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Streaming instability of a dusty plasma in the presence of mass and charge variation

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Abstract. We study the effect of the mass and charge dynamics on the collective behaviour of a dusty plasma. It is shown that the finite non-zero streaming velocity of the dust grains leads to a novel coupling of the dust mass fluctuation with other dynamic variables of the plasma and the grains. The mass fluctuations causes a collisionless dissipation and provides an alternate channel for the beam mode instability to occur. Physically the negative energy wave associated with the beam mode couples to the mass fluctuation induced dissipative medium to produce the instability. We conclude that the higher value of the ion mass density to the dust mass density ratio reduces the threshold value for the onset of the instability. Its application in the astrophysical context is discussed.

Keywords. Dusty plasma; mass fluctuation; charge fluctuation.

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1. Introduction

Dusty plasma is ubiquitous in the universe. Interstellar media, cometary tails, planetary rings, ionosphere, plasma processing, etc. are a few examples of the space and the laboratory plasmas where the dust plays an important role in determining the dynamical behaviour of the plasmas [1–4]. Dusty plasma is a three component plasma system consisting of the electrons, ions and the charged dust grains. The typical size of the dust grains is a few microns and these can carry ten to million times the electronic charge. In general, the size, mass, and charge on the grains may vary; for example, the mass may vary between 10^{-2} g to a few a.m.u. per dust charge [5, 6], the size may vary between a few micrometer to the molecular size i.e. there exists a smooth transition from dusty to a multi component plasma.

The dust grains in a plasma may be charged through the collisions, photoemission etc. We assume that the collisional process is primarily responsible for the grain charging (in this case the grains are negatively charged). As a result of the collisions, the ions stick to the grain surface and thus causes the variation of the grain charge, mass and radius. Generally, the effect of the mass variation of the grain is ignored since $m_i/m_d \ll 1$. In the light of the recent data of ASPERA for the matter ejected from Phobos [6], we carefully observe that even in the above mass limit, ion mass density may be comparable to the dust mass density leading thereby to an interesting consequence of a new wave instability.

Collective behaviour (waves, instabilities, etc.) of a dusty plasma in the absence of charge fluctuation has been studied by several authors [1–4]. Effect of the charge variation on the collective modes has been recently reported [7–11]. In all these studies, mass on the grains has been assumed to be constant. However, in general, it is not true [12]. Actually the size (radius) of the grains should also be treated as a dynamical variable. But, for the simplicity, it is ignored as the time scale over which the radius varies is larger than that of the mass variation due to accretion [12]. Under these assumptions we find that when $v_{d0}/c_i > \eta_m^{1/2}(1+\delta)$, where $\eta_m = (m_{d\theta} n_{d0})/(m_1/n_{i0})$ is the dust mass density to ion mass density ratio and $\delta = n_{e0}/n_{i0}$ is the equilibrium electron to ion number density ratio, there exists a novel mode of instability. The paper is organized in the following fashion. In § 2, basic governing equations are given. Dispersion relation is derived in § 3. Section 4 includes the results and discussions.

2. Basic equations

We consider a three component plasma consisting of the electrons, ions and the electrically charged dust grains. It is assumed that the mass variation is solely due to the sticking of the ions on the grain surface and the other processes like photoionization, sputtering, etc. are ignored. The dust mass fluctuation equation is derived from the mass conservation relation. The basic linearized equations for the number conservation of the electrons, ions and the dust fluids are:

$$\partial_t n_{e1} + \nabla(n_{e1}\mathbf{v}_{e0} + n_{e0}\mathbf{v}_{e1}) = -\beta_{e0} n_{e1} - \beta_{e1} n_{e0}, \tag{1}$$

$$\partial_t n_{i1} + \nabla(n_{i1}\mathbf{v}_{i0} + n_{i0}\mathbf{v}_{i1}) = -\beta_{i0} n_{i1} - \beta_{i1} n_{i0}, \tag{2}$$

$$\partial_t n_{d1} + \nabla(n_{d1}\mathbf{v}_{d0} + n_{d0}\mathbf{v}_{d1}) = 0, \tag{3}$$

where the subscripts 0 and 1 represent the equilibrium and the perturbed quantities respectively; $n_a, \mathbf{v}_a$ (a = electron, ion and dust) are the number density and velocity of ath species. β_e and β_i denote the attachment frequencies of the electrons and the ions respectively.

