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Ion cyclotron instabilities in a mildly relativistic hydrogen-deuterium fusion plasma

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Abstract. A dispersion relation for the perpendicular propagation of the electromagnetic ion cyclotron wave around the second harmonic of the deuterium ion gyrofrequency in a mildly relativistic, anisotropic Maxwellian plasma with hydrogen as the majority species and deuterium as the minority component has been derived. The work has been carried out in the frame of reference of the majority hydrogen ions; to these ions the waves at $2\Omega_D$ would be at its own gyrofrequency.

Using a small quantity ε to order all relevant parameters of the plasma, it was possible to derive the dispersion relations in a simple form. To the lowest order the relativistic factors do not enter the dispersion relation. The plasma can now support two modes—one above and the other below the hydrogen gyrofrequency in agreement with the assumptions. This was also verified numerically using a standard root solver thereby justifying the correctness of the ordering scheme.

In the next higher order, the dispersion relation is a quartic equation and is sensitively dependent on the relativistic factors. The plasma can now support four modes, both above and below the hydrogen gyrofrequency and consistent with the ordering scheme used. However the modes can now coalesce resulting in complex conjugate roots to the dispersion relation thereby indicating an instability.

The advantage of such a scheme is that two dispersion relations – one of which is independent of the relativistic factors and the other which is sensitively dependent on them can be separated out.

Keywords. Ion cyclotron waves; two-ion relativistic plasma; dispersion relation.

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1. Introduction

Ohmic heating alone is not sufficient to bring a plasma to its ignition conditions. Among the many options available, supplementary heating by waves in the ion cyclotron range of frequencies (ICRF) is probably the most common scheme at present and is being used in a large number of tokamaks and mirror machines. These heating experiments have concentrated on the use of RF waves at frequencies ω both above and below the ion gyrofrequency Ω_i. The method is inherently simple, technologically attractive and has proved to be quite successful in raising plasma temperatures.

In this method heating is accomplished by IC damping where the wave energy is absorbed and converted to particle energy and is therefore a hot plasma effect. In a pure hydrogen plasma for waves around the first harmonic of Ω_i, the lowest order thermal corrections to the cold plasma terms cancel and consequently wave absorption is weak. However, for second harmonic ($\omega \approx 2\Omega_D$) heating, the lowest order thermal corrections are not negligible and a useful amount of wave damping can take place [1].

Fusion plasmas however, often contain more than one ionic species and this can give rise to additional absorption processes. In fact, in early low power experiments, the physics of wave damping at $\omega \approx 2\Omega_D$ in a deuterium plasma was found to be dominated by the presence of a small amount of hydrogen ions (to which this wave is at its fundamental gyrofrequency). These ions played a dominant role in the propagation and absorption of wave energy. Starting with modest powers ICRF heating is at present an important scheme with which impressive powers of a few tens of MW have been achieved [2]. With larger and larger power inputs leading to higher and higher energies, relativistic effects can be expected to play an important role in ICRF heating experiments in future.

As mentioned above, fusion plasmas generally have various types of ions in differing concentrations. Of the many scenarios possible, we consider a mildly relativistic plasma containing hydrogen (H) as the majority species and deuterium (D) as the minority constituent and study IC propagation at the second harmonic of deuterium ion gyrofrequency in it (that is $\omega \approx 2\Omega_D$). Since we propose to work in the frame of the majority hydrogen ions this wave at $\omega \approx 2\Omega_D$ would be at its own gyrofrequency Ω_H.

Past analyses of dispersion relations for IC propagation in multi-ion component plasmas have been essentially numerical probably because of the complexity involved in an analytic calculation. Unfortunately, a fully numeric solution has the drawback that the cold plasma terms dominate the relativistic terms of the dielectric tensor thus masking many interesting features of wave propagation in relativistic plasmas. Guided by the experimental parameters of an early mirror experiment and the numerical solutions of the dispersion relations we have, by means of an ordering parameter ε derived two dispersion relations. To the lowest order we arrive at a quadratic equation which is independent of the relativistic terms. This relation supports two modes, one above and the other below the hydrogen gyrofrequency Ω_H. The next higher order dispersion relation is a fourth order equation which is sensitively dependent on the relativistic terms. This relation yields four modes on either side of Ω_H which can coalesce to make the plasma unstable.

The advantage of such an analytic scheme is that we have been able to separate out two dispersion relations, one of which is independent of the relativistic factors while the other is sensitively dependent on them. Such a separation is not possible in a fully numerical calculation.

