PRAMANA
— journal of
physics

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Vol. 45, No. 6,
December 1995
pp. 561–567

Kinetic description of ion acoustic waves in a dusty plasma

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MS received 29 May 1995; revised 20 October 1995

Abstract. A self-consistent model for the description of the ion sound wave in a dusty plasma is given. We show that proper consideration of the ion attachment to the grains gives rise to the dissipation of the ion acoustic wave in a dusty plasma medium. Dissipation rate is proportional to the sum of the electron and twice the ion attachment frequencies.

Keywords. Ion acoustic wave; dusty plasma; charge fluctuations.

PACS Nos 52·25; 52·35

1. Introduction

Investigations of the dusty plasma in recent times have acquired considerable momentum. Major reason for the spurt in the interest is due to its numerous applications to the industrial (e.g. semiconductor industry, plasma etching, etc.) as well as cosmic plasmas (e.g. planetary rings, cometary tails, etc.). Periodicity of the subject being reviewed at present is 1·5 years [1–4]. Recent interest in the investigation of the dielectric response of the dusty plasma medium focusses around charge fluctuating grains. A consistent formulation of the dynamics of the dusty plasma (with constant mass and radius of the grain) is given by Bhatt and Pandey [5] and Vladmirov [6]. Mass of the grain may also vary e.g. due to coagulation of the grains [7] and thus, grain radius and mass in general, may also become dynamical quantities. Set of equations considering mass as a dynamical variable have been given elsewhere [8]. For the present however, we shall assume that the grain is spherical and of fixed radius and mass and that the grain charging is solely due to the attachment of electrons and ions. Grains will be assumed massive compared to the electrons and ions and therefore, immobile. Dynamical response time of the grain is slowest in comparison with the response time of the plasma particles so that the assumption about their immobileness is justified.

We shall study the ion acoustic waves in a dusty plasma. Ion acoustic mode has been studied, in the past by several authors e.g. Tsytovich and Havnes [9], D'Angelo [10], Bhatt and Pandey [5] and Ma and Yu [11]. Tsytovich and Havnes [9] describe the ion acoustic mode by assuming that the plasma fluctuations maintain their quasineutrality. They demonstrate that the mode is damped by the charge fluctuation and that the charge fluctuation induced damping could be larger than the collisionless Landau damping. In the kinetic description of Ma and Yu [11], ion acoustic mode is described by assuming that the perturbations in the plasma may cause space charge field and that the mode may damp or grow depending upon how strong the deviation of plasma is from quasineutrality. D'Angelo [10] gives a fluid description of the ion acoustic mode and shows that the mode damps at the frequency at which ions are injected in the plasma. He calls it 'creation damping' as fresh ions injected in the plasma counters the loss caused by the sticking of the ion to the grain surface. His description assumes that the plasma always maintains quasineutrality. Bhatt and Pandey [5] investigate the ion-acoustic streaming instability and show that the charge fluctuation narrows the parameter space of instability. Scope of the present work includes the consistent formulation of the kinetic problem for the ion acoustic mode in the spirit of Vladmirov [9] and explore, whether conclusions arrived by previous investigation in general and Ma and Yu [11] in particular, are valid. To that end, we shall investigate the ion acoustic wave in the dusty plasma.

Since we are interested in the low frequency ion acoustic mode, electrons can be assumed to follow Boltzmann distribution i.e. they are in thermodynamic equilibrium. Furthermore, charging time scale is comparable with the attachment time scale of the ions which in turn is comparable with the acoustic time scale leading to appreciable dissipation of the acoustic mode.

This paper is organized in the following fashion. Section 2 describes the basic equations. In § 3 we discuss the perturbed equations and derive the dispersion relation. In § 4 discussion of the result and comparison with the previous work are given.

