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CP violation in a multi higgs doublet model

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Abstract.

We study CP violation in a multi-Higgs doublet model based on a $S_3 \times Z_3$ horizontal symmetry. We consider two mechanisms for CP violation in this model: a) CP violation due to complex Yukawa couplings; and b) CP violation due to scalar-pseudoscalar mixings. We find that the predictions for ϵ'/ϵ, CP violation in B decays and the electric dipole moments of neutron and electron are different between these two mechanisms. These predictions are also dramatically different from the minimal Standard Model predictions.

1. Introduction

The origin of CP violation, is one of the outstanding problems of particle physics today. So far CP violation has only been observed in the neutral Kaon system. The observed CP violation can be explained in many models. It is therefore important to study other CP violating experimental observables and compare the results with different model predictions. Such study may reveal the real origin of CP violation.

In the minimal $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model (MSM), there is only one Higgs doublet. When the Higgs develops valcuum expectation value (VEV) v, all fermion receive masses. In the mass eigenstate basis, the Higgs coupling to fermions is diagonal, it does not mediate CP violating interaction. However, the coupling of the charged current to the W-boson becomes complex. It is given by

$$L_C = \frac{g}{\sqrt{2}} \bar{u}_i V_{ij} \gamma^\mu \frac{1-\gamma_5}{2} d_j W^+_\mu + H.C. , \tag{1}$$

where the matrix V_{ij} is the CKM matrix V_{KM}[1]. For three generations of quarks, there is a non-removable phase in the matrix. This is the source of CP violation in the MSM. This matrix is conveniently parametrized as, following Wolfenstein [2]

$$V_{KM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 - iA^2\lambda^4\eta & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} , \tag{2}$$

where $\lambda = V_{us} = 0.221$. If $\eta \neq 0$, CP is violated. Unitarity constraints on the matrix elements provide very powerful and interesting realtions. The most interesting one is the triangle defined by

$$V_{ud}V^*_{td} + V_{us}V^*_{ts} + V_{ub}V^*_{tb} = 0 . \tag{3}$$

In the Wolfenstein parametrization, $V_{ud} \approx V_{tb} \approx 1$ and $V_{ts} \approx V_{cb}^*$, we have

$$V_{ub} + V_{td}^* \approx V_{ub} V_{cb} \,. \tag{4}$$

This defines the triangle shown in Figure 1 with three angles α, β and γ. The area of the triangle is given by $A^2\lambda^6\eta/2$. CP violation in the neutral Kaon system can be explained by the "box" interaction[3]. If CP violation due to the phase in the CKM matrix is the only source for CP violation, experiments at B factories will be able to determine all the three angles[4].

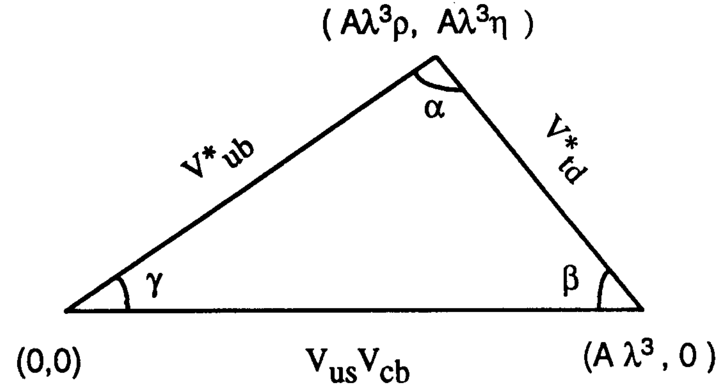

Figure 1. The unitary triangle

Another class of model for CP violation is the multi-Higgs doublet model. If there are more than one Higgs doublets, the neutral scalar couplings to the quarks are not necessarily diagonal, and therefore provide new sources for CP violation[5]. CP violation can arise in three places in this type of models: 1) Non-trivial phase in the V_{KM} matrix; 2)Non-trivial phases in the Yukawa couplings; and 3) Mixings of scalar and pseudoscalar Higgs bosons. In cases 2) and 3), CP violation can occurs at the tree level by exchanging neutral Higgs bosons. In this talk we will present some studies of CP violation in multi-Higgs models with flavour changing neutral currents at the trre level which has CP violation predominantly throug mechanisms 2) and 3).

A most general study suffers from too many free parameters. To have a definite idea, we carry out the study in a $S_3 \times Z_3$ model proposed by Ma[7]. This model has some very interesting predictions for fermion masses and their mixings. It also has interesting predictions for CP violation[8, 9, 10]. We study the predictions for: (i) ϵ'/ϵ; (ii) CP violation in the neutral B system; and (iii) the neutron and electron electric dipole moments (EDM). We compare these predictions with those in the MSM.