2. The dielectric tensor elements

We are interested in the perpendicular propagation of IC waves around the second harmonic of the deuterium ion gyrofrequency ($\omega \approx 2\Omega_D$), in a mildly relativistic plasma with hydrogen as the majority species and deuterium as a minority constituent. The electrons form a neutralising background. The expressions for the relevant dielectric tensor elements have been derived earlier for a single ion plasma [3]; we extend these to a two-ion plasma. We too choose f_0 to be a bi-Maxwellian which is given by [3]

$$f_0 = f_\perp f_\parallel = \frac{1}{\pi^{3/2}\bar{v}_\perp^2 \bar{v}_\parallel} \exp\left(-\frac{v_\perp^2}{\bar{v}_\perp^2} - \frac{v_\parallel^2}{\bar{v}_\parallel^2} \right). \tag{1}$$

The thermal energies $\bar{v}_\perp^2$ and $\bar{v}_\parallel^2$ are related to the temperatures $T_\perp$ and $T_\parallel$, perpendicular and parallel to the magnetic field B_0 respectively by

$$\bar{v}_\perp^2 = \frac{2T_\perp}{M} \text{ and } \bar{v}_\parallel^2 = \frac{2T_\parallel}{M}, \tag{2}$$

where M is the rest mass.

On substituting (1) into the expressions for the dielectric tensor elements and carrying out the various integrations we get the following expressions for K_{xx}, K_{xy} and K_{yy}. A detailed derivation of these elements has been given earlier [3]; we, however, give only the final expressions to facilitate our discussions.

The relevant expressions are,

$$K_{xx} - 1 = \sum_{\mathrm{H,D,e}} \bar{\omega}_p^2 \sum_{n=1}^{\infty} n^2 \left\{ \frac{T_0}{n^2 - Z^2} + \left[\frac{\bar{v}_\perp^2}{c^2} T_1 + \frac{1}{4} \frac{\bar{v}_\parallel^2}{c^2} T_0 \right] \frac{n^2 + Z^2}{(n^2 - Z^2)^2} + \left[\frac{3}{2} \left(\frac{\bar{v}_\perp^2}{c^2} \right)^2 T_2 + \frac{1}{2} \left(\frac{\bar{v}_\perp^2}{c^2} \right) \left(\frac{\bar{v}_\parallel^2}{c^2} \right) T_1 + \frac{3}{16} \left(\frac{\bar{v}_\parallel^2}{c^2} \right)^2 T_0 \right] \frac{Z^2(3n^2 + Z^2)}{(n^2 - Z^2)^3} \right\}. \tag{3a}$$

$$\frac{K_{xy}}{i} = \sum_{\mathrm{H,D,e}} \frac{\bar{\omega}_p^2}{Z} \sum_{n=1}^{\infty} n^2 \left\{ \frac{T'_0}{n^2 - Z^2} + \left[\frac{\bar{v}_\perp^2}{c^2} T'_1 + \frac{1}{4} \frac{\bar{v}_\parallel^2}{c^2} T'_0 \right] \frac{2Z^2}{(n^2 - Z^2)^2} + \left[\frac{3}{2} \left(\frac{\bar{v}_\perp^2}{c^2} \right)^2 T'_2 + \frac{1}{2} \left(\frac{\bar{v}_\perp^2}{c^2} \right) \left(\frac{\bar{v}_\parallel^2}{c^2} \right) T'_1 + \frac{3}{16} \left(\frac{\bar{v}_\parallel^2}{c^2} \right)^2 T'_0 \right] \frac{Z^2(n^2 + 3Z^2)}{(n^2 - Z^2)^3} \right\}. \tag{3b}$$

and

$$K_{yy} - 1 = \sum_{\mathrm{H,D,e}} \bar{\omega}_p^2 \sum_{n=0}^{\infty} \left\{ \frac{T''_0}{n^2 - Z^2} + \left[\frac{\bar{v}_\perp^2}{c^2} T''_1 + \frac{1}{4} \frac{\bar{v}_\parallel^2}{c^2} T''_0 \right] \frac{n^2 + Z^2}{(n^2 - Z^2)^2} + \left[\frac{3}{2} \left(\frac{\bar{v}_\perp^2}{c^2} \right)^2 T''_2 + \frac{1}{2} \left(\frac{\bar{v}_\perp^2}{c^2} \right) \left(\frac{\bar{v}_\parallel^2}{c^2} \right) T''_1 + \frac{3}{16} \left(\frac{\bar{v}_\parallel^2}{c^2} \right)^2 T''_0 \right] \frac{Z^2(3n^2 + Z^2)}{(n^2 - Z^2)^3} \right\}. \tag{3c}$$

In the above the first summation is over the species (H indicating hydrogen, D deuterium and e electrons).

