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Fermilab top mass and modified Fritzsch mass matrices

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Abstract. Fritzsch like mass matrices with non-zero 22-elements both in U sector and D sector have been investigated in the context of latest data regarding $m_t^{\rm phys}$, $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$ and $|V_{ts}|$. Unlike several other phenomenological models, the present model not only accommodates the value of $m_t^{\rm phys}$ in the range 150–240 GeV, encompassing the CDF and D0 values, but is also able to reproduce $|V_{cb}| \cong 0{\cdot}040$ and $|V_{ub}/V_{cb}| = 0{\cdot}08 \pm 0{\cdot}02$ and $|V_{td}|$ is predicted to lie in the range 0·005–0·014. Further, the angles of the unitarity triangle, related to the CP-violating asymmetries, are calculated to be in the ranges $-1{\cdot}0 \leqslant \sin 2\alpha \leqslant -0{\cdot}1$, $0{\cdot}6 \leqslant \sin 2\alpha \leqslant 1{\cdot}0$ and $0{\cdot}48 \leqslant \sin 2\beta \leqslant 0{\cdot}56$, which are in agreement with other recent calculations.

Keywords. Quark mass matrices; quark mixing matrix; unitarity triangle; CP violation; fermion masses.

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1. Introduction

The recently observed evidence for the top quark in $p\bar{p}$ collisions at Fermilab [1, 2] has added another success to the standard model's (SM) already long list of achievements. This, however, is certainly going to accelerate the search for discovering physics beyond the standard model. To this end, a peep into the physics beyond the SM is provided by the fermion mass matrices and mixing angles which essentially enter as free parameters in the SM. In the absence of a dynamical theory for quark mass matrices, several phenomenological models [3–14] have been considered with fair degree of success. The most widely studied as well as the most economical model is that of Fritzsch which, unfortunately, has been ruled out by the data [1, 2, 15]. The observation of rather high $m_t^{\rm phys} = 176 \pm 13$ GeV [1] as well as $m_t^{\rm phys} = 199 \pm 30$ GeV [2] at Fermilab has further complicated the situation regarding phenomenological mass matrices. In fact, rather high value of $m_t^{\rm phys}$, although in agreement with limits on $m_t^{\rm phys}$ expected from radiative corrections within the SM, has posed serious challenge to some of the phenomenological models for mass matrices.

In the absence of a viable dynamical theory for quark mass matrices, it is perhaps desirable to develop phenomenological models which are simple as well as predictive. If such models, while remaining in tune with the data, are able to generate simple relations amongst Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, then these models may provide vital clues for formulating a dynamical theory. We feel that in the formulation of a viable phenomenological scheme, ideas such as the hierarchical structure of mass matrices, relative smallness of CP violation, etc., may play a vital role.

At present we do not attempt to go into the origin of mass matrices, rather try to see the implications of these for the CKM matrix ($V_{\rm CKM}$) phenomenology. In fact, we shall see that these structures not only fit the data but also give very simple relations amongst CKM matrix elements when the above mentioned simplifying assumptions are invoked.

Recently, keeping in mind the above mentioned ideas, we have considered Fritzsch mass matrices which have been modified by additional diagonal elements in both D type and U type mass matrices [16]. It becomes interesting to examine how these mass matrices accommodate recently found value of $m_t^{\rm phys}$ along with the recent refinements in the measurements of $V_{\rm CKM}$ elements. Further, it would be interesting to investigate the implications of such mass matrices for hitherto unknown CKM matrix elements, $|V_{td}|$ and $|V_{ts}|$ as well as the different angles of the unitarity triangle [17, 18].

The plan of the paper is as follows. In §2 we present the essential details of the phenomenological quark mass matrices considered here. In §3, we present definitions as well as relations of angles of the unitarity triangle with the asymmetries in B decays. In §4, we present the details of our calculations and results. Section 5 summarizes our principal findings.

2. Mass matrices

For the sake of completeness and readability we reproduce some of the essential details of Gill and Gupta [16]. To begin with, we consider hermitian mass matrices of the following form

$$M^i = \begin{bmatrix} 0 & A^i & 0 \\ A^{*i} & D^i & B^i \\ 0 & B^{*i} & C^i \end{bmatrix}, \quad [i = u, d], \tag{1}$$

where

$$A^i = |A^i|\exp(i\alpha_i),\ B^i = |B^i|\exp(i\beta_i)$$

and the elements of M^i are supposed to follow the hierarchical structure, e.g., $|A^i| \ll |B^i| \approx D^i < C^i$.