2. Yukawa couplings in the $S_3 \times Z_3$ model

In the $S_3 \times Z_3$ model, there are four Higgs doublets, $\phi_{1,2,3,4}$. The quarks and

Higgs bosons transform under the $S_3 \times Z_3$ symmetry as[7]

$$
\begin{aligned}
&q_{3L}\,, t_R\,, b_R\,, \phi_1 : (1,1) \quad , \quad (q_{1L}, q_{2L})\,, (\phi_3, \phi_4) : (2, \omega)\,, \\
&(c_R, u_R)\,, (s_R, d_R) : (2, \omega^2) \quad , \quad \phi_2 : (1, \omega^2)\,,
\end{aligned} \tag{5}
$$

where $\omega \neq 1$, $\omega^3 = 1$ is the Z_3 element. The Yukawa couplings consistent with the $S_3 \times Z_3$ symmetry are given by

$$
\begin{aligned}
L_Y &= -f_1(\bar{q}_{1L}\tilde{\phi}_3 u_R + \bar{q}_{2L}\tilde{\phi}_4 c_R) - f_2\bar{q}_{3L}\tilde{\phi}_1 t_R - f_3(\bar{q}_{1L}\phi_2 s_R + \bar{q}_{2L}\phi_2 d_R) \\
&- f_4(\bar{q}_{1L}\phi_3 b_R + \bar{q}_{2L}\phi_4 b_R) - f_5(\bar{q}_{3L}\phi_3 d_R + \bar{q}_{3L}\phi_4 s_R) - f_6\bar{q}_{3L}\phi_1 b_R + H(6)
\end{aligned}
$$

where $\tilde{\phi}_i = (\phi_i^{0*}, -\phi_i^-)^T$. Without loss of generality we work in a basis where all VEVs are real. The up-quark mass matrix is diagonal: $\hat{M}^u = Diag(f_1 v_3, f_1 v_4, f_2 v_1)$, and the down-quark mass matrix can be written as, with a suitable choice of quark phases,

$$
M^d = \begin{pmatrix} 0 & a & \xi b \\ a & 0 & b \\ \xi c & c & d \end{pmatrix}\,, \tag{7}
$$

with a, b, c, d real and $\xi = |\xi| e^{i\sigma}$ complex. M^d can be diagonalized by a bi-unitary transformation $M^d = V_L \hat{M}^d V_R^\dagger$. Here $\hat{M}^d$ is the diagonalized down quark mass matrix. V_L and V_R are unitary matrices. Because the up quark mass matrix is already diagonalized, V_L is the CKM matrix V_{KM}.

It is convenient to work in a basis of the Higgs bosons in which the Goldstone bosons are removed. To this end we define the following[9]

$$
\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v_{12} & v_1 v_4/v_{12}v_{124} & -v_1 v_3/v v_{124} \\ v_2/v & -v_1/v_{12} & v_2 v_4/v_{12}v_{124} & -v_2 v_3/v v_{124} \\ v_3/v & 0 & 0 & v_{124}/v \\ v_4/v & 0 & -v_{12}/v_{124} & -v_3 v_4/v v_{124} \end{pmatrix} \begin{pmatrix} G \\ H_1 \\ H_2 \\ H_3 \end{pmatrix}\,, \tag{8}
$$

where $v_{12}^2 = v_1^2 + v_2^2$, $v_{124}^2 = v_1^2 + v_2^2 + v_4^2$, and $v^2 = v_1^2 + v_2^2 + v_3^2 + v_4^2$. The transformation is the same for both the neutral and charged Higgs bosons. For the neutral Higgs bosons, $G = h^0 + iG_Z$, where G_Z is the Goldstone boson 'eaten' by Z, and h^0 is a physical field whose couplings are the same as the Higgs boson in the MSM. For the charged Higgs bosons G is the Goldstone boson 'eaten' by W. In this basis, we have

$$
\begin{aligned}
L_Y &= -(\bar{D}_L \hat{M}^d D_R + \bar{U}_L \hat{M}^u U_R)(1 + \frac{Re h^0}{v\sqrt{2}}) \\
&- \bar{D}_L \tilde{Y}_i^d D_R \frac{h_i^0}{\sqrt{2}} - \bar{U}_L \tilde{Y}_i^u U_R \frac{h_i^{0*}}{\sqrt{2}} \\
&- \bar{U}_L V_{KM} \tilde{Y}_i^d D_R h_i^+ + \bar{D}_L V_{KM}^\dagger \tilde{Y}_i^u U_R h_i^- + H.C.\,,
\end{aligned} \tag{9}
$$

where h_i are the component fields of H_i with $H_i = (h_i^+, h_i^0/\sqrt{2})$. $U_{L,R} = (u, c, t)_{L,R}^T$, and $D_{L,R} = (d, s, b)_{L,R}^T$. The couplings $\tilde{Y}_i$ can be easily expressed in term of quark masses, $V_{L,R}$, and VEVs.

In general $h_i^{0,+}$ are not the mass eigenstates. We can parametrize the mixings as

$$\begin{pmatrix} h^0 \\ Reh_k^0 \\ Imh_k^0 \end{pmatrix} = \begin{pmatrix} \alpha_{00} & \alpha_{0i} & \beta'_{0j} \\ \alpha_{k0} & \alpha_{ki} & \beta'_{kj} \\ \alpha'_{k0} & \alpha'_{ki} & \beta_{kj} \end{pmatrix} \begin{pmatrix} R_0 \\ R_i \\ I_j \end{pmatrix} ,$$
$$h_i^+ = (\gamma_{ij})\eta_j^+ , \tag{10}$$

where R_i, I_i and η_i are the mass eigenstates, the matrix $(\alpha\beta)$ is a 7×7 othogonal matrix, and (γ) is a 3×3 unitary matrix.

