PRAMANA
— journal of
physics

© Printed in India

Vol. 45, No. 4,
October 1995
pp. 343–353

Vacuum structure, chiral symmetry breaking and electric dipole moment of neutron

A R PANDA* and K C ROY
Physics Department, Kendrapara College, Kendrapara 754 211, India
*Present address: Physics Department, Anandapur College, Anandapur 758 021, India

MS received 23 January 1995; revised 22 July 1995

Abstract. Considering a CP-violating QCD interaction, the electric dipole moment of neutron (EDMN) is estimated in a quark model of light mesons with a dynamical breaking of chiral symmetry through a non-trivial vacuum structure. Pion and kaon, being treated consistently within the model, yield to the constituent quark wave functions as well as the dynamical quark masses and thus determine the constituent quark field operators with respect to light quark flavors. Using the translationally invariant hadronic states and these constituent quark field operators, the EDMN estimated here remains well within the recent experimental bound of $D_n < 11 \times 10^{-26}$ e-cm with the CP-violation parameter $|\theta| = 10^{-8}$, which in fact accounts for a strong CP-violation.

Keywords. Strong CP-violation; vacuum structure; chiral symmetry breaking; quark model.

PACS Nos 14·20; 13·40; 12·38; 11·30

1. Introduction

Though electric dipole moment of neutron (EDMN) has been of great interest to physicists since its discovery [1] yet it has drawn much attention of theoreticians as well as of experimentalists after the discovery of CP-violation in $K^0 - \bar{K}^0$ system [2]. Amongst many theoretical models [3] to explain this CP-violation when applied to estimate EDMN, the valency quark model estimates EDMN by adding the valency quark contributions. In the standard model, though one has a non-vanishing contributions towards EDMN due to quarks still it is very much suppressed [4] at the level of three generations [5] and with four generations it does not enhance much. However, when one studies EDMN in QCD it has been observed that due to instanton effects the total divergence term has non-vanishing physical effects [6]. These in turn yield to a CP-violating piece [7] in the QCD Lagrangian due to an equivalent chiral rotation in the quark space which we consider here to describe the strong interaction between quarks in the neutron and hence to estimate its EDMN. However, this being a low energy phenomenon, its study requires a consideration of non-perturbative regime of QCD where one has solutions specifically in the lattice gauge theory [8] or in QCD sum rules [9]. But they have also their own limitations such as non-availability of better computer capabilities in case of former and consideration of non-perturbative vacuum structure through the relationship with perturbative QCD calculations in case of latter.

Recently, an alternative scheme [10] for considering low energy phenomena has been proposed where one realizes, using a variational method [11] the existence of a nontrivial vacuum structure through quark and gluon condensates in the non-perturbative regime [11]. Such a non-trivial vacuum structure has also yielded to a dynamical breaking of chiral symmetry [12] through which it has become possible to determine pion and kaon wave functions and subsequently the constituent quark field operators [13, 14]. The quark field operators obtained here when applied to explain electromagnetic and weak properties of pion, kaon and nucleon [13, 14] and static properties of baryons in general [15], have shown a reasonable amount of success. Further, such a formalism through the extremization of gap function [16] also generated dynamically the quark masses [17]. We may note that the method applied presently is non-perturbative with respect to the use of the equal time algebra for the constituent quark field operators and was also earlier applied to solvable cases in high energy physics [11, 12, 18, 19] and nuclear physics [20] with a reasonable amount of success. In view of its success in a wide ranging phenomena in the present investigation we will be further using these constituent quark field operators as well as the quark masses to estimate the electric dipole moment of neutron (EDMN).

The present article is organized as follows. In § 2, we briefly describe the vacuum structure and XSB used to estimate the constituent quark field operators and fix up our notations to apply them in later sections. In § 3, we estimate EDMN considering the CP-violating fundamental QCD-interaction Lagrangian [7, 21, 22] whereas in § 4, we discuss our results and compare them with the available experimental measurements as well as other model calculations.

2. Chiral symmetry breaking and vacuum structure

Considering the chiral symmetry breaking (XSB) arising from gluon condensates [12] and including such an effect one writes the constituent quark fields as [23]

$$\psi(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int [U(\mathbf{k}) q_I(\mathbf{k}) + V(-\mathbf{k}) \tilde{q}_I(-\mathbf{k})] \exp(i\mathbf{k}\cdot\mathbf{x}) \mathrm{d}\mathbf{k}, \tag{2.1}$$

where the spinors are written as [24, 25]

$$U(\mathbf{k}) = \begin{pmatrix} \cos\frac{\phi(\mathbf{k})}{2} \\ (\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\sin\frac{\phi(\mathbf{k})}{2} \end{pmatrix} \quad \text{and} \quad V(-\mathbf{k}) = \begin{pmatrix} -(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\sin\frac{\phi(\mathbf{k})}{2} \\ \cos\frac{\phi(\mathbf{k})}{2} \end{pmatrix}, \tag{2.2}$$

and the two component quark fields $q_I(\mathbf{k})$ and $\tilde{q}_I(\mathbf{k})$ are

$$q_I(\mathbf{k}) = \sum_r q_{Ir}(\mathbf{k}) u_{Ir} \quad \text{and} \quad \tilde{q}_I(\mathbf{k}) = \sum_r \tilde{q}_{Ir}(\mathbf{k}) v_{Ir} \tag{2.3}$$

with u_{Ir} and v_{Ir} being explicitly written as

$$u_{I\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u_{I-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad v_{I\frac{1}{2}} = \begin{pmatrix} 0 \\ -i \end{pmatrix} \quad \text{and} \quad v_{I-\frac{1}{2}} = \begin{pmatrix} i \\ 0 \end{pmatrix}. \tag{2.4}$$

The function $\phi(\mathbf{k})$ describing the spinors in (2.2) is arbitrary and can be derived in principle from the extremization of the energy functional. The quark field operator $\psi(\mathbf{x})$

in (2.1), through (2.2)–(2.4) is found to satisfy the equal time algebra and also becomes free chiral one when $\phi(\mathbf{k}) = \pi/2$.