2. Basic equation

It is assumed that the grains are spherical conductors and charging of the grains are due to the potential difference between plasma (electrons and ions) and grain surface. Equilibrium currents are given by [12]

$$I_{eo} = -\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_{eo}\exp\left[\frac{e(\phi_g-\phi)}{T_e}\right] \tag{1}$$

$$I_{io} = -\pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_{io}\left[1-\frac{e(\phi_g-\phi)}{T_i}\right] \tag{2}$$

where subscript o denotes the equilibrium quantities; m_α, T_α and $n_{\alpha o}$ are the mass, temperature and equilibrium density of the αth particle; a and ϕ_g are the grain radius and potential respectively; ϕ is the plasma potential and e is the electronic charge. Charge on the grain is a dynamical quantity and its dynamics is described by the following equation

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = I_e + I_i \tag{3}$$

where Q is the average grain charge i.e.

$$Q = \frac{1}{n_d}\int f_d \mathrm{d}q \tag{4}$$

and charge q is an additional independent variable and f_d is the grain distribution function. In equilibrium $I_{eo}+I_{io}=0$ and Q_0 charge is carried by the grains. Equilibrium grain distribution is given by

$$f_{do} = n_d\delta(q+Q_0). \tag{5}$$

Dynamics of the ion is described by kinetic Boltzmann equation

$$\frac{\partial f_i}{\partial t} + \mathbf{v}\cdot\frac{\partial f_i}{\partial \mathbf{r}} + \frac{e}{m_i}\mathbf{E}\cdot\frac{\partial f_i}{\partial \mathbf{v}} = -\int \sigma_i v(f_d f_i - f_{do} f_{io})\mathrm{d}q \tag{6}$$

where,

$$\sigma_i = \begin{cases} \pi a^2\left(1 - \dfrac{2eq}{am_i v^2}\right) & \dfrac{2eq}{am_i v^2} < 1 \\ 0 & \dfrac{2eq}{am_i v^2} \geqslant 1 \end{cases} \tag{7}$$

is the effective collision cross section of the ions with the grains. Moment of eq (5) is described by Bhatt and Pandey [5]. We shall assume that in the equilibrium state, loss of ion is compensated by the external source and the ion distribution function f_{io} is Maxwellian. We should add that assumption of ion following Maxwellian distribution is, in general not valid in the vicinity of the grains. Ion will respond to the electric field of the highly charged grains and therefore in general should be described by the Maxwell Boltzmann distribution.

Electron follows Boltzmann distribution

$$n_e = n_{eo}\exp\left[\frac{e\phi}{T_e}\right]. \tag{8}$$

Localized charge build-up is described by Poisson's equation

$$\nabla^2\phi = 4\pi[en_e - n_d Q] - 4\pi e\int f_i \mathrm{d}\mathbf{v}. \tag{9}$$

Equations (1)–(9) will adequately describe the ion dynamics in a dusty plasma.

3. Dispersion relation

Next, we introduce small perturbation in the distribution function

$$f_d = f_{do} + \hat{f}_d;\quad f_i = f_{io} + \hat{f}_i,\quad \text{with}\quad |\hat{f}_d| \ll f_{do}, |\hat{f}_i| \ll f_{io}.$$

Charge fluctuation equation can be written as

$$\frac{\partial \hat{Q}}{\partial t} + \nu_d^o \hat{Q} = -\frac{e|I_{eo}|}{n_{eo}}\hat{n}_e + e\int_0^\infty \hat{f}_i v\sigma(v, Q_0)\mathrm{d}\mathbf{v} \tag{10}$$

where $\hat{n}_e$ is the perturbed electron density. Last term in the above equation is from the perturbed ion currents. The charge dissipation is described by the charging frequency ν_d^o, which for Maxwellian particle distribution is given by

$$\nu_d^o = \left.\frac{\partial I(q)}{\partial q}\right|_{q=-Q_0} = \frac{1}{\sqrt{2\pi}}\frac{\omega_{pi}^2 a}{v_{Ti}}(1 + \tau + z) \tag{11}$$

with $\tau \equiv T_i/T_e$, $z \equiv eQ_0/aT_e$. Fluctuation in the grain distribution is described by

$$\hat{f}_d = n_{do}[\delta(q + Q_0 - \hat{Q}) - \delta(q + Q_0)]. \tag{12}$$

For the ion distribution, one has

$$\frac{\partial \hat{f}_i}{\partial t} + \mathbf{v}\cdot\frac{\partial \hat{f}_i}{\partial \mathbf{r}} + \frac{e}{m_i}\mathbf{E}\cdot\frac{\partial f_{io}}{\partial \mathbf{v}} = -\nu_{id}^o(v)\hat{f}_i - \hat{\nu}_{id}(v)f_{io} \tag{13}$$