The specific values for the mixings depend on the details of the Higgs potential. Unfortunately they are not determined. To simplify the problem, we will discuss two cases: a) CP violation only comes from complex Yukawa couplings; and b) CP violation only comes from the mixings of real and imaginary h_i^0[10]. Case a) can be realised by constraining certain soft symmetry breaking terms in the potential[9]. We further assume, for simplicity, that Reh_i^0 are the mass eigenstate R_i and consider their effects. The same analysis can be easily carried out for Imh_i^0 in the same way. The source for CP violation is the non-zero value for σ which is a free parameter. We will present our results for $\sigma = 80^0$, which is close to the maximum of the allowed phase. Case b) can be realised by requiring spontaneous CP violation. The value of σ will be zero and CP violation arises due to scalar-pesudoscalar Higgs boson mixing. For illustration, we consider the effects of a neutral mixed state

$$R = cos\theta Reh_2^0 + sin\theta Imh_3^0 , \tag{11}$$

and for the charged Higgs boson we consider mixing

$$\eta^+ = \gamma_{22}h_2^+ + \gamma_{23}h_3^+ , \tag{12}$$

where γ_{ij} are complex numbers, and $|\gamma_{22}|^2 + |\gamma_{23}|^2 = 1$.

The parameters a, b, c, and d are constrained from the down quark masses and the CKM mixings. We take as input parameters $a = 0.04GeV$, $b = 0.25GeV$, $c = 2.66GeV$, $d = 4GeV$. The mass eigenvalues for the down quarks are quite insensitive to the phase σ. For both cases, we have $m_b = 4.8GeV$, $m_s = 149MeV$ and $m_d = 9.5MeV$. These values are well within the allowed regions[11]. The CKM matrix for case a) is

$$V_{KM} = \begin{pmatrix} 0.975 & -0.222 & 0.00476 \\ 0.221 + i0.0033 & 0.974 + i0.014 & 0.043 - i0.0015 \\ -0.014 + i1.2 \times 10^{-5} & -0.041 - i6.8 \times 10^{-4} & 0.998 - i0.034 \end{pmatrix} , \tag{13}$$

and for case b)

$$V_{KM} = \begin{pmatrix} 0.975 & 0.22 & 0.0048 \\ -0.219 & 0.975 & -0.0436 \\ -0.014 & 0.0415 & 0.999 \end{pmatrix} \tag{14}$$

The values for the VEV's are not fixed, we only know $v_3/v_4 = m_u/m_c$. We will use the values: $v_1 = v_2 = 44GeV$, $v_3 = 0.9$ GeV and $v_4 = 238GeV$ for illustration. We shall comment on effects of changing these values later.

3. Constraints on the Higgs masses from the neutral K and B mesons

The $S_3 \times Z_3$ model has very restrictive allowed values for the non-trivial CP violating phase in the CKM matrix. The CP violating measure J[12] is less than 2.5×10^{-6} which is too small to explain CP violation in the neutral Kaon sysytem. Therefore in this model CP violation due to Higgs boson exchange has to be considered.

The CP violating parameter $\bar{\epsilon}$ is given by

$$\bar{\epsilon} = \frac{ImM_{12}^K}{\sqrt{2}\Delta m_K} e^{i\pi/4} , \tag{15}$$

where M_{12}^K is the matrix element which mixes K^0 with $\bar{K}^0$, and Δm_K is the mass difference between m_{K_L} and m_{K_S}. Experimental value for $\bar{\epsilon}$ is $2.3 \times 10^{-3} e^{i\pi/4}$. The $\Delta S = 2$ Hamiltonian, responsible for M_{12}^K, generated by exchanging neutral Higgs bosons R_i is given by

$$H_{eff} = -\frac{1}{2M_{R_i}^2} \left(\bar{d}[(\alpha_{ki} + i\alpha'_{ki})\tilde{Y}_{k,12}^d)\frac{1+\gamma_5}{2} + (\alpha_{ki} - i\alpha'_{ki})\tilde{Y}_{k,21}^{d*})\frac{1-\gamma_5}{2}]s\right)^2 . \tag{16}$$

We obtain

$$\begin{aligned} M_{12}^K &= < K^0|H_{eff}|\bar{K}^0 > \\ &= -\frac{f_k^2 m_K}{2M_{R_i}^2}(-\frac{5}{24}\frac{m_K^2}{(m_s+m_d)^2}[(\alpha_{ki} + i\alpha'_{ki})\tilde{Y}_{k,12}^d)^2 + (\alpha_{ki} - i\alpha'_{ki})\tilde{Y}_{k,21}^{d*})^2] \\ &+ (\alpha_{ki} + i\alpha'_{ki})\tilde{Y}_{k,12}^d(\alpha_{k'i} - i\alpha'_{k'i})\tilde{Y}_{k',21}^{d*}(\frac{1}{12} + \frac{1}{2}\frac{m_K^2}{(m_s+m_d)^2})) . \end{aligned} \tag{17}$$

Here we have used the vaccum saturation and factorization approximation results for the matrix elements[13]. The contribution to the mass difference Δm_K is given by $2ReM_{12}$. Similar formula holds for the neutral B system.