Using (2) and (3) along with the mass conservation equation

$$\partial_t \rho + \nabla\cdot\mathbf{J} = 0, \tag{4}$$

where $\rho = \Sigma m_a n_a$ and $\mathbf{J} = \Sigma m_a n_a \mathbf{v}_a$, one obtains the required equation for the mass fluctuation as

$$\partial_t m_{d1} + (\mathbf{v}_{d0}\cdot\nabla)m_{d1} = (m_{d0}/n_{d0})(\beta_{i0} n_{i1} + \beta_{i1} n_{i0}). \tag{5}$$

In writing the above equation, the term proportional to the electron mass has been neglected. Using similar arguments, the dust charge fluctuation equation can be derived as

$$(\partial_t + \mathbf{v}_{d0}\cdot\nabla)Q_{d1} = -\frac{e}{n_{d0}}[\beta_{e0} n_{e1} + \beta_{e1} n_{e0}) - (\beta_{i0} n_{i1} + \beta_{i1} n_{i0})]. \tag{6}$$

Here the attachment frequencies for the electrons and the ions have been defined as $\beta_a = I_a n_d / q_a n_a$ with 'a' representing for the electron and ion. The expressions for the electron and ion currents are given in the refs [12, 13]. Using the above definition, and the equilibrium constraints $I_{e0} + I_{i0} = 0$, one finds

$$\beta_{a0} = (|I_{a0}| n_{d0})/(q_a n_{a0}) \tag{7}$$

and thus the charge and mass fluctuation equations can be written as:

$$d_t Q_{d1} + \eta q_{d1} = |I_{e0}| \left[\frac{n_{i1}}{n_{i0}} - \frac{n_{e1}}{n_{e0}} \right], \tag{8}$$

$$d_t m_{d1} + \eta \frac{m_i}{e} Q_{d1} = |I_{e0}| \frac{m_i}{e} \frac{n_{i1}}{n_{i0}}, \tag{9}$$

where $d_t = \partial_t + (\mathbf{v}_{d0} \cdot \nabla)$, $\eta = e|I_{e0}|C^{-1}(T_e^{-1} + w_0^{-1})$ and $\hat{\eta} = e|I_{e0}|C^{-1}w_0^{-1}$ with C as the capacitance of the grain and $w_0 = T_i - e\phi_{f0}$, T_i being the ion temperature and ϕ_{f0} the equilibrium floating potential of the grain surface. Equilibrium quasi neutrality $q_i n_{i0} + Q_{d0} n_{d0} - q_e n_{e0} = 0$ has been assumed.

Equation of motion for the electron, ion and dust grain fluids are

$$m_e n_e \mathrm{d}_t \mathbf{v}_e = -\nabla P_e + q_e n_e \mathbf{E} - m_e n_e \nu_e (\mathbf{v}_e - \mathbf{v}_d). \tag{10}$$

$$m_i n_i d_t \mathbf{v}_i = -\nabla P_i + q_i n_i \mathbf{E} - m_i n_i \nu_i (\mathbf{v}_i - \mathbf{v}_d).$$

$$d_t (m_d \mathbf{v}_d) = Q_d \mathbf{E}. \tag{11}$$

In the equation of the motion of the dust, collisional terms are not included and the dust grains are assumed to be cold. The Poisson's equation closes the set of basic equations and is given as

$$\nabla^2 \phi = -4\pi e(n_i - n_e) - 4\pi Q_d n_d. \tag{12}$$

Now (8–12) form the basic set of equations for the study of the various wave phenomena in dusty plasma with dynamical behaviour of the dust mass and charge fluctuations.

3. Dispersion relation

We are interested in the low frequency response of the plasma i.e. $\omega \ll kv_{ti}, kv_{te}$. The equilibrium flow of the dust grains is non-zero i.e. $v_{d0} \neq 0$. Electrons and ions are assumed to follow the Boltzmann distribution;

$$n_e = n_{e0} \exp\left(-\frac{q_e \phi}{T_e}\right),$$

$$n_i = n_{i0} \exp\left(-\frac{q_i \phi}{T_i}\right), \tag{13}$$

where n_{e0}, n_{i0} are the equilibrium electron and ion number densities respectively. Considering the fluctuating quantities to vary as $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, (8–12) can be Fourier analyzed to yield the desired dispersion relation as

$$1 + \frac{1}{k^2 \lambda_e^2}\left(1 + \frac{i\beta}{\hat{\omega} + i\eta}\right) + \frac{1}{k^2 \lambda_i^2}\left(1 + \frac{i\beta\delta}{\hat{\omega} + i\eta}\right) + \frac{i\beta \omega_{ed}^2 v_{d0}}{k\hat{\omega}^2 c_i^2 \left[\frac{i\hat{\eta}(1+\tau)}{\hat{\omega} + i\eta} + 1\right]} - \frac{\omega_{pd}^2}{\hat{\omega}^2} = 0, \tag{14}$$