where attachment frequencies $\nu_{id}^o(v)$ and $\hat{\nu}_{id}(v)$ are given by

$$\nu_{id}^o(v) = \int \sigma_i v f_{do}\,\mathrm{d}q \tag{14}$$

$$\hat{\nu}_{id}(v) = \int \sigma_i v \hat{f}_d\,\mathrm{d}q. \tag{15}$$

Assuming that the fluctuations can be represented as the normal mode $\sim \exp i(\omega t - \mathbf{kx})$, ion distribution function is given by

$$\hat{f}_i = \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v} - i\nu_{id}^o(v)}\left[-\left(\frac{ek\hat{\phi}}{m_i}\right)\frac{\partial f_o}{\partial \mathbf{v}} + i\hat{\nu}_{id}(v)f_{io}\right] \tag{16}$$

and the charge fluctuation $\hat{Q}$ is given by

$$\hat{Q} = \frac{ie|I_{eo}|\hat{\phi}}{(\omega - i\nu_d^o)\,T_t} + \frac{i}{\omega - i\nu_d^o}\left[\frac{e^2k\hat{\phi}}{m_i}\int\frac{(\partial f_{io}/\partial \mathbf{v})v\sigma_i\mathrm{d}\mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v} - i\nu_{id}^o(v)} + e\int\frac{\hat{\nu}_{id}(v)f_{io}}{\omega - \mathbf{k}\cdot\mathbf{v} - i\nu_{id}^o(v)}v\sigma_i\mathrm{d}\mathbf{v}\right]. \tag{17}$$

Last term in the above expression is $O(\nu_{id}^o\nu_d^o/\omega)$ and thus can be ignored. Then taking note of the fact that for $\omega \sim \omega_{pi} \gg \max(kv_{Ti}, \nu_d^o, \nu_{id}^o)$, where $\omega_{pi} = \sqrt{4\pi n_{io}e^2/m_i}$, we can approximate the following integral as

$$ie\int\frac{\hat{\nu}_{id}(v)f_{io}}{\omega - \mathbf{k}\cdot\mathbf{v} - i\nu_{id}^o(v)}\mathrm{d}\mathbf{v} \approx n_{io}\left(\frac{i\nu_d^o}{\omega}\right)\hat{Q}. \tag{18}$$

Then plugging back (16) and (17) in the Poisson's equation (9) and taking note of (18), one gets the following dispersion relation (DR)

$$\frac{1 + k^2\lambda_e^2}{k^2\lambda_e^2} + \chi_i - \frac{i\left(1 + \frac{i\nu_{id}^o}{\omega}\frac{n_{io}}{n_{do}}\right)}{(\omega - i\nu_d^o)}\left[\frac{\alpha}{k^2\lambda_e^2} - \beta\frac{\omega_p i^2}{\omega^2}\right] = 0 \tag{19}$$

where $\lambda_e^2 = T_e/4\pi n_{eo}e^2$, $\alpha = |I_{eo}|n_{d_o}/en_{eo}$. We note that α is the electron attachment frequency, as have been shown by Bhatt and Pandey [5]. One may calculate ν_{ed}^o for a Maxwellian electron

$$\nu_{ed}^o = \frac{n_{do}}{n_{eo}}\int \sigma_e v f_{eo}\mathrm{d}\mathbf{v} = \frac{n_{do}|I_{eo}|}{en_{io}}. \tag{20}$$

Similarly ion attachment frequency is given by

$$\nu_{id}^o = \frac{n_{do}}{n_{io}}\int \sigma_i v f_{io}\mathrm{d}\mathbf{v} = \frac{n_{do}I_{io}}{en_{io}}. \tag{21}$$