The above matrices M^i can be expressed as

$$M^i = P^i \bar{M}^i P^{i\dagger}, \tag{2}$$

where real matrices $\bar{M}^i$ are

$$\bar{M}^i = \begin{bmatrix} 0 & |A^i| & 0 \\ |A^i| & D^i & |B^i| \\ 0 & |B^i| & C^i \end{bmatrix}, \tag{3}$$

and

$$P^i = \mathrm{diag}\{1, \exp(-i\alpha_i), \exp(-i(\alpha+\beta)_i)\}. \tag{4}$$

The matrices $\bar{M}^i$ can be diagonalized exactly by orthogonal transformations, for example,

$$\bar{M}^i = O^i M^i_{\rm diag}(O^i)^\dagger \tag{5}$$

where

$$M^i_{\rm diag} = \mathrm{diag}(m_1, -m_2, m_3), \tag{6}$$

with subscripts 1, 2, 3 referring to u, c, t in U sector and d, s, b in D sector.

Using $\mathrm{tr}(M^i)$, $\mathrm{tr}(M^i)^2$, $\det(M^i)$, the values of matrix elements A^i, B^i, C^i are expressed in terms of quark masses as

$$C^i = (m_1 - m_2 + m_3 - D^i),\ A^i = (m_1 m_2 m_3 / C^i)^{1/2}$$
$$B^i = (-(A^i)^2 + C^i D^i + m_1 m_2 + m_2 m_3 - m_1 m_3)^{1/2}. \tag{7}$$

The diagonalising transformations O^i can be expressed as

$$O^i = \begin{bmatrix} (m_2 m_3 f_1/\Delta_1)^{1/2} & -(m_1 m_3 f_2/\Delta_2)^{1/2} & (m_1 m_2 f_3/\Delta_3)^{1/2} \\ (C^i m_1 f_1/\Delta_1)^{1/2} & (C^i m_2 f_2/\Delta_2)^{1/2} & (C^i m_3 f_3/\Delta_3)^{1/2} \\ -(m_1 f_2 f_3/\Delta_1)^{1/2} & -(m_2 f_1 f_3/\Delta_2)^{1/2} & (m_3 f_1 f_2/\Delta_3)^{1/2} \end{bmatrix} \tag{8}$$

where

$$f_1 = m_3 - m_2 - D^i; f_2 = m_3 + m_1 - D^i;\ f_3 = m_2 - m_1 + D^i;$$
$$\Delta_i = C^i(m_3 - m_1)(m_2 + m_1);\ \Delta_2 = C^i(m_3 + m_2)(m_2 + m_1)$$
$$\Delta_3 = C^i(m_3 + m_2)(m_3 - m_1). \tag{9}$$

To facilitate comparison with other similar approaches as well as for better physical understanding of the structure of V_{CKM}, we present here the approximate form of O^u. For example, by considering $m_u \ll m_c < D^u < m_t$ as well as $m_d \ll m_s < D^d < m_b$, the structure for O^u can be simplified and expressed as

$$O^u \cong \begin{bmatrix} 1 & -a_0 & a_0 d_0^2 R_u \\ a_0(1-R_t)^{1/2} & (1-R_t)^{1/2} & R_t'^{1/2} \\ -a_0 R_t'^{1/2} & -R_t'^{1/2} & (1-R_t)^{1/2} \end{bmatrix} \tag{10}$$

where

$$a_0^2 = (m_u/m_c),\ d_0^2 = (m_c/m_t),\ R_t = D^u/m_t,\ R_t' = \left(\frac{m_c + D^u}{m_t}\right)$$

and

$$R_u = \left(\frac{m_c + D^u}{m_t - D^u}\right)^{1/2}.$$

The matrix O^d can be obtained simply by changing $u \to d$, $c \to s$, $t \to b$ with $a_0 \to b_0$ and $d_0 \to c_0$ where $b_0^2 = (m_d/m_s)$ and $c_0^2 = (m_s/m_b)$.

The mixing matrix V_{CKM} in terms of $O^{u,d}$ can be expressed as

$$V_{\mathrm{CKM}} = O^{u\dagger} P^{ud} O^d, \tag{11}$$

where

$$P^{ud} = \mathrm{diag}\{1, \exp(i\phi_1), \exp(i\phi_2)\};\quad \phi_1 = \alpha_u - \alpha_d \text{ and } \phi_2 - \phi_1 = \beta_u - \beta_d.$$

Using (10) and retaining terms of leading order, (11) can be simplified and written as

$$V_{\mathrm{CKM}} \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$
$$\cong \begin{bmatrix} 1 + a_0 b_0 g_1 & -b_0 + a_0 g_1 & b_0 c_0^2 R_d + a_0 g_2 \\ -a_0 + b_0 g_1 & a_0 b_0 + g_1 & g_2 \\ a_0 d_0^2 R_u + b_0 g_4 & g_4 & g_3 \end{bmatrix} \tag{12}$$