The definitions of the other parameters are

$$\omega_p^2 = \frac{4\pi N e^2}{M}, \quad \Omega_0 = \frac{eB_0}{Mc} = \gamma\Omega,$$

$$\bar{\omega}_p^2 = \frac{\omega_p^2}{\Omega_0^2} \quad \text{and} \quad Z = \frac{\omega}{\Omega_0} \tag{4}$$

where c is the velocity of light, N, the plasma density and γ the relativistic factor.

The Ts arise from the $\mathrm{d}v_\perp$ integrations and are, in general, functions of the modified Bessel function $I_n(l_\perp)$ [3]. The argument $l_\perp$ of I_n is defined by

$$l_\perp = \frac{k_\perp^2 T_\perp}{\Omega_0^2 M}. \tag{5}$$

The explicit functional forms of the Ts are given in [3]. Also the prime on the summation in K_{yy} indicates that the $n = 0$ contribution has to be divided by 2.

3. The ordering procedure

We are interested in the perpendicular propagation of IC waves around $\omega \approx 2\Omega_D$ in a hydrogen-deuterium plasma. As mentioned, to the hydrogen ions these waves at $\omega \approx 2\Omega_D$ would be at its own gyrofrequency Ω_H. We shall thus work in the frame of the hydrogen ions.

The tensor elements (3a) to (3c) are all infinite summations and hence if used as such, the dispersion relation would become very complicated. Therefore, to arrive at a dispersion relation in a simple form, the dielectric tensor elements K_{ij} must contain only a finite number of terms to begin with. Such a truncation is possible only if the individual members of the tensor elements become progressively lesser than 1. We, therefore, define a small parameter ε to scale all the relevant parameters in the individual tensor elements and the final dispersion relation.

A quantity which is small and therefore convenient for the definition of ε is the ratio of the rest mass of the electron to that of the proton; thus $\varepsilon \approx (M_e/M_H)^{1/2}$. Having defined ε we next characterize the wave in terms of this small parameter, conforming with experiments over a wide range of parameters. Theoretically, for a resonance, the frequency ω must be equal to Ω_H; experimentally, however, the waves usually have a small range of frequencies around Ω_H. It is this deviation from the exact value of Ω_H that we scale by ε.

In the presently available ICRF heating experiments the parameter (5) is much less than unity [4] and hence we shall scale this factor also in terms of ε; that is $l_{\perp D} \approx \varepsilon$. Such an ordering, while agreeing with experiment, also has the advantage that the exponential and modified Bessel functions that occur in (3a) to (3c) can be expanded as a power series. We also make the simplifying assumption that the two ionic species are approximately in temperature equilibrium while the electrons are relatively colder than the ions. Thus $T_{\perp H}/T_{\perp D} \approx 1$ (so that $l_{\perp H} \approx \varepsilon$) while $T_{\perp e}/T_{\perp H} \approx \varepsilon$. However the electrons in spite of being colder will, because of their smaller mass, be more energetic and thus we shall assume that $\frac{\bar{v}_{\perp H}^2}{c^2} \approx \frac{\bar{v}_{\perp D}^2}{c^2} \approx \varepsilon^2$ while $\frac{\bar{v}_{\perp e}^2}{c^2} = \varepsilon$. And finally in many typical fusion experiments the ion plasma frequency is much larger than the ion gyrofrequency so that $(1/\bar{\omega}_{pH}^2) \approx \varepsilon$. Summarising, we have

$$M_e/M_H \approx \varepsilon^2, \quad \Gamma = 1 - Z_H^2 \approx \varepsilon, \quad T_{\perp H}/T_{\perp D} \approx 1, \quad T_{\perp e}/T_{\perp H} \approx \varepsilon,$$

$$\frac{\bar{v}_{\perp H}^2}{c^2} \approx \frac{\bar{v}_{\perp D}^2}{c^2} \approx \varepsilon^2, \quad \frac{\bar{v}_{\perp e}^2}{c^2} \approx \varepsilon, \quad l_{\perp H} \approx l_{\perp D} \approx \varepsilon \quad \text{and} \quad \frac{1}{\bar{\omega}_{PH}^2} \approx \varepsilon \tag{6}$$

We shall use this ordering scheme in the derivation of our dispersion relations.

4. The tensor elements and the dispersion relation

The expressions for the dielectric tensor elements K_{xx}, K_{xy} and K_{yy} can be derived using the ordering scheme of (6) and the relevant expressions from (3a) to (3c). In fact they are given, to order ε^2, in [3]. However, when the tensor elements are of this order the relativistic terms do not enter the dispersion relation [5]. We have therefore extended the tensor elements to order ε^3 while also considering contributions from our second ionic species. Unfortunately the expressions for the tensor elements now become very lengthy and hence we shall not give them here; however, their derivation is straight forward.