Further, associating the quark field operators as in (2.1) with the presence of quark condensates through the quark–antiquark pair creation operator [13]

$$B_Q^\dagger = \int h(\mathbf{k}) q_I^\dagger(\mathbf{k})(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\tilde{q}_I(-\mathbf{k})\mathrm{d}\mathbf{k} \tag{2.5}$$

and a unitary transformation

$$U_Q = \mathrm{e}^{B_Q^\dagger - B_Q} \tag{2.6}$$

one obtains with $\tilde{\psi}(\mathbf{k}) = U_Q^\dagger \tilde{\psi}_0(\mathbf{k}) U_Q$ the relationship between the arbitrary function $\phi(\mathbf{k})$ and the correlation function $h(\mathbf{k})$ as [13, 18]

$$\frac{\phi(\mathbf{k})}{2} = \frac{\phi_0}{2} - h(\mathbf{k}), \tag{2.7}$$

where $\tilde{\psi}_0(\mathbf{k})$ is the chiral field operator. Thus, the form of quark field operator in (2.1) with (2.2) corresponding to XSB can have an interpretation of destabilizing the chiral vacuum or perturbative vacuum $|\mathrm{vac}\rangle$ (i.e. the vacuum state associated with the expansion of free field operators) through (2.6) when $\phi_0 = \pi/2$. Such a destabilization also leads to the possibility of a non-perturbative vacuum $|\mathrm{vac}'\rangle$ (i.e. the vacuum state obtained via minimization of energy density) [9] obtained through the unitary transformation in (2.6) as

$$|\mathrm{vac}'\rangle = U_Q|\mathrm{vac}\rangle. \tag{2.8}$$

Such a non-perturbative vacuum is also found to be at a lower energy than the perturbative or chiral vacuum [10–12, 13, 18]. Such a change of basis from $|\mathrm{vac}\rangle$ to $|\mathrm{vac}'\rangle$ is also corresponding to the change of parameters of the theory and can be treated as a vacuum realignment [10].

Further, with the quark field operators (2.1) one also obtains the expectation value for flavor 'i' as,

$$\langle \mathrm{vac}'|\bar{\psi}_i(\mathbf{x})\psi_i(\mathbf{y})|\mathrm{vac}'\rangle = \frac{6}{(2\pi)^3}\int \cos\phi_i(\mathbf{k})\exp[i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})]\mathrm{d}\mathbf{k}. \tag{2.9}$$

where the factor 6 is due to color and spin summation. One also has here from (2.7) a relationship between the function $\phi_i(\mathbf{k})$ and the quark–antiquark correlation function $h_i(\mathbf{k})$ as $\cos\phi_i(\mathbf{k}) = \sin(2h_i(\mathbf{k}))$ which in fact is an explicit description of the vacuum structure or vacuum realignment [10, 11, 13, 18].

As noted earlier, though in principle the function $\phi(\mathbf{k})$ can be obtained from the extremization of energy functional still, one does not have a closed form for it. However, one uses here a phenomenological Gaussian function, for its simplicity as

$$\sin(2h_i(\mathbf{k})) = \cos\phi_i(\mathbf{k}) = \exp(-R_i^2\mathbf{k}^2/2) \tag{2.10}$$

which has been earlier useful to consider hadronic phenomena associated with the vacuum structure [13, 18].

Next the function $\phi_i(\mathbf{k})$ can also be related to the pion wave function using Goldstone theorem [13], where one considers the chiral charge operator for pion (say π^+) as

$$Q_5^{\pi^+} = \int \psi_1^\dagger(\mathbf{x})\gamma^5\psi_2(\mathbf{x})\mathrm{d}\mathbf{x} \tag{2.11}$$

with the fields $\psi_1(\mathbf{x})$ and $\psi_2(\mathbf{x})$ written for u and d quarks respectively. With the help of such a chiral charge operator one also has chiral symmetry and its breaking through $Q_5^{\pi^+}|\mathrm{vac}\rangle = 0$ and $Q_5^{\pi^+}|\mathrm{vac}'\rangle \neq 0$ respectively. One may also note here that such a XSB corresponds to a pion state of zero momentum and zero energy as a Goldstone mode, and is also written explicitly as

$$|\pi^+(\mathbf{0})\rangle = \frac{N_\pi}{\sqrt{6}}\int \mathrm{d}\mathbf{k}\left(\cos\frac{\phi_1(\mathbf{k})}{2}\cos\frac{\phi_2(\mathbf{k})}{2} - \sin\frac{\phi_1(\mathbf{k})}{2}\sin\frac{\phi_2(\mathbf{k})}{2}\right)$$
$$\times\, \mathrm{u}_I^+(\mathbf{k})\tilde{\mathrm{d}}_I(-\mathbf{k})|\mathrm{vac}'\rangle \tag{2.12}$$

with N_π as the normalization constant. Further, assuming the initial $SU(2)_L \times SU(2)_R$ chiral symmetry breaking into custodial symmetry $SU(2)_V$ one has here $R_1 = R_2 = R$ and hence $\phi_1(\mathbf{k}) = \phi_2(\mathbf{k}) = \phi(\mathbf{k})$ which when substituted in (2.12) yields to it as

$$|\pi^+(\mathbf{0})\rangle = N_\pi\frac{1}{\sqrt{6}}\int \mathrm{d}\mathbf{k}\cos\phi(\mathbf{k})u_I^\dagger(\mathbf{k})\tilde{\mathrm{d}}_I(\mathbf{k})|\mathrm{vac}'\rangle. \tag{2.13}$$