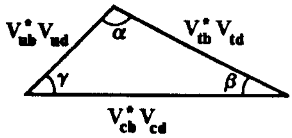

Figure 1. The CKM unitarity triangle in the complex plane.

where

$$g_1 = ((1 - R_t)(1 - R_b))^{1/2}\exp(i\phi_1) + (R_t' R_b')^{1/2}\exp(i\phi_2), \tag{13}$$

$$g_2 = (R_b'(1 - R_t))^{1/2}\exp(i\phi_1) - (R_t'(1 - R_b))^{1/2}\exp(i\phi_2), \tag{14}$$

$$g_3 = g_1(\phi_1 \Leftrightarrow \phi_2) \tag{15}$$

and

$$g_4 = -g_2(\phi_1 \Leftrightarrow \phi_2). \tag{16}$$

The above expressions for V_{CKM} are approximate, however, for the purpose of calculations, we have employed exact expressions.

3. Unitarity triangle

The unitarity of CKM matrix leads to six relations involving complex CKM matrix elements. These six relations represent six triangles in the complex plane with their angles constituting weak observable phases. Out of the six triangles, there is one triangle where all the three angles are naturally large and is expressed as

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0, \tag{17}$$

which is represented in figure 1. Interestingly, the sides of this triangle correspond to the decays $B_d \to \pi\pi$, $B_d \to D\pi$ and $B_d - \bar{B}_d$ mixing represented by $(V_{ud}V_{ub}^{*})$, $(V_{td}V_{tb}^{*})$ and $(V_{cd}V_{cb}^{*})$ respectively. The three angles of the triangle in terms of CKM matrix elements are defined as [18]

$$\alpha \equiv \arg(-V_{td}V_{tb}^{*}/V_{ud}V_{ub}^{*}), \tag{18}$$

$$\beta \equiv \arg(-V_{cd}V_{cb}^{*}/V_{td}V^{*}{}_{tb}), \tag{19}$$

$$\gamma \equiv \arg(-V_{ud}V_{ub}^{*}/V_{cd}V_{cb}^{*}). \tag{20}$$

Table 1. CP violating asymmetry parameter, ImΛ, for different decay modes and its corresponding relations to angles of unitarity triangle.

Quark subprocess	Decay mode	ImΛ
$\bar{b} \to \bar{u}u\bar{d}$	$B_d^0 \to \pi^+\pi^-$	$\sin 2\alpha$
$\bar{b} \to \bar{c}c\bar{s}$	$B_d^0 \to \psi K_s$	$-\sin 2\beta$
$\bar{b} \to \bar{u}u\bar{d}$	$B_s^0 \to \rho K_s$	$-\sin 2\gamma$

The CP violating asymmetry parameter Λ, for the respective decays, is related to the angles of the unitarity triangle [18], expressed in table 1. However, the relation of the asymmetry parameter for the respective decays in terms of V_{CKM} elements is given as

$$\operatorname{Im}\Lambda(B_d^0\to\pi^+\pi^-)=\operatorname{Im}\left(\frac{V_{tb}^*\,V_{td}\,V_{ub}\,V_{ud}^*}{V_{tb}\,V_{td}^*\,V_{ub}^*\,V_{ud}}\right)=\frac{2\bar\eta[\bar\eta^2+\bar\rho(\bar\rho-1)]}{[\bar\eta^2+(1-\bar\rho)^2][\bar\eta^2+\bar\rho^2]}; \tag{21}$$

$$\operatorname{Im}\Lambda(B_d^0\to\psi K_s)=\operatorname{Im}\left(\frac{V_{tb}^*\,V_{td}\,V_{cb}\,V_{cd}^*}{V_{tb}\,V_{td}^*\,V_{cb}^*\,V_{cd}}\right)=\frac{2\bar\eta(\bar\rho-1)}{[\bar\eta^2+(1-\bar\rho)^2]}; \tag{22}$$

and

$$\operatorname{Im}\Lambda(B_s^0\to\rho K_s)=\operatorname{Im}\left(\frac{V_{tb}^*\,V_{ts}\,V_{cd}\,V_{cs}^*\,V_{ub}\,V_{ud}^*}{V_{tb}\,V_{ts}^*\,V_{cd}^*\,V_{cs}\,V_{ub}^*\,V_{ud}}\right)=-\frac{2\bar\eta\bar\rho}{[\bar\eta^2+\bar\rho^2]}. \tag{23}$$

In (21)–(23) we have also shown relations in terms of modified Wolfenstein parameters ($\bar\rho=\rho(1-\lambda^2/2)$, $\bar\eta=\eta(1-\lambda^2/2)$) of CKM matrix as suggested by Buras *et al* [19].