Substituting the tensor elements into the formula for dispersion relation (equation (21) of [3]) and simplifying we arrive at our dispersion relation which we shall, for the moment, write as

$$\mathrm{D}(\omega, k_\perp) = \mathrm{D}_1(\omega, k_\perp;\ \approx \varepsilon^2) + \mathrm{D}_2(\omega, k_\perp;\ \approx \varepsilon^4) = 0. \tag{7}$$

Relation (7) indicates that our dispersion relation has to be written as a sum of two parts—the first one, namely D_1, containing terms of order ε^2 and D_2 terms of order ε^4. Obviously the two have to be separately equated to zero and solutions obtained.

The dispersion relation, to order ε^2, has the form

$$\begin{aligned} \mathrm{D}_1(\omega, k_\perp;\ \approx \varepsilon^2) = \Gamma^2 &\left\{ - N_{\mathrm{DH}}\left[\frac{1}{3}N_{\mathrm{DH}} + \frac{4}{9} - \frac{2}{3}N_{\mathrm{eH}} + \frac{2}{3}\frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}}\right] + N_{\mathrm{eH}}^2\right\} \\ &+ \Gamma\left\{\frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}} - 1 - \frac{4}{3}N_{\mathrm{DH}} - \delta\right\} - \frac{1}{2}l_{\perp\mathrm{H}}^2 \\ &+ \left[\frac{4\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{c^2}\right]\left[\frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}} - 1 - \frac{4}{3}N_{\mathrm{DH}}\right] = 0 \end{aligned} \tag{8}$$

where $N_{\mathrm{eH}} = N_e/N_{\mathrm{H}}$ etc, and

$$\begin{aligned} \delta = &\left\{\frac{2}{\bar{\omega}_{\mathrm{pH}}^2} - l_{\perp\mathrm{D}}\left[\frac{1}{2}N_{\mathrm{DH}}\frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}} + \frac{1}{3}N_{\mathrm{DH}}^2 + \frac{2}{3}N_{\mathrm{DH}} - N_{\mathrm{eH}}N_{\mathrm{DH}}\right]\right. \\ &\left. + l_{\perp\mathrm{H}}\left[\frac{1}{3} - 2N_{\mathrm{eH}} + \frac{2}{3}N_{\mathrm{DH}}\right] - l_{\perp\mathrm{H}}\left[1 - \frac{l_{\perp\mathrm{H}}}{\beta_\perp} + \frac{4}{3}N_{\mathrm{DH}}\right]\right\} \end{aligned}$$

and

$$\beta_{\perp\mathrm{H}} = \frac{4\pi N_{\mathrm{H}} T_{\perp\mathrm{H}}}{B_0^2} \quad \text{with} \quad \frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}} \approx 1 + N_{\mathrm{DH}}$$

As a check on (8) we note that for $N_{\mathrm{D}} = 0$, it reduces to the dispersion relation in [3]; the small differences being due to our slightly different ordering. Inspecting (8), we find that for the first three terms to be of order ε^2 we need to set [6]

$$\left[1 - \frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}} + \frac{4}{3}N_{\mathrm{DH}}\right] \approx \varepsilon. \tag{9}$$

With this the last term and a factor similar to (9) in δ in (8) is of the order ε^3 and hence drops out of it.

The dispersion relation, to order ε^2, is thus given by

$$\mathrm{D}_1(\omega, k_\perp;\ \approx \varepsilon^2) = \Gamma^2\left\{N_{\mathrm{eH}}^2 - N_{\mathrm{DH}}\left[\frac{1}{3}N_{\mathrm{DH}} + \frac{4}{9} - \frac{2}{3}N_{\mathrm{eH}} + \frac{2}{3}\frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}}\right]\right\}$$

$$-\Gamma\left\{1-\frac{l_{\perp H}}{\beta_{\perp H}}+\frac{4}{3}N_{DH}+\delta\right\}-\frac{1}{2}l_{\perp H}^2=0 \tag{10}$$

where

$$\delta=\left\{\frac{2}{\omega_{pH}^2}-l_{\perp D}\left[\frac{1}{2}N_{DH}\frac{l_{\perp H}}{\beta_{\perp H}}+\frac{1}{3}N_{DH}^2+\frac{2}{3}N_{DH}-N_{cH}N_{DH}\right]\right.$$
$$\left.+\,l_{\perp H}\left[\frac{1}{3}-2N_{cH}+\frac{2}{3}N_{DH}\right]\right\}$$

Adding the higher order terms in (8) and in δ to $D_2(\omega,k_\perp:\ \approx\varepsilon^4)$ the dispersion relation, to order ε^4, is given by

$$a_4\Gamma^4+a_3\Gamma^3+a_2\Gamma^2+a_1\Gamma+a_0=0. \tag{11}$$

The expressions for the coefficients in (11) are lengthy and are therefore given in an appendix, namely Appendix A. As a check on these expressions we note that for $N_{DH}=0$ they reduce to the corresponding expressions for a single ion plasma [5]. Here too a term

$$\left[\frac{24(\bar{v}_{\perp H}^2)^2+8\bar{v}_{\perp H}^2\bar{v}_{\parallel H}^2+3(\bar{v}_{\parallel H}^2)^2}{4c^4}\right]\left(1+\frac{4}{3}N_{DH}-\frac{l_{\perp H}}{\beta_{\perp H}}\right)$$

drops out of (11) as it is now of order ε^5.