Thus, we see that using the constraint $I_{eo} + I_{io} = 0$, one can write $v^o_{id} = v^o_{ed}(1 + (n_{eo}/n_{io}))$. This result is in agreement with the result derived on the intuitive ground by Bhatt and Pandey [5]

$$\beta = -\left(\frac{\omega^2}{k^2}\right)\frac{n_{do}}{n_{io}}\int_0^{\infty}\frac{(\partial f_{io}/\partial \mathbf{v})v\sigma(v,Q_0)}{\omega - \mathbf{k}\cdot\mathbf{v} - iv^o_{id}(v)}\mathrm{d}\mathbf{v} \tag{22}$$

$$\chi_i = -\frac{\omega^2_{p_i}}{\omega^2}\frac{1}{n_{io}}\int\frac{(\partial f_{io}/\partial \mathbf{v})}{\omega - \mathbf{k}\cdot\mathbf{v} - iv^o_{id}(v)}\mathrm{d}\mathbf{v}. \tag{23}$$

Without loss of generality, assuming $\mathbf{k}\parallel\hat{\mathbf{x}}$ and going to spherical polar coordinate to evaluate (19) with $\omega \sim \omega_{pi} \gg (kv_{Ti}, v^o_d, v^o_{id})$, one gets

$$\beta = \frac{4}{3}\pi a^2\sqrt{\frac{8T_i}{\pi m_i}}n_{do}\left(1 - \frac{eQ_0}{2CT_i}\right) = \frac{2}{3}\frac{I_{io}}{e}\frac{n_{do}}{n_{io}}A \tag{24}$$

where $A = 1 + (1 - (eQ_0/CT_i))^{-1}$. Ion susceptibility χ_i becomes

$$\chi_i \approx -\frac{\omega^2_{pi}}{\omega^2}\left(1 + \frac{2iv^o_{id}}{\omega}\right). \tag{25}$$

The above expression has been derived by assuming $v^o_{id}(v) \approx v_{id}$ which tantamounts to assuming that $Z_d e^2/am_i v^2 \ll 1$ i.e. ratio of the dust potential to the kinetic energy is much less than 1. In the other limit, one cannot take v^o_{id} out of integral and the result will renormalize the ion acoustic speed.

While evaluating (23) we have not considered the resonant contribution to the ion susceptibility. For $\omega \ll Pv^o_d$, where $P = (n_d/n_e)(aT_e/e^2)$ is the ratio of dust space change to the electron space change (Havnes *et al* [13]) resonant denominator is evaluated by the usual Landau rule. It has been demonstrated by Tsytovich and Havnes [9], that the change fluctuation induced damping rate may exceed the Landau damping.

Thus, DR can be written as

$$(\omega^2 - k^2C_s^2)(\omega - iv^{o'}_d) - i(\varepsilon + 2v^o_{id})k^2C_s^2 = 0 \tag{26}$$

where $v^{o'}_d \equiv v^o_d + \alpha/(1 + k^2\lambda_e^2)$, $C_s = \lambda_e\omega_{pi}/(\sqrt{1 + k^2\lambda_e^2})$, $\varepsilon = \alpha(1 - (2n_{eo}A/3n_{io}))$. In writing DR (26) we have ignored terms $O(v^o_d\alpha, v^o_d\beta, v^o_{id}v^o_d)$. We see that DR (26) reduces to DR of [11] in $v^o_{id} \to 0$ limit. As $\varepsilon \approx v^o_{id}$, we see that the proper consideration of ion attachment causes the mode to damp faster than have been reported [11].

4. Discussion

DR (26) can be solved analytically. However, since $\omega \gg (v^o_{id} \sim \varepsilon)$, it is instructive to solve the above equation perturbatively. Then the roots are

$$\omega_{1,2} = kc_s \pm i\varepsilon\left(\frac{1}{2} + \frac{v^o_{id}}{\varepsilon}\right)$$

$$\omega_3 \approx i\varepsilon\left(\frac{1}{2} + \frac{v^o_{id}}{\varepsilon}\right). \tag{27}$$

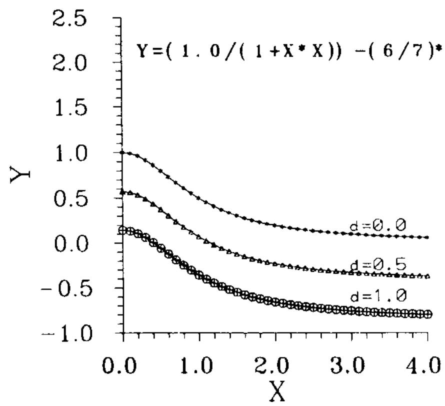

Figure 1. This figure shows variation of $y = (2\omega_i)/\alpha$ against $x = k\lambda_e$ for different values of $d = (n_{eo}/n_{io})$ and $(eQ_o/CT_i) = -2{\cdot}5$ when $\nu^o_{id} = 0$. It is evident from the figure that for certain values of x, mode becomes unstable.