4. Results and discussion

Before we present our results, a brief discussion about various inputs which have gone into the analysis is perhaps in order. As a first step, we have considered quark masses at 1 GeV [20], for example, $m_u=0{\cdot}0051\pm0{\cdot}0015$ GeV, $m_d=0{\cdot}0089\pm0{\cdot}0026$ GeV, $m_s=0{\cdot}175\pm0{\cdot}055$ GeV, $m_c=1{\cdot}35\pm0{\cdot}05$ GeV, $m_b=5{\cdot}3\pm0{\cdot}1$ GeV. Further, in order to simplify the analysis, we have fixed the two phases, viz., $\phi_1=\phi_2=90°$ in accordance with the values considered elsewhere [14, 15]. Noting the fact that the CKM matrix elements $|V_{us}|$ and $|V_{cb}|$ are well known and have weak dependence on m_3's and D^i's, we have restricted the parameter space by first reproducing $|V_{us}|\cong 0{\cdot}22$ and $|V_{cb}|\cong0{\cdot}040$ [18], ignoring the spread in the values of $|V_{cb}|$ as the calculated quantities hardly show any dependence on these. After having fixed the values of $|V_{us}|$ and $|V_{cb}|$, we have calculated $|V_{ub}|$, $|V_{td}|$, $|V_{ts}|$ and other phenomenological quantities related to V_{CKM}, for different values of R_t and m_t (1 GeV), however, for the purpose of discussion we have converted m_t (1 GeV) values into m_t^{phys} values. The values of R_b are constrained by the relation,

$$|V_{cb}|=(R_b'(1-R_t))^{1/2}-(R_t'(1-R_b))^{1/2}. \tag{24}$$

In order to have a better appreciation of the significance of our results, we first discuss our results when (i) $R_t\neq0$, $R_b=0$ (ii) $R_b\neq0$, $R_t=0$, corresponding to 22-element being nonzero in U and D sectors respectively. Keeping in mind that V_{CKM} can be characterized by any of the four elements [17], in table 2 we have presented our results pertaining to $|V_{ub}|$ and $|V_{td}|$. This is primarily to examine whether in the above mentioned cases the present form of mass matrices can accommodate m_t^{phys} in the range 150–240 GeV and the latest data regarding $R_{ub}(=|V_{ub}/V_{cb}|)$ which is in the range 0·06–0·10 [18]. From table 2, it is evident that 0·07 is the maximum value of R_{ub} for m_t^{phys} in the range 150–240 GeV in agreement with the conclusions of other similar calculations [14, 15]. Further, one can easily check that the above value of R_{ub} cannot be increased beyond 0·07 by any variation of R_t (or R_b), m_t^{phys}, ϕ_1, ϕ_2 and other quark

Table 2. Calculated values of $|V_{cb}|$, R_{ub} and $|V_{td}|$ when (i) $R_b = 0$, $R_t \neq 0$ and (ii) $R_t = 0$ and $R_b \neq 0$ for certain representative values of the sets (R_t, m_t^{phys}) and (R_b, m_t^{phys}), m_t^{phys} in GeV units. In ascending order, the corresponding m_t (1 GeV) values in GeV units are 250, 300, 350 and 400.

$R_b = 0$				$R_t = 0$			
(R_t, m_t^{phys})	$\|V_{cb}\|$	R_{ub}	$\|V_{td}\|$	(R_b, m_t^{phys})	$\|V_{cb}\|$	R_{ub}	$\|V_{td}\|$
(−0·030, 150)	0·036	0·07	0·008	(0·012, 150)	0·043	0·06	0·011
(−0·020, 150)	0·037	0·06	0·009	(0·014, 150)	0·036	0·06	0·009
(−0·017, 150)	0·045	0·06	0·011	(0·042, 150)	0·041	0·07	0·008
(−0·020, 180)	0·038	0·06	0·010	(0·013, 180)	0·039	0·06	0·010
(−0·031, 210)	0·042	0·07	0·009	(0·041, 180)	0·039	0·07	0·007
(−0·021, 210)	0·039	0·06	0·009	(0·013, 210)	0·045	0·06	0·011
(−0·020, 210)	0·043	0·06	0·010	(0·015, 210)	0·038	0·06	0·009
(−0·031, 240)	0·038	0·06	0·008	(0·044, 240)	0·045	0·07	0·008
(−0·023, 240)	0·035	0·07	0·008	(0·016, 240)	0·036	0·07	0·009
(−0·021, 240)	0·045	0·07	0·010	(0·044, 240)	0·043	0·07	0·008

masses. Therefore, when $R_b = 0$ (or $R_t = 0$), it is clear that it is not possible to achieve simultaneously $m_t^{\text{phys}} > 150\,\text{GeV}$ as well as R_{ub} in the entire range 0·06–0·10. From a cursory look at table 2, it is clear that the value of $|V_{td}|$, for the considered range of parameters, is predicted in a narrow range of 0·008–0·011, therefore, a precise measurement of $|V_{td}|$ will have an important consequence for the mass matrices wherein either $R_b = 0$ or $R_t = 0$.

