the quarks in $SU(15)$ wheras they do in the 331 model.

Now we are ready to describe the structure of the model. The gauge group is extended to $SU(3)_C \times SU(3)_L \times U(1)_X$. The triplets of $SU(3)_L$ have electric charges (X-1,X,X+1) while antitriplets have charges (X+1,X,X-1). In either case the X value is the electric charge of the middle component.

For the leptons we assign (e^-, ν_e, e^+) to an antitriplet and similarly for the μ and τ families. The left handed quarks are assigned to triplets for the first two families and to an antitriplet for the third family. The right-handed quarks are taken to be singlets as in the standard model. No new leptons have been introduced but there is a new quark per family: D , S (Q=-4/3) in the first and second, and T (Q=+5/3) in the third.

Anomaly cancellation occurs in a nontrivial manner *between* the families. One can check in turn all the relevant gauge anomalies: $(3_C)^3, (3_L)^3, (3_C)^2X, (3_L)^2X, X^3, X(T_{\mu\nu})^2$. Each extended family has non-vanishing anomaly for the 2nd,4th and 5th of these, while the three families taken together cancel with anomalies proportional to $+1 + 1 - 2$ as expected. The 1st, 3rd and 6th anomalies *do* cancel family by family.

In the Higgs sector there is a triplet Φ with X = +1 whose VEV breaks the 331 gauge group to the 321 gauge group of the standard model. This also allows D,S,T quarks as well as the Z_2 and (Y^{--}, Y^-), (Y^{++}, Y^+) gauge bosons to acquire mass. The electroweak breaking involves three $SU(2)$ doublets embedded in two triplets $\phi(X = 0)$ and $\phi'(X = -1)$ and a sextet $H^{\alpha\beta}(X = 0)$. The sextet is necessary to give a symmetric (in family space) contribution to the charged fermion masses. Note that ϕ gives masses to d,s,t (and an antisymmetric contribution to the charged lepton masses) while ϕ' gives masses to u,c and b. Let the scale of breaking of 331 to 321, the VEV of Φ, be denoted by U. What can the scale U be? There is a lower bound coming from precision electroweak data: what makes the 331 model more interesting is that there is also an upper bound.

The lower bound comes from the limits on $Z - Z'$ mixing and from the absence of flavor-changing neutral currents. The latter requires that it is the third family which is treated asymmetrically rather than the first or second. A lower bound on the Y mass comes from data on polarized muon decay. The Y would give a component (V+A) compared to the W which is (V-A). The strong limits on the parameter ξ defined as the coefficient of (-A), namely $1 \geq \xi \geq 0.997$ imply that the mass of the dilepton must exceed 300 GeV. More accurate measurements of muon decay, and of muonium-antimuonium conversion will put better limit on the dilepton mass. Incidentally, it is fascinating that these low energy experiments rather than, say, LEP data place the most stringent limits on this new physics.

The upper limit on U arises because the electroweak mixing angle must be less than 0.25 in order that the 321 standard model can be embedded in 331. It is easy to see that if $SU(2) \times U(1)$ is embedded just in $SU(3)$ then the value of $sin^2\theta$ is exactly 1/4, and if this value is exceeded then the X coupling constant becomes imaginary. Since the value is already close to one quarter at the Z mass and it increases with energy the consequence is that the scale U cannot be more than a few TeV. Requiring that α_X not exceed 2π then enforces that $M(Z_2) \leq 2.2$ TeV.

(2) Cubic See-Saw for Neutrino Masses

In the minimal 331 model neutrinos are massless since $\Delta L = 0$ and there are no right-handed neutrino fields. We have examined (with P. Krastev and J.T.Liu) the effects of soft breaking of L on the neutrino masses.

Spontaneous breaking of L would lead to a massless Majoron. Therefore we employ explicit breaking by terms:

$$m_1 H^{\alpha_1\beta_1} H^{\alpha_2\beta_2} H^{\alpha_3\beta_3} \epsilon_{\alpha_1\alpha_2\alpha_3} \epsilon_{\beta_1\beta_2\beta_3} + m_2(H^{\alpha\beta}\bar{\phi}_\alpha\bar{\phi}_\beta + h.c.) \tag{1}$$

The Yukawa couplings to the leptons are given by:

$$h_1^{ij} L^i_\alpha L^j_\beta H^{\alpha\beta} + h_2^{ij}(L^i_\alpha L^j_\beta \bar{\phi}_\gamma \epsilon^{\alpha\beta\gamma} + h.c.) \tag{2}$$

The couplings h_1,h_2 are symmetric, antisymmetric respectively in family space.

At one loop level there are diagrams where the soft breaking of L through the parameters m_1, m_2 gives rise to neutrino masses. What is unique to the 331 model is the presence of contributions of the general form:

$$M(\nu_i) = CM(l_i)^3/M_W^2 \tag{3}$$

This is the cubic see-saw. $M(l_i)$ is the mass ofthe corresponding charged lepton.It arises because all the Yukawa couplings are related by $SU(3)$.

If we take, merely as an example, Dennis Sciama's favorite value for the tau neutrino mass of 29.3 eV, then the mu neutrino mass is 6.2meV (that is, 0.062 eV), and the electron neutrino mass is 690 peV. This last tiny mass gives rise to a contribution to neutrinoless double beta decay but it is many orders of magnitude below present experimental limits.

As is usual with this type of formula, one can fit any two out of Cold Dark Matter, the Atmospheric Neutrino Problem and the MSW Mechanism for the Solar Neutrino Problem but not all three simultaneously.

Our formula is called a cubic see-saw but really it is not a see-saw at all because there is no right-handed neutrino involved.

(3) Phenomenology of the 331 Model for Hadronic and Leptonic Colliders

In hadronic colliders, for example pp at 10 TeV center of mass, dileptons can be both pair produced and associatedly produced in conjunction with new quarks (which themselves carry lepton number). Explicit calculations show that such events have a healthy detectable rate for any luminosity sufficiently high to be otherwise interesting in looking for Higgs bosons or low-energy supersymmetry.

In an e^-e^- collider at 300-1000 GeV center of mass energy the Y^{--} could appear as a striking narrow direct channel resonance. One would best look at the channel $e^-e^- \rightarrow \mu^-\mu^-$ for which the standard model background is zero.

This work was supported in part by the US Department of Energy under Grant No. DE-FG05-85ER-40219.