5. Discussion

In the derivation of our dispersion relations (10) and (11), the scaling adopted was that of $l_{\perp H}\approx l_{\perp D}\approx\varepsilon$ as the two ionic species were assumed to be in temperature equilibrium. It would, however, be interesting to also consider the case where the two ionic species have appreciably different temperatures and the consequent changes in our dispersion relation. Such a situation, though algebraically tedious, can easily be studied by changing the ordering of $l_{\perp H}$ and $l_{\perp D}$. We discuss below the cases where the majority hydrogen ions are much colder than deuterium and vice versa.

We first consider $T_{\perp H}\to 0$ so that $l_{\perp H}\approx\varepsilon^2$ and $\bar{v}_{\perp H}^2/c^2\approx\varepsilon^3$. However, $l_{\perp H}/\beta_{\perp H}$ is finite as it is independent of temperature. The quadratic equation (10) now has one solution at $Z_H(=\omega/\Omega_H)=1{\cdot}0$. This can be anticipated physically as the majority species hydrogen is now effectively cold. The other solution is still dependent on the finite temperature effects introduced by the second species deuterium. On the other hand the quartic equation (11) reduces to a cubic equation, the characteristics of which now depend on the deuterium density and temperature. A solution of this dispersion relation, for the parameters given below, gives one real root and a pair of complex conjugate roots around $Z_H=1{\cdot}0$. No major changes in the form of these equations were introduced when $T_{\parallel H}\to 0$.

No major changes in (10) and (11) were found for the other limiting case of $T_{\perp D}\to 0$ $(l_{\perp D}\approx\varepsilon^2,\ \bar{v}_{\perp D}^2/c^2\approx\varepsilon^3)$.

6. Results

The dispersion relations have been plotted for conditions of a typical mirror experiment [7].

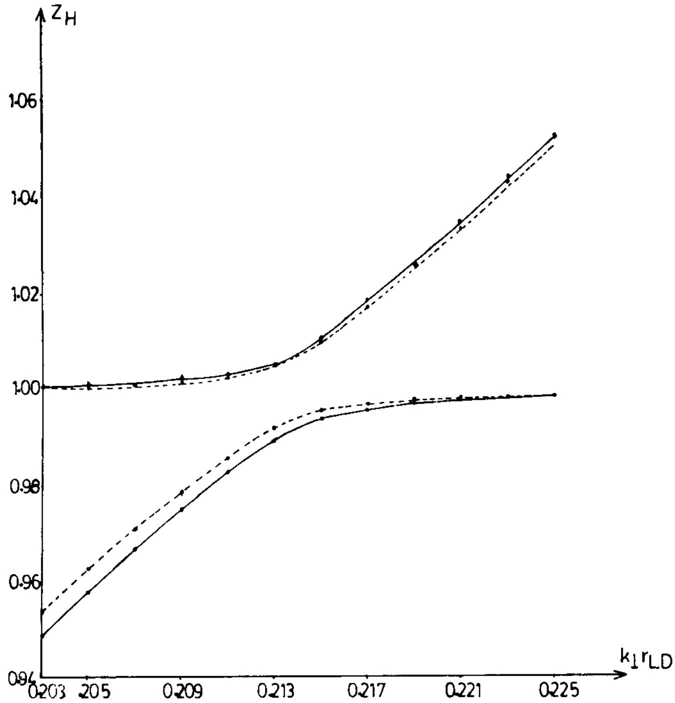

Figure 1. A plot of Z_H versus $k_\perp r_{LD}$ for $T_{\perp e}/T_{\perp H} = 0{\cdot}023$ and $T_{\perp D}/T_{\perp H} = 0{\cdot}5$ and $T_{\perp H} = 50\,\text{keV}$. The dotted lines are the solutions of (10) for $N_H = 2{\cdot}67 * 10^{13}\,\text{cm}^{-3}$ and a deuterium density 30% of that of hydrogen. The solid lines are for a single ion plasma ($N_H = 3{\cdot}76318 * 10^{13}\,\text{cm}^{-3}$ and $N_D = 0{\cdot}0$). The dots on the curves are the solutions obtained numerically.