In tables 3(a, b) we have presented the results of calculations where in general $R_b \neq 0$ for different R_t values. For the sake of uniformity, we have presented in the tables the ratios of V_{CKM} matrix elements, e.g., R_{ub}, $R_{td} = |V_{td}/V_{cb}|$, $R_{ts} = |V_{ts}/V_{cb}|$. A general survey of the tables brings out easily that we are able to obtain R_{ub} from 0·06–0·10 by varying R_t for various values of m_t^{phys}. This can also be checked from the expressions of $|V_{ub}|$ and $|V_{cb}|$. An important prediction of the model is that $|V_{ts}| \lesssim |V_{cb}|$ for the entire range of m_t^{phys} and R_t. This is in accordance with expectations from the unitarity of V_{CKM}. Similarly the results of $|V_{td}|$ can encompass the presently expected range [18]. Coming to the angles of unitarity triangle, α and β, we find that the present values are in accordance with similar calculations by other authors [19, 21, 22]. In the tables 3(a, b), we have not shown values of J-rephasing invariant measure of CP-violation [17]. However, one can easily show that J, related to the area of the unitarity triangle, lies in the range $(1{\cdot}7\text{–}2{\cdot}8) \times 10^{-5}$, which is in agreement with the data and other similar calculations [14, 15, 23].

A closer scrutiny of our results reveals several interesting points. One finds that tables 3(a) correspond to positive values of $\sin 2\alpha$ whereas 3(b) correspond to the negative values. This sign ambiguity is in fact reflection of the uncertainty regarding the quadrant of the phase of the CKM matrix. This is also reflected in the fact that $|V_{td}|$ has essentially two corresponding ranges, e.g., negative values of $\sin 2\alpha$ correspond to $|V_{td}| < 0{\cdot}009$ whereas positive values correspond to $|V_{td}| \geqslant 0{\cdot}009$. This ambiguity, however, in the present formulation, arises from (24) wherein corresponding to a given value of R_t there are two values of R_b which can reproduce $|V_{cb}|$. Therefore, a measurement of $\sin 2\alpha$, in the B decay as mentioned in table 1, would help in fixing the quadrant

Table 3a. Calculated values of $R_{ub}, R_{td}, R_{ts}, S_{2\alpha}(=\sin 2\alpha)$ and $S_{2\beta}(=\sin 2\beta)$ for different m_t^{phys} and R_t values when $|V_{cb}| = 0{\cdot}040$ with $S_{2\alpha}$ taking positive values.