To constrain the Higgs boson masses, we require that the neutral Higgs boson contributions to the mass differences in the neutral K and B systems to be less than the experimental values: $\Delta m_K/m_K = 7 \times 10^{-15}$, and $\Delta m_B/m_B = 8 \times 10^{-14}$. We find that for case a) the tightest constraints on the masses of $Reh_{1,2}^0$ are from the mass difference ΔM_B of the neutral B mesons which gives $M_{h_1} > 2.9TeV$ and $M_{h_2} > 3.1TeV$. With these masses, $Reh_{1,2}^0$ can not produce large enough $\bar{\epsilon}$. Similar consideration yields $M_{h3} > 3.5TeV$, and we find the experimental value of $\bar{\epsilon}$ can now be produced if the mass is about $5.6TeV$. The mass difference ΔM_K of the neutral K mesons gives weaker bounds in all cases. For case b), the experimental value of ΔM_B constrains $M_R > 3TeV$. From the experimental value of $\bar{\epsilon}$, we obtain $sin\theta cos\theta/M_R^2 = 1.1 \times 10^{-8}\,\mathrm{GeV}^{-2}$ which implies $M_R < 7TeV$.

4. Predictions for ϵ'/ϵ.

In this section we study the direct CP violation in $K_{L,S} \to 2\pi$ decays. CP violation in these processes is characterized by the value of ϵ'/ϵ. ϵ'/ϵ is defined as

$$\frac{\epsilon'}{\epsilon} = \frac{\omega\xi - ImA_2/ReA_0}{\xi + ImM_{12}/\Delta M_K} , \tag{18}$$

where $\omega = ReA_2/ReA_0 = 1/20$, $\xi = ImA_0/ReA_0$. Here A_0 and A_2 are the $\Delta I = 1/2$, $3/2$ decay amplitudes for $K_{L,S} \to 2\pi$.

In the MSM, the contribution to ϵ'/ϵ is donminantly from the gluon penguin. However, for large top quark mass of order 200 GeV, the electroweak penguin also contribute significantly and may even cancel the gluon penguin contribution. There are large uncertainties from hadronic matrix evaluation, $\Lambda_4^{(QCD)}$ dependce and errors in the CKM matrix. The range for ϵ'/ϵ is predicted to be between 10^{-4} to 10^{-3} for $\Lambda_4^{(QCD)} = 300$ MeV[14]. This is consisten with the experimental constraints from Fermilab, $(7.4 \pm 6.0) \times 10^{-4}$ and CERN, $(23 \pm 6.5) \times 10^{-4}$[15].

In the $S_3 \times Z_3$ model there are several contributions to ϵ'/ϵ. Due to large neutral Higgs masses, the neutral Higgs boson contributions to ϵ'/ϵ are very small. However there may be large contributions from the charged Higgs bosons. The dominant contribution is from the charged Higgs boson mediated gluon penguin. The relevant $\Delta S = 1$ effective Larangian is given by

$$L_{\Delta S=1} = i\bar{d}\sigma^{\mu\nu}(\tilde{f}_1\frac{1+\gamma_5}{2} + \tilde{f}_2\frac{1-\gamma_5}{2})\lambda^a s G^a_{\mu\nu}\ , \tag{19}$$

where $G^a_{\mu\nu}$ is the gluon field strength, λ^a are the $SU(3)_C$ generators, and

$$\begin{aligned}
\tilde{f}_1 &= \frac{g_s(\mu)}{32\pi^2}\frac{m_l}{M^2_{h_j^+}}(\frac{3}{2} - ln\frac{m_l^2}{M^2_{h_j^+}})Im\{(V_{KM}\tilde{Y}_i^d\gamma_{ij})_{l1}(\tilde{Y}_k^{u\dagger}V_{KM}\gamma_{kj})^*_{l2}\}\zeta_f\ ,\\
\tilde{f}_2 &= \frac{g_s(\mu)}{32\pi^2}\frac{m_l}{M^2_{h_j^+}}(\frac{3}{2} - ln\frac{m_l^2}{M^2_{h_j^+}})Im\{(V_{KM}\tilde{Y}_i^d\gamma_{ij})^*_{l2}(\tilde{Y}_k^{u\dagger}V_{KM}\gamma_{kj})_{l1}\}\zeta_f\ ,
\end{aligned} \tag{20}$$