The ions in the experiment had a temperature $T_{\perp H} = 50\,\text{keV}$ and to conform to our ordering scheme we assume a slightly lower electron temperature of $T_{\perp e} = 1{\cdot}15\,\text{keV}$ compared to the observed value of 9 keV. Since the electron contributions to (10) and (11) are only minimal this assumption does not greatly alter our results. The other relevant parameter is $B_0 = 4 * 10^4\,\text{G}$.

Figure 1 is a plot of Z_H versus $k_\perp r_{LD}$(r_{LD} is deuterium ion Larmour radius) for $T_{\perp e}/T_{\perp H} = 0{\cdot}023$ and $T_{\perp D}/T_{\perp H} = 0{\cdot}5$. Since $l_\perp \approx \varepsilon (= 0{\cdot}023)$, we are restricted to a small range of $k_\perp r_{LD}$ from 0·205 to 0·225. Corresponding to the midpoint of this interval, $\beta_{\perp H}$ was calculated using (9) and finally N_H from the definition of $\beta_{\perp H}$. The dotted lines are the solutions of (10) for $N_H = 2{\cdot}67 * 10^{13}\,\text{cm}^{-3}$ and a deuterium density of 30% of that of hydrogen. We find that there are two modes on either side of the hydrogen ion gyrofrequency Ω_H in agreement with our assumptions. The lower frequency (LF) mode approaches Ω_H while the other high frequency (HF) mode deviates away from it for increasing $k_\perp r_{LD}$. For a comparative study we also plot these modes for a single ion plasma ($N_H = 3{\cdot}76318 * 10^{13}\,\text{cm}^{-3}$ and N_D is 0·0) and these are depicted as the pair of solid lines. We find that the curves are similar; however the point of their closest approach decreases with increasing deuterium density.

Extensive numerical calculations were also performed to confirm the above results. The routines for the calculation of the modified Bessel functions in the tensor elements (3a) to (3c) were obtained from the well known text numerical recipes [8]. The returned values of $e^{-l_\perp} I_n(l_\perp)$ were then rechecked for their accuracy against their values available in literature [9]. The dispersion relations were then set up using (3a) to (3c) with the relativistic terms equal to zero ((10) is a non-relativistic dispersion relation) and solved iteratively using the standard root solver ZANLYT. The dots on the curves in figure 1 are the solutions obtained numerically. We find the agreement to be good especially for the LF mode where they are almost identical with the analytic solutions. Very slight variations occur for the HF mode. However, on the whole the numerical solutions confirm the soundness of our ordering scheme and the correctness of relation (10).

Figure 2 is a plot of our quartic relation (11) for $N_H = 2{\cdot}9563 * 10^{13}\,\mathrm{cm}^{-3}$ (with a deuterium density 20% of this value and depicted by solid lines) and $2{\cdot}67 * 10^{13}\,\mathrm{cm}^{-3}$ (deuterium density is equal to 30% of this value; depicted by dotted lines), the temperature $T_{\perp H}$ being $= 50\,\mathrm{keV}$ and $T_{\perp D}/T_{\perp H} = 0{\cdot}5$. As can be seen, the plasma can now support four modes on either side of Ω_H. However there is only a small region of wavelength where there are four distinct modes. The LF and HF modes then coalesce