	m_t^{phys}(GeV) = 150						m_t^{phys}(GeV) = 180						m_t^{phys}(GeV) = 210						m_t^{phys}(GeV) = 240					
R_t	R_b	R_{ub}	R_{td}	R_{ts}	$S_{2\alpha}$	$S_{2\beta}$	R_b	R_{ub}	R_{td}	R_{ts}	$S_{2\alpha}$	$S_{2\beta}$	R_b	R_{ub}	R_{td}	R_{ts}	$S_{2\alpha}$	$S_{2\beta}$	R_b	R_{ub}	R_{td}	R_{ts}	$S_{2\alpha}$	$S_{2\beta}$
0·00	−0·02	0·06	0·24	0·97	0·63	0·51	−0·02	0·06	0·24	0·97	0·60	0·51	−0·02	0·06	0·24	0·97	0·58	0·51	−0·02	0·06	0·24	0·97	0·56	0·51
0·01	−0·00	0·07	0·25	0·97	0·82	0·50	−0·01	0·07	0·25	0·97	0·81	0·50	−0·01	0·07	0·25	0·97	0·80	0·51	−0·01	0·07	0·25	0·97	0·79	0·51
0·02	0·01	0·07	0·26	0·97	0·90	0·50	0·01	0·07	0·26	0·97	0·90	0·50	0·01	0·07	0·26	0·97	0·89	0·50	0·01	0·07	0·26	0·97	0·89	0·50
0·03	0·02	0·08	0·27	0·97	0·95	0·50	0·02	0·07	0·26	0·97	0·95	0·50	0·02	0·07	0·26	0·97	0·95	0·50	0·02	0·07	0·26	0·97	0·94	0·50
0·04	0·03	0·08	0·27	0·97	0·98	0·50	0·03	0·08	0·27	0·97	0·98	0·50	0·03	0·08	0·27	0·97	0·98	0·50	0·03	0·08	0·27	0·97	0·97	0·50
0·05	0·05	0·08	0·28	0·97	0·99	0·49	0·04	0·08	0·28	0·97	0·99	0·50	0·04	0·08	0·28	0·97	0·99	0·50	0·04	0·08	0·28	0·97	0·99	0·50
0·06	0·06	0·08	0·28	0·97	1·00	0·49	0·06	0·08	0·28	0·97	1·00	0·50	0·06	0·08	0·28	0·97	1·00	0·50	0·05	0·08	0·28	0·97	1·00	0·50
0·07	0·07	0·09	0·28	0·96	1·00	0·49	0·07	0·08	0·28	0·96	1·00	0·49	0·07	0·08	0·28	0·96	1·00	0·50	0·07	0·08	0·28	0·96	1·00	0·50
0·08	0·08	0·09	0·29	0·96	0·99	0·49	0·08	0·09	0·29	0·96	1·00	0·49	0·08	0·09	0·29	0·96	1·00	0·50	0·08	0·09	0·29	0·96	1·00	0·50
0·09	0·09	0·09	0·29	0·96	0·99	0·49	0·09	0·09	0·29	0·96	0·99	0·49	0·09	0·09	0·29	0·96	0·99	0·49	0·09	0·09	0·29	0·96	0·99	0·50
0·10	0·10	0·09	0·30	0·96	0·98	0·49	0·10	0·09	0·30	0·96	0·98	0·49	0·10	0·09	0·30	0·96	0·98	0·49	0·10	0·09	0·30	0·96	0·98	0·49
0·11	0·11	0·09	0·30	0·96	0·97	0·49	0·11	0·09	0·30	0·96	0·97	0·49	0·11	0·09	0·30	0·96	0·97	0·49	0·11	0·09	0·30	0·96	0·97	0·49
0·12	0·13	0·10	0·30	0·96	0·95	0·49	0·12	0·10	0·30	0·96	0·96	0·49	0·12	0·10	0·30	0·96	0·96	0·49	0·12	0·10	0·30	0·96	0·96	0·49
0·13	0·14	0·10	0·31	0·96	0·94	0·49	0·14	0·10	0·31	0·96	0·95	0·49	0·14	0·10	0·31	0·96	0·95	0·49	0·13	0·10	0·31	0·96	0·95	0·49

Table 3b. Same as in table 3(a) with $S_{2\alpha}$ taking negative values.

m_t^{phys}(GeV) = 150

R_t	R_b	R_{ub}	R_{td}	R_{ts}	$S_{2\alpha}$	$S_{2\beta}$
0·00	−0·03	0·06	0·21	0·98	−0·16	0·52
0·01	−0·02	0·06	0·20	0·98	−0·43	0·52
0·02	−0·02	0·07	0·20	0·98	−0·59	0·53
0·03	−0·01	0·07	0·19	0·98	−0·70	0·53
0·04	0·00	0·07	0·19	0·99	−0·78	0·53
0·05	0·01	0·07	0·18	0·99	−0·84	0·54
0·06	0·02	0·07	0·17	0·99	−0·88	0·54
0·07	0·03	0·08	0·17	0·99	−0·91	0·54
0·08	0·03	0·08	0·17	0·99	−0·94	0·54
0·09	0·04	0·08	0·17	0·99	−0·96	0·54
0·10	0·05	0·08	0·16	0·99	−0·97	0·55
0·11	0·06	0·09	0·16	0·99	−0·98	0·55
0·12	0·07	0·09	0·16	0·99	−0·99	0·55
0·13	0·08	0·09	0·16	0·98	−0·99	0·55
0·14	0·09	0·09	0·15	0·99	−0·99	0·53
0·15	0·10	0·09	0·15	0·99	−1·00	0·56
0·16	0·11	0·10	0·15	0·99	−1·00	0·56
0·17	0·12	0·10	0·14	0·99	−1·00	0·56