where $\zeta_f = (\alpha_s(m_h)/\alpha_s(\mu))^{14/23} \approx 0.17$ is the QCD correction factor, and l is summed over u, c and t. We will use $\alpha_s(\mu) \approx 4\pi/6$ for $\mu = 1GeV$. The above effective Lagrangian will generate a non-zero value for ImA_0[16]. $L_{\Delta S=1}$ also generates a non-zero value $\bar{\epsilon}_{LD}$ for CP violation in $K^0 - \bar{K}^0$ mixing due to long distance interactions through K^0 and π, η, η' mixings[17]. One obtains[17, 18]

$$\begin{aligned}
\frac{\xi}{\bar{\epsilon}_{LD}} &\approx -0.196D\ ,\\
2m_K ImM^K_{12,LD} &\approx 0.8 \times 10^{-7}(\tilde{f}_1 + \tilde{f}_2)(GeV^3)\ ,
\end{aligned} \tag{21}$$

where D is a supression factor of order $O(m_K^2, m_\pi^2)/\Lambda^2$. $\xi/\bar{\epsilon}_{LD}$ is of order -0.014 to -0.1.

We find that in both a) and b) cases, the donimant contributions are from the top quark in the loop arising from mixing in the charged Higgs boson couplings. For case a), we have

$$\bar{\epsilon}_{LD}(h_i^+) \approx a_i\frac{GeV^2}{m^2_{h_i^+}}\frac{m_t}{150GeV}ln\frac{m_t^2}{m^2_{h_i^+}}\ , \tag{22}$$

with $a_1 = 18$, $a_2 = 25$ and $a_3 = -7$.

And for case b), we have

$$\bar{\epsilon}_{LD} \approx -7.35 \times 10^3 Im(\gamma_{22}\gamma_{23}^*)\frac{GeV^2}{m^2_{\eta^+}}\frac{m_t}{150GeV}ln\frac{m_t^2}{m^2_{\eta^+}}\ . \tag{23}$$

The contributions to $\bar{\epsilon}$ can be significant in both cases depending on the Higgs boson masses and the CP violating parameter $Im(\gamma_{22}\gamma_{23}^*)$. We will study constraints on these parameters in Sec.VI. When these constraints are taken into account, $\bar{\epsilon}_{LD}$ is generally constrained to be less than 3×10^{-5} for case a) and ϵ'/ϵ to be less than 3×10^{-5}. However, for case b), $\bar{\epsilon}_{LD}$ can still be as large as 10^{-3} and ϵ'/ϵ can be 10^{-3}.

5. CP violation in the neutral B system.

There are many processes which can test CP violation in the neutral B system. Some particularly interesting ones are[4]

$$B_d \to J/\psi K_S\ ,\ B_d \to \pi^+\pi^-\ ,\ B_s \to \rho K_S\ . \tag{24}$$

The differences of time variation of decay rates for the above processes and their CP tranformed states are given by

$$\begin{aligned} a_{fCP} &= \frac{\Gamma(B^0(t)\to f_{CP})-\Gamma(\bar{B}^0(t)\to f_{CP})}{\Gamma(B^0(t)\to f_{CP})+\Gamma(\bar{B}^0(t)\to f_{CP})} \\ &= \frac{(1-|\lambda|^2)cos(\Delta M_B t)-2Im\lambda sin(\Delta M_B t)}{1+|\lambda|^2}\ , \end{aligned} \tag{25}$$

where f_{CP} indicates the final states. λ is defined as

$$\lambda = \left(\frac{q}{p}\right)_B \frac{\bar{A}}{A} S\ , \tag{26}$$

where $(q/p)_B = \sqrt{M_{12}^{B*}/M_{12}^B}$, A and $\bar{A}$ are the decay amplitudes. If the final state contains K_S, $S=(q/p)_K$ which has a phase of order 10^{-3}. For other cases S is equal to one.

Non-zero asymmetry a_{fCP} signals CP violation. If $|\lambda|$ is not equal to one, it indicates that CP is violated in the decay amplitudes. In the MSM $|\lambda|$ is equal to one to a very good approximation for the above three processes. The asymmetries are proportional to $Im\lambda$. In the MSM, the processes in Eq.(24) measure the three angles α,β and γ,

$$\begin{aligned} &Im\lambda(B_d\to J/\psi K_S) = -sin2\beta\ , \\ &\quad Im\lambda(B_d\to \pi^+\pi^-) = sin2\alpha\ , \\ &\quad Im\lambda(B_s\to \rho K_S) = -sin2\gamma\ , \end{aligned} \tag{27}$$

In the $S_3\times Z_3$ model, the situation is very different. Although the CP violating decay amplitudes A and $\bar{A}$ are small, the phase of $\sqrt{M_{12}^{B*}/M_{12}^B}$ in the $B-\bar{B}$ mixing due to neutral Higgs boson exchange can be large. In case a), there is CP violation arising from the phase in Yukawa coupling of Higgs bosons, as well as CKM matrix, but the former is much larger. The three meaurements in Eq.(24) do not measure the angles α, β and γ anymore. The first two processes will mostly measure the phases in $M_{12}^{B_d}$. We have

$$\begin{aligned} Im\lambda(B_d\to\pi^+\pi^-) &\approx Im\lambda(B_d\to J/\psi K_S)\le 0.42\ ,\ from\ Reh_1^0, \\ Im\lambda(B_d\to\pi^+\pi^-) &\approx Im\lambda(B_d\to J/\psi K_S)\le 0.19\ ,\ from\ Reh_2^0\ , \\ Im\lambda(B_d\to\pi^+\pi^-) &\approx Im\lambda(B_d\to J/\psi K_S)\approx 0.19\ ,\ form\ Reh_3^0\ . \end{aligned} \tag{28}$$