The normalization constant N_π also, one obtains here, from the relation

$$N_\pi^2\int \cos^2\phi(\mathbf{k})\mathrm{d}\mathbf{k} = 1 \tag{2.14}$$

which with the $\cos\phi(\mathbf{k})$ as in (2.10) becomes

$$N_\pi = \left(\frac{R_\pi^2}{\pi}\right)^{3/4}, \tag{2.15}$$

where $R_\pi^2 = R^2$ for the pion has been used. The estimation of R_π^2 as $23{\cdot}58\,\mathrm{GeV}^{-2}$ [13] through the pionic properties such as pion decay constant has in fact led to a determination of the vacuum structure for u or d constituent quark through $R_1^2 = R_2^2 = R_\pi^2$ [13]. Such a vacuum structure has also estimated pion charge radius in reasonable agreement with its experimental measurement [13].

Next, in a similar manner, defining the chiral charge generator $Q_5^{K^+}$ for K-meson as [14]

$$Q_5^{K^+} = \int \psi_1^\dagger(\mathbf{x})\gamma^5\psi_3(\mathbf{x})\mathrm{d}\mathbf{x} \tag{2.16}$$

with $\psi_3(\mathbf{x})$ being taken for s-quark, one evaluates from it using (2.1) the K^+ state as [14]

$$|K^+(\mathbf{0})\rangle = \frac{N_K}{\sqrt{6}}\int u_1^\dagger(\mathbf{k})\tilde{s}_I(-\mathbf{k})$$
$$\times\left(\cos\frac{\phi_1(\mathbf{k})}{2}\cos\frac{\phi_3(\mathbf{k})}{2} - \sin\frac{\phi_1(\mathbf{k})}{2}\sin\frac{\phi_3(\mathbf{k})}{2}\right)\mathrm{d}\mathbf{k}|\mathrm{vac}'\rangle, \tag{2.17}$$

where N_K is the normalization constant for K-meson with the functions $\phi_1(\mathbf{k})$ and $\phi_3(\mathbf{k})$ taken for u and s quarks respectively. One may note here that with $s \to d$ (i.e. $\phi_3(\mathbf{k}) \to \phi_2(\mathbf{k})$) the kaon state in (2.17) reduces to the pion state of zero momentum in (2.12).. However, (2.17) has two unknown parameters N_K and R_3^2 (through $\cos\phi_3(\mathbf{k})$) related to each other. Thus, the estimation of them requires two relations which are taken to be the normalization condition for K-meson state as described in (2.17) and the kaon decay constant through Van Royen and Weisskopf relations [26]. The normalization condition for K-meson as

$$N_K^2 \int \left(\cos\frac{\phi_1(\mathbf{k})}{2} \cos\frac{\phi_3(\mathbf{k})}{2} - \sin\frac{\phi_1(\mathbf{k})}{2} \sin\frac{\phi_3(\mathbf{k})}{2} \right)^2 \mathrm{d}\mathbf{k} = 1 \tag{2.18}$$

and the Va eisskopf relation for K-meson in momentum space as [14].

$$\frac{{}_1(\mathbf{k})}{2} \cos\frac{\phi_3(\mathbf{k})}{2} - \sin\frac{\phi_1(\mathbf{k})}{2} - \sin\frac{\phi_3(\mathbf{k})}{2} \Bigg) \mathrm{d}\mathbf{k} \tag{2.19}$$

yields to

$$\frac{1}{(2\pi)^{3/2}} \frac{1}{N_K} = \frac{f_K (m_K)^{1/2}}{\sqrt{6}} \tag{2.20}$$

which in fact with the experimentally measured value of f_K gives N_K as [14]

$$N_K = 1{\cdot}9935\,\mathrm{GeV}^{3/2}. \tag{2.21}$$

Now a substitution of this value of N_K in (2.21) and the earlier obtained $\phi_1(\mathbf{k})$ through $R_1^2 = 23{\cdot}58\,\mathrm{GeV}^{-2}$ in (2.18) yields to R_3^2 *as*

$$R_3^2 = 4{\cdot}084\,\mathrm{GeV}^{-2}. \tag{2.22}$$

Thus such an estimation of R_3 or R_s determines $\phi_3(\mathbf{k})$ and hence the vacuum structure for s-quark. Such a vacuum structure when applied to estimate the kaon charge radius is also found [14] to estimate in reasonable agreement with its experimental measurement.

Further, through such a formalism, on extremization of the gap function, one can generate dynamically the quark masses as [16, 17]

$$m_i = \lim_{k \to 0} |\mathbf{k}| \tan(2h_i(\mathbf{k})) \tag{2.23}$$

which with (2.10) yields to

$$m_i = \frac{1}{R_i}, \tag{2.24}$$

which, in fact, with the quark radii as mentioned earlier gives $m_u = m_d = 0{\cdot}206\,\mathrm{GeV}$ and $m_s = 0{\cdot}495\,\mathrm{GeV}$.