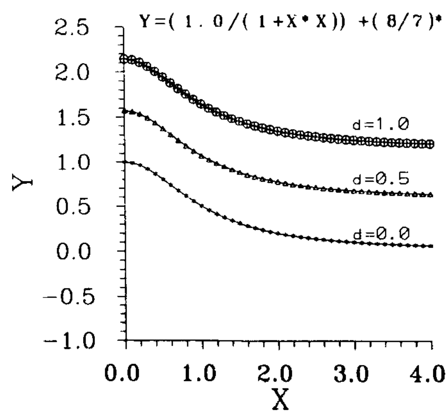

Figure 2. This figure shows the same when $\nu^o_{id} \neq 0$. It is clear from figure 2 that for all wavelengths, charge fluctuation will cause the damping of ion acoustic mode.

Thus, we see that the ion acoustic mode acquires a damping rate proportional to $\sim \varepsilon/2 + \nu^o_{id} \approx \nu^o_{id}$. The above result modifies the damping rate predicted by Ma and Yu [11]. Energy transfer between ion acoustic wave and dust charge fluctuation leads to this damping. Our result is broadly in agreement with the conclusions drawn by Tsytovich and Havnes [9] and D'Angelo [10]. It must be mentioned however, that the dissipation derived by D'Angelo [10] is caused primarily by the injection of the ions and thus is proportional to the injection frequency of the ion in the plasma whereas in the present analysis dissipation is due to the attachment of ions to the grains.

It is interesting to note that without the ion attachment frequency ν^o_{id} ion acoustic mode may become unstable [11], if $\varepsilon < 0$, i.e. $\lambda_e^2 k^2 > 3/2(n_{io}/n_{eo})A$. However, self-consistent treatment of the charge fluctuation shows that even when $\varepsilon < 0$, there is no instability unless additional condition $(\nu^o_{id} - \varepsilon/2) < 0$ is satisfied. As is evident from (20) and (21) and the discussion therein $v_{id}^{o}/\varepsilon \approx d = n_{eo}/n_{io}$. The number density ratio d may have different values, e.g. for diffuse cloud in the interstellar media $\delta \approx 0$ [10] whereas in the planetary rings and cometary tails $d \leqslant 1$ [1–4]. Thus we see that the charge fluctuation will invariably cause the damping of ion acoustic response of the plasma. The instability suggested in ref. [11] is, thus highly unlikely in a dusty plasma. Figure 1 depicts the plot of $y = (2\omega_i/\alpha)$ vs. $x = k\lambda_e$ for $d = 0, 0{\cdot}5, 1$ and $(eQ_0/C T_i) = -2{\cdot}5$ when ion attachment frequency has been ignored. We see that for $d = 0{\cdot}5$ and 1, mode becomes unstable for some critical wavelength. Figure 2 depicts the same for non-zero ion attachment frequency. We see from figure 2 that there is no instability, and ion acoustic mode can only damp in a dusty plasma.

We conclude that the self-consistent treatment of charge dynamics leads to the damping of the acoustic wave. Further, Landau damping will also cause the mode to dissipate [9] and thus, there is no window whatsoever contrary to the claim mode in ref. [11] where the mode may become unstable. The parameter regime in which the acoustic mode becomes unstable is highly unlikely. Our analysis demonstrates that the role of ion attachment in the charge dynamics of a dusty plasma is crucial. It is through this attachment that the ion loses its energy to the grains and causes the damping.

Acknowledgements

One of the authors [BP] thanks Prof. S Vladmirov for discussion during the preparation of the manuscript. We thank the referee for bringing to our notice the work of N D'Angelo [10].

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