where $\omega_{\rm ed}^2 = \dfrac{4\pi Z e^2 n_{\rm e0}}{m_{\rm d0}}$, $c_{\rm i}^2 = \dfrac{T_{\rm i}}{m_{\rm i}}$, $\lambda_{\rm e,i}^2 = \dfrac{T_{\rm e,i}}{4\pi n_{\rm e0,i0} e^2}$, $\delta = \dfrac{n_{\rm e0}}{n_{\rm i0}}$, $\tau = \dfrac{T_{\rm i}}{T_{\rm e}}$, $\omega_{\rm pd}^2 = \dfrac{4\pi n_{\rm d0} Q_{\rm d0}^2}{m_{\rm d0}}$, $\hat{\omega} = \omega - k v_{\rm d0}$. Except for the 4th term in (14), rest of the terms are identical to the dispersion relation of Bhatt and Pandey [11]. The 4th term is caused by the mass fluctuation dynamics of the grains. We note that this term vanishes when the dust drift is zero. It is quite natural because in the absence of the finite flow velocity of the dust grains, the mass fluctuation equation becomes redundant and it does not get coupled with the other dynamic variables of the plasma and dust fluids. Defining $C_{\rm ds}^2 = \lambda_{\rm eff}^2 \omega_{\rm pd}^2$ where $\lambda_{\rm eff}^{-2} = \lambda_{\rm e}^{-2} + \lambda_{\rm i}^{-2}$; assuming $T_{\rm e} \approx T_{\rm i} \approx T$, $n_{\rm e0} \approx n_{\rm i0} \approx n_0$ (a situation relevant to the planetary rings [14]), eq. (14) in the limit of $k^2 \lambda_{\rm eff}^2 \ll 1$ can be written as

$$(\hat{\omega}^2 - k^2 C_{\rm ds}^2)(\hat{\omega} + i\eta) + i\beta\hat{\omega}^2(1+\delta) = \alpha\beta - i\beta\varepsilon(\hat{\omega} + i\eta), \tag{15}$$

where

$$\alpha = \varepsilon\hat{\eta}(1+\tau),\ \varepsilon = \frac{\omega_{\rm ed}^2 \lambda_{\rm eff}^2 k v_{\rm d0}}{c_{\rm i}^2}.$$

4. Results and discussion

We are interested in a physical situation where mass variation, charging and collision frequency satisfy the condition $\hat{\eta} \sim \eta, \beta < \omega$. Since in this limit, charge and mass dynamics can have appreciable effect and the damping effect on the wave is not very severe, the root of (15) can be calculated perturbatively. Accordingly the solution can be given as

$$\omega = k(v_{\rm d0} - C_{\rm ds}) + (\beta/2)\frac{\alpha + i\varepsilon k C_{\rm ds} - i k^2 C_{\rm ds}^2(1+\delta)}{k^2 C_{\rm ds}^2}. \tag{16}$$

If one assumes that $\varepsilon > k C_{\rm ds}(1+\delta)$ i.e. $\left(\dfrac{v_{\rm d0}}{c_{\rm i}}\right) > \eta_{\rm m}^{1/2}(1+\delta)$, an instability sets in. Thus we see that the threshold condition for the onset of the beam driven instability is modified over the earlier condition $v_{\rm d0}/c_{\rm i} > 1$ as derived by Rosenberg [14]. Obviously the multiplicative factor $\eta_{\rm m}^{1/2}$ in our threshold condition originates from the dust mass fluctuation dynamics.

In many astrophysical plasmas $\eta_{\rm m}^{1/2} = \left(\dfrac{n_{\rm d0} m_{\rm d0}}{n_{\rm i0} m_{\rm i0}}\right)^{1/2} \gg 1$ owing to the large ratio of the dust to ion mass. Thus the mass fluctuation dynamics of the dust grains has a major role to play in deciding the onset condition for the beam driven mode instability in dusty plasmas. As we will see in the following discussions, the Mass Fluctuation Driven Streaming Instability (MFDSI) may have favourable conditions in many of the astrophysical systems.

(a) *Mars*: The formation of the dust belt around the mars is according to the present belief caused by the material ejected from its moon Phobos. Assuming that the grains initially move along the circular Keplerian orbit with $v_{\rm d0} \approx v_{\rm k}^{\rm phobos} \sim 2{\cdot}64$ km/sec; for typical plasma parameters of the Mars, $n_{\rm i0} \sim 0{\cdot}1\,{\rm cm}^{-3}$, $T \sim 0{\cdot}1$ eV, $m_{\rm i} = 10^{-24}$ g [15], one finds that $v_{\rm d0}/c_{\rm i} \sim 0{\cdot}2$. Taking $n_{\rm d0} \sim 10^{-9} - 10^{-10}\,{\rm cm}^{-3}$, $m_{\rm d0} \sim 10^3 - 10^7$ a.m.u. [6], one finds the value of $\eta_{\rm m}^{1/2} \sim 10^{-5} - 10^{-6}$. Thus the onset condition for the MIDIS is satisfied and its existence in Martian ring is possible.