For case b), we find

$$Im\lambda(B_d \to \pi^+\pi^-) \approx Im\lambda(B_d \to J/\psi K_S) \approx -0.25 \,. \qquad (29)$$

$Im\lambda$ for $B_s \to \rho K_S$ is different for a) and b). For case a), the neutral Higgs boson contributions to the asymmetry are small. However $Im\lambda(B_s \to \rho K_S)$ due to CP violation in the KM-matrix can be about 0.1. For case b), $Im\lambda(B_s \to \rho K_S)$ from neutral Higgs boson exchange is only about 0.02.

If interpreted as in Eq.(27), we find for case a), $sin2\alpha = -sin2\beta$, $sin\gamma = 0.05$, and $\alpha + \beta + \gamma \neq 180^0$. For case b), we have, $sin2\alpha = -sin2\beta$, $sin\gamma = 0.01$. We again find, $\alpha + \beta + \gamma \neq 180^0$.

6. The neutron and electron electric dipole moments.

The EDMs of neutron and electron in the MSM model are extremely small. The neutron EDM D_n can only be generated at three loop level. It is predicted to be less than 10^{-31} ecm[19]. The electron EDM is even smaller ($< 10^{-36}$ ecm)[20]. The experimental upper bound on the neutron EDM is 1.2×10^{-25} ecm[21]. For the electron the bound is about 10^{-26} ecm[22]. If future measurement will obtain an EDM larger than the MSM model prediction, it will be an indication for new physics beyond the MSM.

The prediction for the EDMs in the $S_3 \times Z_3$ are very different from the MSM. They may reach the experimental bounds.

At the one loop level, the neutral Higgs contributions to the neutron EDM are small. For case a) we find that $D_n < 2 \times 10^{-28}$ ecm For case b), we have $D_n(d) \approx 2 \times 10^{-29} ecm$. The u quark EDM is zero at the one loop level.

There may be large contributions to the neutron EDM at the two loop level from the Weinberg operator[23] $D_n(W)$ and from the color dipole moment of gluon due to Bar-Zee type of diagrams[24, 25] $D_n(BZ)$. In our model, we have

$$\begin{aligned}
D_n(W) &\approx e\zeta_W \Lambda \frac{1}{64\pi^2} ImZ^i_{tt} \frac{m_t^2}{m^2_{h^0_i}} ln \frac{m_t^2}{m^2_{h^0_i}} \,, \\
D_n(BZ,q) &\approx \frac{m_q}{64\pi^3} \frac{c_q}{9} \alpha_s(\mu) \zeta_{bz} \frac{m_t^2}{m^2_{h^0_i}} \left(ln \frac{m_t^2}{m^2_{h^0_i}} \right)^2 ImZ^i_{tq} \,, \qquad (30)
\end{aligned}$$

where $\zeta_W \approx 6 \times 10^{-6}$, and $\zeta_{bz} \approx 10^{-2}$ are the QCD correction factors, $c_u = 2$ and $c_d = 4$, and $\Lambda \approx 1GeV$ is the chiral symmetry breaking scale. The parameters ImZ are given by

$$\begin{aligned}
ImZ^i_{tt} &= \frac{1}{m_t^2} Re(\tilde{Y}^u_{k,33}(\alpha_{ki} - i\alpha'_{ki})) Im(\tilde{Y}^u_{k',33}(\alpha_{k'i} - i\alpha'_{k'i})) \,, \\
ImZ^i_{tu} &= \frac{1}{m_u m_t} Im(\tilde{Y}^u_{k,33}(\alpha_{ki} - i\alpha'_{ki}) \tilde{Y}^u_{k',11}(\alpha_{k'i} - i\alpha'_{k'i})) \\
ImZ^i_{td} &= \frac{1}{m_d m_t} Im(\tilde{Y}^u_{k,33}(\alpha_{ki} - i\alpha'_{ki}) \tilde{Y}^d_{k',11}(\alpha_{k'i} + i\alpha'_{k'i})) \,. \qquad (31)
\end{aligned}$$

For case a), because there is no CP violation in the up quark sector only down quark loops contribute, $D_n(W)$ from the Weinberg operator at the two loop level is small. There are non-zero $D_n(BZ)$ from d-quark due to Bar-Zee mechanism. We find that the contributions from $Reh^0_{1,2}$ is also small ($< 4 \times 10^{-28} ecm$). Reh^0_3 contribution is even smaller.