We, in fact, will use the field operators and the masses of the constituent quarks obtained in this section to obtain the EDMN in later sections.

3. Electric dipole moment of neutron

In this section we consider the 'strong' CP-violation present in the fundamental QCD Lagrangian to estimate the electric dipole moment of neutron. In QCD, consideration of a total divergence term constructed from the gluon field strength tensor $G_{\mu\nu}$ as $(1/2)\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}G^{\mu\nu}$ has been found to have some non-vanishing physical effects due to instanton effects [6]. Further, it is parameterized in terms of a constant parameter as $-\theta g^2\tilde{G}_{\mu\nu}G^{\mu\nu}/32\pi^2$, with θ being zero or non-zero means no CP-violation or CP-violation. Also, through appropriate global rotations of the quark fields the effect of $\tilde{G}_{\mu\nu}G^{\mu\nu}$ is transferred to the quark sector and thus yielding to a CP-violating Lagrangian upto first order in θ, written as [7,22]

$$\delta\mathscr{L}_{\mathrm{CP}}(x) = i\theta\tilde{m}\bar{\psi}_q(x)\gamma^5\psi_q(x), \tag{3.1}$$

where

$$\tilde{m} = \frac{m_{\mathrm{u}}m_{\mathrm{d}}m_{\mathrm{s}}}{(m_{\mathrm{u}}m_{\mathrm{d}} + m_{\mathrm{d}}m_{\mathrm{s}} + m_{\mathrm{s}}m_{\mathrm{u}})}, \tag{3.2}$$

with m_{u}, m_{d} and m_{s} being the masses of u, d and s quarks respectively. Thus with the CP-violating interaction in (3.1) and the electromagnetic current $J_\mu(x)$ one has from current algebra an expression involving EDMN (D_n) as [21]

$$T\langle n_{1/2}(\mathbf{P}_f)|J_\mu(0)i\int \mathrm{d}^4x\,\delta\mathscr{L}_{\mathrm{CP}}(x)|n_{1/2}(\mathbf{P}_i)\rangle$$
$$= -D_n(\bar{u}_{1/2}(\mathbf{P}_f)\sigma_{\mu\nu}k^\nu\gamma^5u_{1/2}(\mathbf{P}_i) + O(\mathbf{k}^2), \tag{3.3}$$

where $\mathbf{k} = \mathbf{P}_f - \mathbf{P}_i$ and is the momentum carried by the electromagnetic current J_μ. Thus we estimate D_n through (3.3), and to do so, we consider the matrix element in (3.3) as

$$T\sum_x \langle n_{1/2}(\mathbf{P}_f)|J_\mu(0)|X\rangle\langle X|i\int \mathrm{d}^4x\,\delta\mathscr{L}_{\mathrm{CP}}(x)|n_{1/2}(\mathbf{P}_i)\rangle, \tag{3.4}$$

where $|X\rangle$ indicates intermediate states arising due to $\delta\mathscr{L}_{\mathrm{CP}}(x)$ and is an admixture of opposite parity states consistent with the quantum numbers of neutron which may be a pion-nucleon system in the ground state. Thus of the two possible intermediate states in this regard i.e. $|p\pi^-\rangle$ and $|n\pi^0\rangle$; only $|p\pi^-\rangle$ one contributes as π^0 at an elementary level does not couple to photon at low momenta. Further, $|X\rangle$ being an odd parity state, the space part of the electromagnetic current $J_\mu(0)$ only contributes to the matrix element which when used in (3.4) yields in the Breit frame the expression as [21]

$$T\sum_X \langle n_{1/2}(-\mathbf{p})|J_i(0)|X\rangle\langle X|i\int \mathrm{d}^4x\,\delta\mathscr{L}_{\mathrm{CP}}(x)|n_{1/2}(\mathbf{p})\rangle$$
$$= -D_n\bar{u}_{1/2}(-\mathbf{p})\sigma_{i\nu}k^\nu\gamma^5u_{1/2}(\mathbf{p}), \tag{3.5}$$

where in the r.h.s., terms of $O(\mathbf{k}^2)$ and higher have been neglected due to the small momentum transfer. Further, replacing the possible intermediate states $|X\rangle$ by $(\sqrt{2/3})|p_{1/2}(\mathbf{p}')\pi^-(p'')\rangle$ with $\sqrt{2/3}$ as the appropriate C.G. factor, and simplifying the r.h.s., (3.5) becomes,

$$2\cdot\frac{2}{3}\int M_{\rm em}(\mathbf{p};\mathbf{p}'',\mathbf{p}')M_{\rm CP}(\mathbf{p}',\mathbf{p}'',\mathbf{p})\mathrm{d}\mathbf{p}'\mathrm{d}\mathbf{p}''=D_n(2i/m_n)p_z p_i, \tag{3.6}$$

where in (3.6) the energy conservation $k^0=p_f^0=p_i^0$ has been used. The matrix elements due to electromagnetic and CP-violating interactions as $M_{\rm em}(\mathbf{p};\mathbf{p}'',\mathbf{p}')$ and $M_{\rm CP}(\mathbf{p}',\mathbf{p}'',\mathbf{p})$ respectively are explicitly written as

$$M_{\rm em}(\mathbf{p};\mathbf{p}'',\mathbf{p}')=\langle n_{1/2}(-\mathbf{p})|J_i(0)|p_{1/2}(\mathbf{p}')\pi^-(\mathbf{p}'')\rangle \tag{3.7}$$

and

$$M_{\rm CP}(\mathbf{p}',\mathbf{p}'';\mathbf{p})=\langle p_{1/2}(\mathbf{p}')\pi^-(\mathbf{p}'')|i\int \mathrm{d}^4x\,\delta\mathscr{L}_{\rm CP}(x)|n_{1/2}(\mathbf{p})\rangle. \tag{3.8}$$