m_t^{phys}(GeV) = 180

R_t	R_b	R_{ub}	R_{td}	R_{ts}	$S_{2\alpha}$	$S_{2\beta}$
0·00	−0·03	0·06	0·22	0·98	−0·12	0·52
0·01	−0·02	0·06	0·20	0·98	−0·42	0·52
0·02	−0·02	0·07	0·20	0·98	−0·58	0·52
0·03	0·00	0·07	0·19	0·99	−0·78	0·53
0·04	0·01	0·07	0·18	0·99	−0·83	0·53
0·05	0·02	0·07	0·18	0·99	−0·88	0·53
0·06	0·02	0·08	0·18	0·99	−0·91	0·54
0·07	0·03	0·08	0·17	0·99	−0·94	0·54
0·08	0·04	0·08	0·17	0·99	−0·96	0·54
0·09	0·05	0·08	0·17	0·99	−0·97	0·54
0·10	0·06	0·08	0·16	0·99	−0·99	0·54
0·11	0·07	0·09	0·16	0·99	−0·99	0·55
0·12	0·08	0·09	0·16	0·99	−1·00	0·55
0·13	0·09	0·09	0·15	0·99	−1·00	0·55
0·14	0·10	0·09	0·15	0·99	−1·00	0·55
0·15	0·11	0·10	0·15	0·99	−1·00	0·55
0·16	0·12	0·10	0·14	0·99	−1·00	0·56
0·17	0·13	0·10	0·14	1·0	−0·99	0·56

m_t^{phys}(GeV) = 210

R_t	R_b	R_{ub}	R_{td}	R_{ts}	$S_{2\alpha}$	$S_{2\beta}$
0·00	−0·03	0·06	0·22	0·98	−0·10	0·52
0·01	−0·03	0·06	0·21	0·98	−0·41	0·52
0·02	−0·02	0·07	0·20	0·98	−0·57	0·52
0·03	−0·01	0·07	0·19	0·98	−0·69	0·53
0·04	0·00	0·07	0·19	0·99	−0·77	0·53
0·05	0·01	0·07	0·18	0·99	−0·83	0·53
0·06	0·02	0·07	0·18	0·99	−0·88	0·53
0·07	0·02	0·08	0·18	0·99	−0·92	0·53
0·08	0·03	0·08	0·17	0·99	−0·94	0·53
0·09	0·04	0·08	0·17	0·99	−0·96	0·54
0·10	0·05	0·08	0·17	0·99	−0·98	0·54
0·11	0·06	0·08	0·16	0·99	−0·99	0·54
0·12	0·07	0·09	0·16	0·99	−0·99	0·54
0·13	0·08	0·09	0·16	0·99	−1·00	0·54
0·14	0·09	0·09	0·15	0·99	−1·00	0·54
0·15	0·10	0·09	0·15	0·99	−1·00	0·55
0·16	0·11	0·10	0·15	0·99	−1·00	0·55
0·17	0·12	0·10	0·14	0·99	−1·00	0·55

m_t^{phys}(GeV) = 240

R_t	R_b	R_{ub}	R_{td}	R_{ts}	$S_{2\alpha}$	$S_{2\beta}$
0·00	−0·03	0·06	0·22	0·98	−0·07	0·52
0·01	−0·03	0·06	0·21	0·98	−0·39	0·52
0·02	−0·02	0·06	0·20	0·98	−0·57	0·52
0·03	−0·01	0·07	0·19	0·98	−0·69	0·52
0·04	0·00	0·07	0·19	0·99	−0·77	0·53
0·05	0·01	0·07	0·18	0·99	−0·83	0·53
0·06	0·01	0·07	0·18	0·99	−0·88	0·53
0·07	0·02	0·08	0·18	0·99	−0·92	0·53
0·08	0·03	0·08	0·17	0·99	−0·94	0·53
0·09	0·04	0·08	0·17	0·99	−0·96	0·53
0·10	0·05	0·08	0·17	0·99	−0·98	0·54
0·11	0·06	0·08	0·16	0·99	−0·99	0·54
0·12	0·07	0·09	0·16	0·99	−0·99	0·54
0·13	0·08	0·09	0·16	0·99	−1·00	0·54
0·14	0·09	0·09	0·15	0·99	−1·00	0·54
0·15	0·10	0·09	0·15	0·99	−1·00	0·54
0·16	0·11	0·09	0·15	0·99	−1·00	0·54
0·17	0·12	0·10	0·14	0·99	−1·00	0·55

of the phase of the CKM matrix. Further, in view of the very narrow range of sin 2β, in the present as well as other similar calculations [19, 21, 22], a measurement of the decay $B_d^0 \rightarrow \psi K_s$ would provide a good test of present form of mass matrices as well as the CKM mechanism in general.

It needs to be brought out clearly that the present analysis is crucially dependent on D^u and D^d. In the V_{CKM} matrix elements, however, these manifest through R_t and R_b. Such mass matrices, in the language of Ramond *et al* [24], correspond to texture 4 zeros. Also, it seems that the analysis of Ramond *et al* finds it difficult to accommodate $|V_{cb}| \cong 0{\cdot}04$ as well as $R_{ub} = 0{\cdot}08 \pm 0{\cdot}02$ with texture 5 and 6 zeros mass matrices. Therefore, texture 4 zeros mass matrices seem to be essential for fitting the V_{CKM} phenomenology. It may be of interest to mention that such mass matrices have been shown, by Joyce and Turok [25], to maintain their basic structure as they evolve from GUT scale to the low energy scale.