For case b), the two loop contributions to the EDM are significantly larger because in this case there is CP violation in the top quark interaction. We have

$$D_n(BZ,u) \approx (2 \sim 8) \times 10^{-26} ecm\ ,$$
$$D_n(BZ,d) \approx (2 \sim 8) \times 10^{-27} ecm\ , \tag{32}$$

for m_t between 100 GeV to 200 GeV. The contribution from the Weinberg operator is small, $D_n(W) \leq 10^{-30} ecm$.

The charged Higgs bosons can also contribute to the neutron EDM. At the one loop level, the u and d quark EDM are given by

$$d_u = -\frac{1}{48\pi^2}\frac{m_l}{m^2_{h^+_i}} ln \frac{m_l^2}{m^2_{h^+_i}} Im[\gamma_{ji}\gamma^*_{ki}(V_{KM}\tilde{Y}^d_j)_{1l}(V^\dagger_{KM}\tilde{Y}^u_k)_{l1}]\ ,$$
$$d_d = \frac{1}{24\pi^2}\frac{m_l}{m^2_{h^+_i}} ln \frac{m_l^2}{m^2_{h^+_i}} Im[\gamma_{ji}\gamma^*_{ki}(V_{KM}\tilde{Y}^d_j)_{l1}(V^\dagger_{KM}\tilde{Y}^u_k)_{1l}]\ . \tag{33}$$

For d_u, l is summed over d, s, and b; and for d_d, l is summed over u, c, and t. At the two loop level, there is a large contribution from the Weinberg operator,

$$D_n(W) \approx e\zeta'_W \Lambda \frac{1}{32\pi^2} Im Z'^i_{tt} \frac{m_t^2}{m^2_{h^+_i}} ln \frac{m_t^2}{m^2_{h^+_i}}\ , \tag{34}$$

where $\zeta'_W = 3 \times 10^{-4}$ is the QCD correction factor, and

$$Im Z'^i_{tt} = \frac{1}{m_b m_t} Im[\gamma_{ji}\gamma^*_{ki}(V_{KM}\tilde{Y}^d_j)_{33}(V_{KM}\tilde{Y}^u_k)_{33}]\ . \tag{35}$$

We find that in case a) the dominant contributions are from the two loop Weinberg operator. We have

$$D_n(W) \approx b_i \times 10^{-19}\frac{GeV^2}{m^2_{h^+_i}} ln \frac{m_t^2}{m^2_{h^+_i}} \frac{m_t^2}{(150GeV)^2}\ ecm\ , \tag{36}$$

where $b_1 = 1.6$, $b_2 = 1.4$ and $b_3 = 1.2 \times 10^{-6}$.

Requiring the contributions to be less than the experimental value, we find the masses of $h^+_{1,2}$ have to be larger than $2.5 TeV$. There is no constraint on h^+_3 mass. Combining this information with those from Eqs.(22) and (23), we find the charged Higgs boson contributions to $\bar{\epsilon}_{LD}$ is less than 3×10^{-5}, and ϵ'/ϵ is less than 3×10^{-5}.

For case b), we find the dominant contribution is from the one loop d quark EDM. We have

$$D_n(d) \approx 5.4 \times 10^{-19} Im(\gamma_{22}\gamma^*_{23})\frac{GeV^2}{m^2_{\eta^+}} ln \frac{m_t^2}{m^2_{\eta^+}} \frac{m_t}{150GeV}\ ecm\ . \tag{37}$$

Requiring $D_n(d)$ to be less than the experimental value, $\bar{\epsilon}_{LD}$ is constrained to be less than 10^{-3}, and ϵ'/ϵ can still be of order 10^{-3}. Assuming maximum mixing, the mass of η^+ is constrained to be larger than $5\ TeV$.

The $S_3 \times Z_3$ model may also have interesting CP violating signatures in the lepton sector. We assume the same $S_3 \times Z_3$ assignments for the left handed and the charged right handed leptons as their quark partnenrs[7]. The mass matrix and Yukawa couplings for the charged leptons are similar to the down quarks. One simply changes the parameters (a, b, c, d , and ξ) for quarks to (a_l , b_l , c_l , d_l , and $\xi_l = |\xi| e^{i\sigma'}$) for leptons. We use[8]: $a_l = 0.106 GeV$, $b_l = 0$, $c_l = 1.781 GeV$, $d_l = 8.6 \times 10^{-3} GeV$. For this set of parameters, we have $m_e = 0.511 MeV$, $m_\mu = 106 MeV$ and $m_\tau = 1784 MeV$ which are in good agreement with experimental data.

The calculation for the electron EDM is similar to the neutron EDM. For case a) we find that the one loop contributions are small ($< 10^{29} ecm$) with $\sigma' = 80^0$. However the two loop contribution due to Bar-Zee mechanism[24, 26] can be as large as 10^{-27} ecm, for $m_t < 200 GeV$. For case b), we find that the one loop and two loop contributions are small ($< 10^{-33} ecm$).