The factor '2' in (3.6), has in fact arisen due to the interchange of the two vertices i.e. electromagnetic and CP-violating ones. However, the estimations of the matrix elements in (3.7) and (3.8) need the translationally invariant states for proton and neutron which we write in consistent with their SU(6) quantum numbers as

$$\begin{aligned}|p_{1/2}(\mathbf{p})\rangle=\frac{\varepsilon_{ijk}}{3\sqrt{2}}\int &\mathrm{d}\mathbf{k}_1\mathrm{d}\mathbf{k}_2\mathrm{d}\mathbf{k}_3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\tilde{u}_p(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\\ &\times[u_{I\frac{1}{2}}^{i\dagger}(\mathbf{k}_1+\mathbf{p}/3)u_{I\frac{1}{2}}^{j\dagger}(\mathbf{k}_2+\mathbf{p}/3)d_{I-\frac{1}{2}}^{k\dagger}(\mathbf{k}_3+\mathbf{p}/3)\\ &-u_{I-\frac{1}{2}}^{i\dagger}(\mathbf{k}_1+\mathbf{p}/3)u_{I\frac{1}{2}}^{j\dagger}(\mathbf{k}_2+\mathbf{p}/3)d_{I\frac{1}{2}}^{k\dagger}(\mathbf{k}_3+\mathbf{p}/3)]|\mathrm{vac}'\rangle\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}|n_{1/2}(\mathbf{p})\rangle=\frac{\varepsilon_{ijk}}{3\sqrt{2}}\int &\mathrm{d}\mathbf{k}_1\mathrm{d}\mathbf{k}_2\mathrm{d}\mathbf{k}_3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\tilde{u}_n(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\\ &\times[u_{I\frac{1}{2}}^{i\dagger}(\mathbf{k}_1+\mathbf{p}/3)d_{I\frac{1}{2}}^{j\dagger}(\mathbf{k}_2+\mathbf{p}/3)d_{I-\frac{1}{2}}^{k\dagger}(\mathbf{k}_3+\mathbf{p}/3)\\ &-u_{I-\frac{1}{2}}^{i\dagger}(\mathbf{k}_1+\mathbf{p}/3)d_{I\frac{1}{2}}^{j\dagger}(\mathbf{k}_2+\mathbf{p}/3)d_{I\frac{1}{2}}^{k\dagger}(\mathbf{k}_3+\mathbf{p}/3)]|\mathrm{vac}'\rangle,\end{aligned} \tag{3.10}$$

where the wave functions $\tilde{u}_{n,p}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ for neutron and proton are described by normalized harmonic oscillator wave functions as

$$\tilde{u}_{n,p}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(3R_{n,p}^4/\pi^2)^{3/4}\exp\left[(-R_{n,p}^2/6)\sum_{i<j}(\mathbf{k}_i-\mathbf{k}_j)^2\right] \tag{3.11}$$

with $R_{n,p}^2$ as their harmonic oscillator radii. However, in the earlier applications of the present formalism to explain the static properties of nucleons the two radii were taken to be the same [13], and here also we take them to be the same i.e. $R_n^2=R_p^2=R_N^2$. Thus, with the pion state as described in (2.3) and the electromagnetic current as $J_i(0)=\sum_{q,\alpha}e_q\bar{\psi}_q^\alpha(0)\gamma^i\psi_q^\alpha(0)$, we estimate the matrix element (3.7) to be

$$\begin{aligned}M_{\rm em}(\mathbf{p};\mathbf{p}'',\mathbf{p}')&=\langle n_{1/2}(-\mathbf{p})|\sum_{q,\alpha}e_q\bar{q}^\alpha(0)\gamma^i q^\alpha(0)|p_{1/2}(\mathbf{p}')\pi^-(\mathbf{p}'')\rangle\\ &=-\frac{e_u}{(2\pi)^3}\frac{1}{3\sqrt{6}}\left(\frac{3R_N^4}{\pi^2}\right)^{3/2}\left(\frac{R_\pi^2}{\pi}\right)^{3/4}\int\mathrm{d}\mathbf{k}_1\mathrm{d}\mathbf{k}_3\end{aligned}$$

$$\times \exp\left[-\frac{R_\pi^2}{2}\left(\mathbf{k}_2^2-\mathbf{k}_2\cdot\mathbf{p}''+\frac{\mathbf{p}''^2}{4}\right)\right]$$

$$\times \exp\left[-2R_N^2(\mathbf{k}_2^2+\mathbf{k}_3^2+\mathbf{k}_2\cdot\mathbf{k}_3+\frac{\mathbf{k}_2\cdot\mathbf{p}'}{2}+\frac{\mathbf{p}'^2}{6})\right]$$

$$\times [4V^\dagger_{-1/2}(\mathbf{k})\alpha_i U_{1/2}(\mathbf{k}')+V^\dagger_{1/2}(\mathbf{k})\alpha_i U_{-1/2}(\mathbf{k}')]$$

$$\times \frac{1}{|-\mathbf{k}_2+\mathbf{p}''||\mathbf{k}_2+\mathbf{p}'|}\sin\frac{\phi(\mathbf{k}_2+\mathbf{p}')}{2}\sin\frac{\phi(-\mathbf{k}_2+\mathbf{p}'')}{2}, \tag{3.12}$$