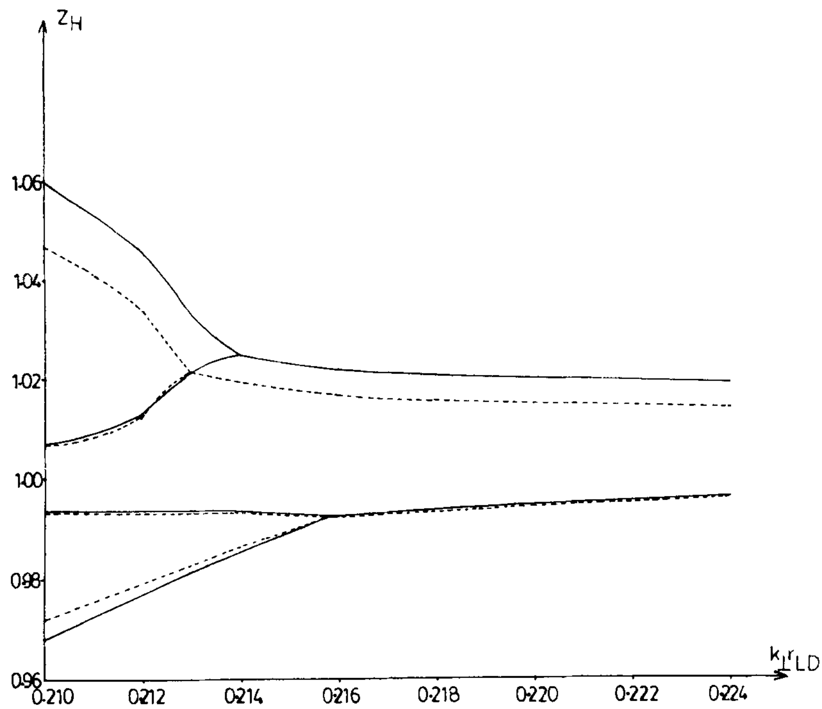

Figure 2. A plot of quartic equation (11). The solid lines are for $N_H = 2{\cdot}9563 * 10^{13}\,\mathrm{cm}^{-3}$ and a deuterium density 20% of that of hydrogen. The dotted lines are for $N_H = 2{\cdot}6700 * 10^{13}\,\mathrm{cm}^{-3}$ and a deuterium density 30% of that of hydrogen. The other parameters are the same as in figure 1.

to give a pair of complex conjugate roots indicating an instability. As can be seen the variations for the two values of density are minor except for the early onset of the instability for the HF mode for the latter case. Unfortunately we are not able to demonstrate any numerical confirmation of figure 2, due to the reasons mentioned in § 1. The non-relativistic terms "interfere" with and even dominate the relativistic terms in a fully numeric calculation; whereas analytically we have been successful in isolating the relativistic from the non-relativistic dispersion relation.

We now briefly compare these results with that obtained for a single ion plasma [5]. A weakly relativistic single ion plasma can support three modes—a pair of LF modes which coalesce to produce an instability and a HF mode. However, for a two ion plasma we find that in addition to the three modes, the second ion component introduces another HF mode.

7. Conclusions

We have in this paper studied the propagation and stability of IC modes in weakly relativistic two-ion component plasma with hydrogen as the majority species and deuterium as the minority species. We have, by means of an ordering parameter, been able to separate out two dispersion relations – one of which is independent of the relativistic factors (and having stable roots) and the other which depends on the relativistic terms. The non-relativistic relation has two solutions—a HF mode and a LF mode, with frequencies respectively higher and lower than the ion gyrofrequency. The latter dispersion relation supports four modes—a pair of HF and LF modes which can coalesce to make the plasma unstable. The relativistic terms are responsible for the instability. The results for the non-relativistic case have also been verified numerically using a standard root solver.

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Appendix A

$$a_4 = N_{\mathrm{DH}}\left[\frac{8}{9}\frac{l_{\perp \mathrm{H}}}{\beta_{\perp \mathrm{H}}} + \frac{16}{27} + \frac{4}{9}N_{\mathrm{DH}} - \frac{8}{9}N_{\mathrm{eH}}\right]$$

$$a_3 = l_{\perp \mathrm{H}}\left[-\frac{1}{3}\frac{l_{\perp \mathrm{H}}}{\beta_{\perp \mathrm{H}}} - \frac{4}{9} + \frac{4}{9}N_{\mathrm{DH}} + \frac{4}{3}N_{\mathrm{eH}}\right]$$

$$+ l_{\perp \mathrm{D}}N_{\mathrm{DH}}\left[\frac{4}{3}N_{\mathrm{eH}} - \frac{2l_{\perp \mathrm{H}}}{3\beta_{\perp \mathrm{H}}} - \frac{8}{9} - \frac{4}{9}N_{\mathrm{DH}}\right]$$

$$-\frac{1}{\bar{\omega}_{\mathrm{pH}}^2}\left[\frac{l_{\perp \mathrm{H}}}{\beta_{\perp \mathrm{H}}} + 2 + \frac{4}{3}N_{\mathrm{DH}}\right]$$

$$a_2 = -l_{\perp \mathrm{H}}\left[\frac{5}{8}\frac{l_{\perp \mathrm{H}}}{\beta_{\perp \mathrm{H}}}l_{\perp \mathrm{H}} + \frac{53}{48}l_{\perp \mathrm{H}} + \frac{4}{\bar{\omega}_{\mathrm{pH}}^2} + \frac{9}{4}l_{\perp \mathrm{H}}N_{\mathrm{DH}} + 2\frac{T_{\perp \mathrm{e}}}{T_{\perp \mathrm{H}}}N_{\mathrm{eH}}\right.$$

$$-\frac{15}{4}l_{\perp\mathrm{H}}N_{\mathrm{eH}}\Bigg] + N_{\mathrm{DH}}l_{\perp\mathrm{D}}\left[\frac{1}{2}\frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}}l_{\perp\mathrm{D}} + \frac{3}{5}l_{\perp\mathrm{D}} + \frac{1}{3}l_{\perp\mathrm{H}} + \frac{1}{\bar{\omega}_{\mathrm{pH}}^2}\right.$$