7. Conclusions

We have studied in detail some effects due to two different CP violating mechanisms in the $S_3 \times Z_3$ model. Both mechanisms discussed in this paper can explain the observed CP violation in the neutral K system. CP violation in the neutral K system and the mass difference in the neutral B system constrain the neutral Higgs boson masses to be in the multi TeV region. In the previous discussions we have chosen a particular set of parameters. The detailed predictions will depend on all the parameters, but the general features will remain to be the same. We have checked the predictions using another set of parameters, we find the changes are not significant except that the electron EDM for case b) can reach 10^{-28} ecm. The predictions presented here represent the typical values for the observables. We summarize our results for ϵ'/ϵ, CP violation in B decays and the neutron and electron EDMs in the table below. It is clear from the table, that the predictions for the observables considered are very different in the MSM and in multi-Higgs doublet models. Future experiments will be able to rule out some models.

Observable	MSM	Case a)	Case b)
$\bar{\epsilon}$	Input	Input	Input
ϵ'/ϵ	$10^{-4} \sim 10^{-3}$	$\sim 10^{-5}$	$\sim 10^{-3}$
B Decay Asymmetry	$\alpha + \beta + \gamma = 180^0$	$\alpha + \beta + \gamma \neq 180^0$ $sin2\gamma \approx -sin2\beta \approx$ $0.2 \sim 0.4$ $sin\gamma < 0.1$	$\alpha + \beta + \gamma \neq 180^0$ $sin2\gamma \approx -sin2\beta \approx$ 0.25 $sin\gamma \approx 0$
D_n(ecm)	$10^{-31} \sim 10^{-33}$	can reach 10^{-25}	can reach 10^{-25}
D_e(ecm)	$< 10^{-36}$	$< 10^{-27}$	$< 10^{-28}$

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References

[1] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49**, 652(1973).

[2] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945(1983).

[3] T. Inami and C.S. Lim, Prog. Theor. Phys. **65**, 297(1981).

[4] For a recent review, see, e.g., Y. Nir and H.R. Quinn in B Decays, edited by S. Stone (Worl Scientific, Singapore, 1992), P. 362; I. Dunietz, ibid, P. 393.

[5] Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. **D25**, 1895(1982); T. Brown, N.G. Deshpande, S. Pakvasa and H. Sugawara, Phys. Lett. **B144**, 95(1984);J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1(1987); L. Hall and S. Weinberg, Phys. Rev. **D48**, R979(1993).

[6] S. Weinberg, Phys. Rev. Lett. **37**, 657(1976).

[7] E. Ma, Phys. Rev. **D43**, 2761(1991); ibid **D44**, 587(1991).

[8] N.G. Deshpande, M. Gupta and P. B. Pal. Phys. Rev. **D45**, 953(1992).

[9] L. Lavoura, Phys. Rev. **D46**, 4101(1992).

[10] N.G. Deshpande and Xiao-Gang He, Phys. Rev. **D49**, 4812 (1994).

[11] J. Gasser and H. Leutwyler, Nulc. Phys. **B250**, 465(1985).

[12] C. Jarlskog, Phys. Rev. Lett. **55**, 1039(1985); O.W. Greenberg, Phys. Rev. **D32**, 1941(1985); D.-D. Wu, Phys. Rev. **D33**, 860(1986).

[13] J. Trampetic, Phys. Rev. **D27**, 1565(1983).

[14] A. Buras, M. Jamin and M. Lautenbacher,Nucl. Phys. **B408**, 209(1993).

[15] A. Wagner, talk presented at the European Physics Symposium on High Energy Physics, Marseille, July (1993); L.K. Gibbons et al., Phys. Rev. Lett. **70**, 203(1993).

[16] N.G. Deshpande, Phys. Rev. **D23**, 2654(1981); A. Sanda, ibid, **23**, 2647(1981).

[17] J. Donoghue and B. Holstein, Phys. Rev. **D32**, 1152(1985).

[18] H.-Y. Cheng, Int. J. Mod. Phys. **A7**, 1059(1992).

[19] B. McKellar, S. Choudhury, X.-G. He, and S. Pakvasa, Phys. Lett. **B197**, 556(1987); X.-G. He, B. McKellar and S. Pakvasa, Int. J. Mod. Phys. **A4**, 5011(1989).

[20] M.E. Pospelov and I.B. Khriplovich, Sov. J. Nucl. Phys. **53**, 638(1991).

[21] K. Smith et al., Phys. Lett. **B234**, 234(1990); I.S. Altarev et al., Phys. Lett. **B276**, 242(1992).

[22] K. Abdullah et al., Phys. Rev. Lett. **65**, 2347(1990).

[23] S. Weinberg, Phys. Rev. lett. **63**, 2333(1989).

[24] S.M. Barr and A. Zee, Phys. Rev. Lett. **65**, 21(1990).

[25] J. Gunion and D. Wyler, Phys. Lett. **B248**, 170(1990); D. Chang, W.-Y. Keung and T.C. Yuan, Phys. Lett. **B251**, 608(1990).

[26] D. Chang, W.-Y. Keung and T.C. Yuan, Phys. Rev. **D43**, R14(1991); R.G. Leigh, S. Paban and R.M. Xu, Nucl. Phys. **B352**, 45 (1991).