where $\mathbf{k}=-\mathbf{k}_2+\mathbf{p}''$ and $\mathbf{k}'=\mathbf{k}_2+\mathbf{p}'$ have been substituted, and with a further simplification of the spinor part (3.12) becomes

$$M_{\rm em}(\mathbf{p};\mathbf{p}'',\mathbf{p}')=\frac{5e_u}{(2\pi)^3 3\sqrt{6}}\frac{4ip_z p_i}{9}\left(\frac{3R_N^4}{\pi^2}\right)^{3/2}\left(\frac{R_\pi^2}{\pi}\right)^{3/4}$$

$$\times\int d\mathbf{k}_2 d\mathbf{k}_3 \sin\frac{\phi(\mathbf{k}_2+\mathbf{p}')}{2}\sin\frac{\phi(-\mathbf{k}_2+\mathbf{p}'')}{2}$$

$$\times\frac{1}{|-\mathbf{k}_2+\mathbf{p}''||\mathbf{k}_2+\mathbf{p}'|}\exp\left[-\frac{R_\pi^2}{2}(\mathbf{k}_2^2-\mathbf{k}_2\cdot\mathbf{p}''+\frac{\mathbf{p}''^2}{4}\right]$$

$$\times\exp\left[-2R_N^2\left(\mathbf{k}_2^2+\mathbf{k}_3^2+\mathbf{k}_2\cdot\mathbf{k}_3+\frac{\mathbf{k}_2\cdot\mathbf{p}'}{2}+\frac{\mathbf{p}'^2}{6}\right)\right], \tag{3.13}$$

The matrix element (3.8) involving CP-violating interaction $\delta\mathscr{L}_{\rm CP}(x)$ with the use of the time translational invariance becomes,

$$M_{\rm CP}(\mathbf{p}',\mathbf{p}'';p)=-\frac{1}{E_\pi(\mathbf{p}'')}\langle p_{1/2}(\mathbf{p}')\pi^-(\mathbf{p}'')|i\int d\mathbf{x}\,\delta\mathscr{L}_{\rm CP}(\mathbf{x},0)|n_{1/2}(\mathbf{p})\rangle, \tag{3.14}$$

with the pion energy $E_\pi(\mathbf{p}'')=(\mathbf{p}''^2+m_\pi^2)^{1/2}$. However, a substitution of the CP-violating Lagrangian (3.1) in (3.14) yields it as,

$$M_{\rm CP}(\mathbf{p}',\mathbf{p}'';p)=-\frac{i\theta\tilde{m}}{\mathrm{E}_\pi(\mathbf{p}'')}\langle p_{1/2}(\mathbf{p}')\pi^-(\mathbf{p}'')|\int d\mathbf{k}\bar{q}^\alpha(\mathbf{k})\gamma^5 q^\alpha(\mathbf{k})|n_{1/2}(\mathbf{p})\rangle. \tag{3.15}$$

which in fact with the proton, neutron and pion states described earlier yields to

$$M_{\rm CP}(\mathbf{p}',\mathbf{p}'';p)=\frac{i\theta\tilde{m}}{E_\pi(\mathbf{p}'')}\frac{1}{6\sqrt{6}}\left(\frac{3R_N^4}{\pi^2}\right)^{3/2}\left(\frac{R_\pi^2}{\pi}\right)^{3/4}\exp\left[-\frac{R_\pi^2 p'^2}{8}\right]$$

$$\times\exp\left[-\frac{R_N^2}{24}(5\mathbf{p}'^2+9\mathbf{p}''^2-6\mathbf{p}'\cdot\mathbf{p}'')\right]\int d\mathbf{k}'_3$$

$$\times\exp\left[-2R_N^2\left(\mathbf{k}'_3+\frac{\mathbf{p}'+3\mathbf{p}''}{12}\right)^2\right]$$

$$\times[4U^\dagger_{1/2}(\mathbf{k})\gamma^0\gamma^5 V_{-1/2}(\mathbf{k})+U^\dagger_{-1/2}(\mathbf{k})\gamma^0\gamma^5 V_{1/2}(\mathbf{k})], \tag{3.16}$$

where, $\mathbf{k}$ has been substituted for $(\mathbf{p}' + \mathbf{p}'')/2$. Thus, after some simplifications and integration in (3.16) we obtain it as,

$$M_{\mathrm{CP}}(\mathbf{p},\mathbf{p}'',\mathbf{p}) = -\frac{\theta\tilde{m}}{E_\pi(\mathbf{p}'')}\frac{1}{2\sqrt{6}}\left(\frac{3R_N^4}{\pi^2}\right)^{3/2}\left(\frac{R_\pi^2}{\pi}\right)^{3/4}\left(\frac{\pi}{2R_N^2}\right)^{3/2}$$
$$\times \exp\left[-\frac{5R_N^2+3R_\pi^2}{24}\mathbf{p}'^2 - \frac{3R_N^2}{8}\mathbf{p}''^2 + \frac{R_N^2}{4}\mathbf{p}'\cdot\mathbf{p}''\right]\cos\phi(\mathbf{k}). \tag{3.17}$$