$$\left. + \frac{5}{12}l_{\perp\mathrm{D}}N_{\mathrm{DH}} - \frac{3}{2}l_{\perp\mathrm{D}}N_{\mathrm{eH}}\right] + \left[\frac{4\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right]\left[\frac{l_{\perp\mathrm{H}}}{2\beta_{\perp\mathrm{H}}} + 1{\cdot}0\right] + 2\frac{M_{\mathrm{e}}}{M_{\mathrm{H}}}N_{\mathrm{eH}}$$

$$+ \frac{2}{9}N_{\mathrm{DH}}\left[\frac{4\bar{v}_{\perp\mathrm{D}}^2 + \bar{v}_{\parallel\mathrm{D}}^2}{2c^2}\right] + \left[\frac{4\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right]\left[\frac{4}{9}N_{\mathrm{DH}} - 2N_{\mathrm{eH}}\right]$$

$$- l_{\perp\mathrm{H}}\left[1 + \frac{4}{3}N_{\mathrm{DH}} - \frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}}\right]$$

$$a_1 = \left[\frac{4\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right]\left[1 + \frac{4}{3}N_{\mathrm{DH}} - \frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}}\right] + l_{\perp\mathrm{H}}\left\{\frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}}\left[\frac{6\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right] - l_{\perp\mathrm{H}}^2\right.$$

$$\left. + \frac{4}{3}\left[\frac{7\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right] + \left[\frac{24\bar{v}_{\perp\mathrm{H}}^2 + 4\bar{v}_{\parallel\mathrm{H}}^2}{6c^2}\right]N_{\mathrm{DH}} - 4\left[\frac{6\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right]N_{\mathrm{eH}}\right\}$$

$$+ N_{\mathrm{DH}}l_{\perp\mathrm{D}}\left\{-\frac{1}{2}\frac{l_{\perp\mathrm{H}}}{\beta_{\perp\mathrm{H}}}\left[\frac{6\bar{v}_{\perp\mathrm{D}}^2 + \bar{v}_{\parallel\mathrm{D}}^2}{2c^2}\right] + \frac{1}{6}l_{\perp\mathrm{D}}^2 - \left[\frac{6\bar{v}_{\perp\mathrm{D}}^2 + \bar{v}_{\parallel\mathrm{D}}^2}{4c^2}\right] - \frac{1}{2}l_{\perp\mathrm{H}}l_{\perp\mathrm{D}}\right.$$

$$- \frac{1}{6}\left[\frac{4\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right] + \frac{3}{4}l_{\perp\mathrm{H}}^2 - \frac{1}{3}\left[\frac{6\bar{v}_{\perp\mathrm{D}}^2 + \bar{v}_{\parallel\mathrm{D}}^2}{2c^2}\right]N_{\mathrm{DH}}$$

$$\left. + \left[\frac{6\bar{v}_{\perp\mathrm{D}}^2 + \bar{v}_{\parallel\mathrm{D}}^2}{2c^2}\right]N_{\mathrm{eH}}\right\} + \frac{2}{\bar{\omega}_{\mathrm{pH}}^2}\left[\frac{4\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right]$$

and

$$a_0 = l_{\perp\mathrm{H}}^2\left[\frac{6\bar{v}_{\perp\mathrm{H}}^2 + \bar{v}_{\parallel\mathrm{H}}^2}{2c^2}\right].$$

References

[1] R A Cairns, *Radio frequency heating of plasmas* (Adam Hilger, England, 1991)
[2] A Kaye, T Brown, V Bhatnagar, P Crawley, J Jacquinot, R Lobel, J Plancoulaine, P-H Rebut, T Wade and C Walker, *Fusion Eng. Des.* **24**, 1 (1994)
[3] R W Landau and S Cuperman, *J. Plasma Phys.* **4**, 495 (1970)
[4] V Fuchs and A Bers, *Phys. Fluids* **31**, 3702 (1988)
[5] Chandu Venugopal, P J Kurian and G Renuka, *J. Plasma Phys.* **51**, 371 (1994)
[6] R W Landau and S Cuperman, *J. Plasma Phys.* **6**, 495 (1971)
[7] G R Smith, W M Nevins and W M Sharp, *Phys. Fluids* **27**, 2120 (1984)
[8] W H Press, B P Flannery, S A Teukolsky and W T Vetterling, *Numerical recipes: The art of scientific computing* (Cambridge, University Press, 1986)
[9] M Abromowitz and I A Stegun, *Handbook of Mathematical Functions* (Dover Publications, New York, 1970)