(b) *Jovian ring*: Quasi periodic (~ 28 days) high speed (20–56 km/sec) streams of submicron ($0{\cdot}02 \leqslant a \leqslant 0{\cdot}1\,\mu$m) sized grains of the mass ($1{\cdot}6 \times 10^{-16} \leqslant m_{d0} \leqslant 1{\cdot}1 \times 10^{-14}$ g) were detected by 'Vlysses' mission to Jupiter [4]. Taking typical ring plasma parameters: $n_{i0} \sim 10^2\,\text{cm}^{-3}$, $m_i \sim 10^{-22}$ g for a tenuous dust cloud $n_{d0} \sim 10^{-3}\,\text{cm}^{-3}$ [16], one can calculate $v_{d0}/c_i \sim 3$ and $\eta_m^{1/2} \sim 1$. Obviously the instability condition holds good in the Jovian ring and hence the excitation of MFDSI may not be ruled out in the Jovian atmosphere.

(c) *Saturn ring*: For Saturn ring $v_{d0}/c_i \simeq \dfrac{R(\Omega_s - \Omega_k)}{c_i} \sim O(1)$ [8] for different rings. Here R is the distance from the Saturn to ring and Ω_s, Ω_k represent rotational and Keplerian frequencies respectively. For typical plasma parameters $n_{i0} \sim 10\,\text{cm}^{-3}$, $T \sim 50$ eV, $m_i \sim 10^{-23}$ g for the example for O^+ ions) and $n_{d0}m_{d0} \sim 1\,\text{g/cm}^3$ [8], one gets $\eta_m^{1/2} \sim 10^{11}$. Very clearly the instability condition is not satisfied at all and hence the MFDSI is unlikely in Saturn ring.

(d) *Comets*: Havens [18] reports that the streaming instability may play an important role in interaction between cometary dust and solar wind plasma. Nappi [19] has also pointed out the interesting consequences for a streaming instability if dust is a dominant charge carrier. Considering the dust-plasma parameters inside the ionopause of the Halley's comet as described by Mendis and Rosenberg [4] i.e. $n_{i0} \sim 10^3 - 10^4\,\text{cm}^{-3}$, $T_e \sim T_i = T \sim 0{\cdot}1$ eV, n_{d0}, $10^{-3}\,\text{cm}^{-3}$, $a \sim 0{\cdot}1$–$10\,\mu$m, v_{d0} (of the order of the solar wind speed) ~ 400 km/sec, $v_{d0}/c_i \sim 10^2$. Now considering the dust mass $m_{d0} \sim 10^{-16}$ g, the value of $\eta_m^{1/2} \sim 10^{-3}$. Thus it is concluded that the cometary environment may be susceptible to the MFDSI.

(e) *Interstellar media*: Most interesting region in the interstellar media is the star forming zones where one observes the molecular clouds [17]. The effect of the MHD shocks in such clouds have been studied in the recent past [3]. Assuming that the shock front moves with typical velocity $V_s \approx v_{d0}$; for the typical parameters [3] $T \sim 10$–20 K, $m_i \sim 10^{-22}$ g, one gets the shock speed $V_s \sim 10$ km/sec leading thereby $v_{d0}/c_i \sim 10^4$. For $Zn_{d0} \ll n_{i0}$, one finds that $\eta_m^{1/2} \sim 1$. Thus the interstellar media especially the star forming zones may become important locations for thc existence of the MFDSI.

It is worth adding a few more comments in favour of the aforesaid instability. The inclusion of the mass and charge fluctuation dynamics requires that the time scale of the wave of interest ($\tau_\omega = 2\pi/\omega$) be of the order of the time scales of the mass fluctuation ($\tau = (a^2 w_i n_{i0} A \xi_i)^{-1}$) and charge fluctuation ($\tau_c = (\pi a^2 w_e n_{e0} e)^{-1}$) where πa^2 is the geometrical cross-section of the grains; w_i is the ion thermal velocity; A, ξ_i are the atomic mass number and sticking co-efficient of the ions respectively; w_e is the electron thermal velocity. These requirements restrict the scale size of the instability and is found that in the case of the above listed astrophysical systems the scale sizes are of the order of the size of the respective systems. Thus it is conjectured that the MFDSI could be a possible physical mechanism for the large scale structure formation. Furthermore, it could also be proposed as a triggering mechanism for the onset of the Jeans collapse.

In the light of the above discussions, it becomes clear that the MFDSI has a major role to play in most of the astrophysical situations. The dust mass to ion mass density ratio decides the threshold condition for the onset of the instability.

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