The substitution of (3.17) along with (3.13) in the l.h.s. of (3.6) when compared with its r.h.s. gives rise to an expression for electric dipole moment of neutron after $\mathbf{k}_3$ integration as,

$$D_n = \frac{20}{729}\theta\tilde{m}m_n e(2\pi)^{-3}\left(\frac{3R_N^2}{2\pi}\right)^3\left(\frac{R_\pi^2}{\pi}\right)^{3/2} 2\pi I, \tag{3.18}$$

where the integral 'I' is written as,

$$I = \int |\mathbf{k}_2|^2 \mathrm{d}|\mathbf{k}_2||\mathbf{p}'|^2\mathrm{d}|\mathbf{p}'||\mathbf{p}''|^2\mathrm{d}|\mathbf{p}''|\mathrm{d}\phi'\,\mathrm{d}\phi''\,\mathrm{d}\cos\theta'\,\mathrm{d}\cos\theta''$$
$$\times\left(\frac{\left[1-\exp\left[-\frac{R_\pi^2}{2}(\mathbf{k}_2^2+\mathbf{p}''^2-2k_2p''\cos\theta'')\right]\right]^{1/2}\left[1-\exp\left[-\frac{R_\pi^2}{2}(\mathbf{k}_2^2+\mathbf{p}'^2+2k_2p'\cos\theta')\right]\right]^{1/2}}{|\mathbf{k}_2-\mathbf{p}''||\mathbf{k}_2+\mathbf{p}'|(\mathbf{p}''^2+m_\pi^2)^{1/2}}\right)$$
$$\times\exp\left[-R_N^2\left(\frac{3\mathbf{k}_2^2}{2}+K_2p'\cos\theta'+\frac{\mathbf{p}'^2}{3}\right)\right]\exp\left[-\frac{R_\pi^2}{2}\left(\mathbf{k}_2^2-K_2p''\cos\theta''+\frac{\mathbf{p}''^2}{4}\right)\right]$$
$$\times\exp\left[-\frac{R_\pi^2-R_N^2}{4}(\sin\theta'\sin\theta''\cos(\phi''-\phi')+\cos\theta'\cos\theta'')p'p''\right]$$
$$\times\exp\left[-\frac{5R_N^2+6R_\pi^2}{24}\mathbf{p}'^2+\frac{R_\pi^2+3R_N^2}{8}\mathbf{p}''^2\right], \tag{3.19}$$

where $k_2 = |\mathbf{k}_2|$, $p' = |\mathbf{p}'|$ and $p'' = |\mathbf{p}''|$. However, the integral 'I' in (3.19) is to be evaluated numerically. Thus, in the following section we will be using (3.18) to estimate D_n, where also we will discuss and compare it with other theoretical estimations as well as the experimental measurements.

4. Results and discussion

The estimation of EDMN from (3.18) of the earlier section primarily requires the radius parameters R_n^2, R_p^2 and R_π^2 for the neutron, proton and pion wave functions in addition to the quark masses and neutron mass. However, they were obtained from the earlier applications [13] of the formalism as $R_n^2 = R_p^2 = R_N^2 = 13{\cdot}52\,\mathrm{GeV}^{-2}$ and $R_\pi^2 = 23{\cdot}58\,\mathrm{GeV}^{-2}$, and thus to this effect the present estimation involves no free parameters.

With these parameters the integral in (3.19) is numerically obtained to be

$$I = 1{\cdot}300547 \times 10^{-3}\,\mathrm{GeV}^6, \tag{4.1}$$

which when substituted in (3.18) along with the dynamically generated quark masses as described in (2.24) and the neutron mass from its experimental measurement [27] determines the EDMN as

$$D_n = -7{\cdot}46 \times 10^{-8}|\theta|\,\text{e-cm}. \tag{4.2}$$

Thus, with a value of the parameter $|\theta| = 10^{-8}$ [7, 21, 22] the EDMN becomes

$$D_n = -7{\cdot}46 \times 10^{-26}\,\text{e-cm}, \tag{4.3}$$

which lies within the uncertainty of its recently measured value of $(3 \pm 5) \times 10^{-26}$ e-cm [28] and well within the most recent experimental upper bound of $|D_n| < 11 \times 10^{-26}$ e-cm [27].

We now compare our result with the other model estimations. In standard model, EDMN comes out to be within the range of 10^{-30} to 10^{-32} e-cm [4] which is smaller compared to experiments. Even with four generations in such a model, the estimation does not go beyond 10^{-29} e-cm [29]. On the other hand, the result of Weinberg–Higgs model of spontaneous CP-violation [30] with dominant hadronic loops is 10^{-25} e-cm, which is larger than experimental upper bound [27]. But, in the Left–Right Symmetry model EDMN is found to be in the range of 10^{-27} to 10^{-25} e-cm [31] in which also the present estimation lies. However, in bag model, with the QCD interaction at the valency quark core level with the add parity N^{*-} states EDMN was obtained to be $2{\cdot}7 \times 10^{-16}|\theta|$ e-cm [7] whereas the same in a current algebra analysis with a CP-violating πN-interaction was estimated to be $-3{\cdot}8 \times 10^{-16}|\theta|$e-cm [21]. But both these estimations when compared with the experimental measurement [27] are pointing to a rather weaker CP-violation. In the framework of CBM, Morgan and Miller [22], taking both valency quark core and pionic contributions, have estimated EDMN as $-4 \times 10^{-17}\theta$ e-cm, which also with the experimental measurement [27] yields to a weaker CP-violation, though it is relatively stronger than the earlier two estimations [7, 21]. Further, in some recent attempts [32–34] in the context of removal of $U(1)$ anomaly problem; under certain approximations θ has also been predicted in QCD to be vanishing. In view of these diverse model predictions we believe an accurate measurement in this regard can provide the actual dynamics concerned with the EDMN and thus be in a position to distinguish different theoretical models as well as the approximations involved in them. Nevertheless, we observe that the present investigation in its attempt to explain EDMN has an agreement with the Left–Right Symmetry model and has a more reasonable explanation for the same compared to the standard model. However, the present nonperturbative mechanism has yielded a limited applicability with respect to light flavors only and we believe when the same is extended to heavier flavors can be able to describe hadron dynamics in a unified manner.