5. Conclusions

Fritzsch like mass matrices, with non-zero 22-elements in both U and D sectors, have been considered to accommodate current experimental constraints on m_t^{phys} and CKM matrix elements. We have shown that such mass matrices can accommodate CDF and D0 m_t^{phys} values, e.g., 176 ± 13 GeV and 199 ± 30 GeV respectively, apart from reproducing $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}/V_{cb}| = 0{\cdot}08 \pm 0{\cdot}02$. Further, $|V_{td}|$ is predicted to lie in the range 0·005–0·014, in agreement with the latest conclusions of Buras [26]. The calculated values of the angles of unitarity triangle, α and β, are also in agreement with other authors [19, 21, 22], however two different ranges for these highlight the ambiguity regarding the quadrant of CP-violating phase of CKM matrix.

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References

[1] F Abe *et al* (CDF Collab.), *Phys. Rev.* **D50**, 2966 (1994); *Phys. Rev. Lett.* **73**, 225 (1994); *Phys. Rev. Lett.* **74**, 2626 (1995)
[2] S Abachi *et al* (DO Collab.), *Phys. Rev. Lett.* **74**, 2632 (1995)
[3] H Fritzsch, *Phys. Lett.* **B73**, 317 (1978); **B166**, 423 (1986); *Nucl. Phys.* **B155**, 189 (1979)
[4] B Stech, *Phys. Lett.* **B130**, 189 (1983)
[5] M Shin, *Phys. Lett.* **B145**, 285 (1984)
[6] M Gronau, R Johnson and J Schechter, *Phys. Rev. Lett.* **54**, 2176 (1985)
[7] L Lavoura, *Phys. Lett.* **B228**, 245 (1989)
E Ma, *Phys. Rev.* **D43**, R2761 (1991)
[8] C H Albright Preprint No. FERMILAB-CONF-90/196-T(1990); FERMILAB-PUB-89/258-T(1989)
[9] X-G He and W S Hou, *Phys. Rev.* **D41**, 1517 (1990)
[10] M Tanimoto, *Phys. Rev.* **D41**, 1586 (1990)
[11] M T Yamawaki and W W Wada, *Phys. Rev.* **D43**, 2432 (1991)
[12] E Ma, *Phys. Rev.* **D43**, R2761 (1991); T V Duong and E Ma, *J. Phys.* **G19**, 339 (1993)
[13] Suraj N Gupta and James M Johnson, *Phys. Rev.* **D44**, 2110 (1991)
[14] R E Shrock, *Phys. Rev.* **D45**, R10 (1992)

[15] F Abe *et al*, *Phys. Rev. Lett.* **64**, 142 (1990)
[16] P S Gill and M Gupta, PU-PHY-REPORT No. 1/93 (Unpublished)
[17] C Jarlskog (ed), in *CP Violation* (World Scientific, Singapore, 1989) p. 3
[18] Particle Data Group, *Phys. Rev.* **D50**, 1315 (1994)
[19] A J Buras, M E Lautenbacher and G Ostermaier, *Phys. Rev.* **D50**, 3433 (1994)
[20] J Gasser and H Leutwyler, *Phys. Rep.* **87**, 77 (1982)
S Narison, *Phys. Lett.* **B197**, 405 (1987)
[21] G Buchalla and A J Buras, MPI-PhT/94-19 (1994)
[22] A Ali, CERN-TH 7123/93
A Ali and D London, DESY-022/93 (1993)
I Bigi, CERN-TH 7207/94 (1993)
Y Nir and Uri Sarid, *Phys. Rev.* **D50**, 2818 (1993)
[23] C Jarlskog, *Phys. Rev. Lett.* **55**, 1039 (1985) and *Z. Phys.* **C29**, 491 (1985)
J F Donoghue, T Nakada, E A Paschos and D Wyler, *Phys. Lett.* **B195**, 285 (1987)
G Bélanger, E Boridy, C Hamazaoui and G G Jakimov, *Phys. Rev.* **D48**, 4275 (1993)
[24] P Ramond, R G Robert and G G Ross, *Nucl. Phys.* **B406**, 19 (1993)
[25] M Joyce and M Turok, Princeton University Preprint-TH-1374 (1993)
[26] A J Buras, *Acta Phys. Pol.* **B26**, 755 (1995)