Acknowledgements

The authors are thankful to Profs S P Misra and N Barik for valuable discussions. They also thank A Nath, P Das and R K Sahoo for discussions.

References

[1] N F Ramsay, *Ann. Rev. Nucl. Part. Sci.* **32**, 211 (1982) and references therein
[2] J H Christenson, J W Cronin, V L Fitch and R Turley, *Phys. Rev. Lett.* **13**, 138 (1964)
[3] X G He, B H J McKeller and S Pakvasa, *Int. J. Mod. Phys.* **A4**, 5011 (1989) and references therein
[4] E P Shabalin, *Sov. J. Nucl. Phys.* **28**, 75 (1978); **31**, 864 (1980); **32**, 228 (1980) and references therein
[5] J O Eeg and I Picek, *Phys. Lett.* **B130**, 508 (1983); *Nucl. Phys.* **B244**, 77 (1984)
[6] S M Bar and A Zee, *Phys. Rev. Lett.* **55**, 2253 (1985)
A A Belavin, A M Polyakov, A S Schwartz and Yu S Tyupkin, *Phys. Lett.* **B59**, 85 (1975)
R Jackiw and C Rebii, *Phys. Rev. Lett.* **37**, 172 (1976)
C G Callan Jr., R F Dashen and D J Gross, *Phys. Lett.* **B63**, 334 (1976)
[7] V Baluni, *Phys. Rev.* **D19**, 2227 (1979)
[8] K G Wilson, *Phys. Rev.* **D10**, 2445 (1974)
J B Kogut, *Rev. Mod. Phys.* **51**, 659 (1979); **55**, 775 (1983)
M Creutz, *Phys. Rev. Lett.* **45**, 313 (1980); *Phys. Rev.* **D21**, 2308 (1980)
[9] M A Shifman, A I Vainstein and V I Zakharov, *Nucl. Phys.* **B147**, 365, 448, 519 (1979)
R A Bertlman, *Acta. Phys. Aust.* **53**, 305 (1981)
[10] S P Misra, *Phys. Rev.* **D35**, 2607 (1987)
[11] H Mishra, S P Mishra and A Mishra, *Int. J. Mod. Phys.* **A3**, 2331 (1988)
A Mishra, H Mishra, S P Mishra and S N Nayak, *Phys. Lett.* **B251**, 541 (1990)
[12] A Mishra, H Mishra, S P Misra and S N Nayak, *Pramana – J. Phys.* **37**, 59 (1991); *Z. Phys.* **C57**, 241 (1993)
[13] A Mishra and S P Misra, *Z. Phys.* **C58**, 325 (1993)
A Mishra, H Mishra and S P Misra, *Z. Phys.* **C57**, 241 (1983)
[14] S P Misra, Preprint IP/BBSR/94-90; hep-ph/9405402
[15] A R Panda and K C Roy, Submitted for publication
[16] S L Adler and A C Davis, *Nucl. Phys.* **B244**, 469 (1984)
M G Mitchard, A C Davis and A J Macfarlane, *Nucl. Phys.* **B325**, 470 (1989)
[17] H Mishra and S P Misra, *Phys. Rev.* **D48**, 5376 (1993)
[18] A Mishra, H Mishra, S P Misra and S N Nayak, *Phys. Rev.* **D44**, 110 (1990)
[19] A Mishra, H Mishra and S P Misra, *Int. J. Mod. Phys.* **A5**, 3391 (1990)
H Mishra, S P Misra, P K Panda and B K Parida, *Int. J. Mod. Phys.* **E1**, 405 (1992)
[20] P K Panda, R Sahu and S P Misra, *Phys. Rev.* **C45**, 2079 (1992)
[21] R J Crewther, P Di-Vecchia, G Veneziano and E Witten, *Phys. Lett.* **B88**, 123 (1979)
[22] M G Morgan and G A Miller, *Phys. Lett.* **B179**, 379 (1986)
[23] S P Misra, *Phys. Rev.* **D18**, 1661, 1673 (1978)
[24] A Le Yaouanc *et al*, *Phys. Rev. Lett.* **54**, 506 (1986)
[25] S P Misra and S Panda, *Pramana – J. Phys.* **27**, 523 (1985)
[26] R Van Royen and V F Weisskopf, *Nuovo Cimento* **A51**, 617 (1965)
[27] Particle Data Group, *Phys. Rev.* **D50**, 1173 (1994)
[28] Smith *et al*, *Phys. Lett.* **B234**, 192 (1990)
[29] A Barroso, R Chalabi, N Dombey and C Hanzaoui, *Phys. Lett.* **B196**, 269 (1987)
[30] S Weinberg, *Phys. Rev. Lett.* **37**, 657 (1976)
H Y Cheng, *Phys. Rev.* **D42**, 2329 (1990)
[31] K Babu and R N Mohapatra, Preprint Ud Mp PP 89–063
[32] A Abada *et al*, *Phys. Lett.* **B234**, 191 (1990)
[33] S Aoki and T Hatsuda, *Phys. Rev.* **D45**, 2427 (1992)
[34] H Banerjee, P Mitra and D Chatterjee, *Z. Phys.* **C62**, 511 (1994)