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Neutrino emissivity of quark matter at finite temperatures

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Abstract. We evaluate the emissivity rates for d-decay and s-decay by exactly solving the angular integrals involved and without assuming the degeneracy of electrons. We have also studied the effects of QCD coupling constant as well as the s-quark mass on the emissivity rates. We find that these parameters are important in determining the threshold and extinction densities for d- and s-decays.

Keywords. Quark stars; bag model; emissivity rate; mean free path.

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1. Introduction

The possibility of dense stars having a core of essentially free quarks rather than nucleons has been studied for a long time now [1, 2]. If such stars do exist they have to be isolated observationally from other types of neutron stars. One distinguishing feature, as is well-known, is the surface emission of radiation which leads to stringent conditions on the temperature of the quark matter stars. Again, as is well-known, the neutrino emission plays an important role in the cooling of quark stars in the early stages of star evolution. The formalism for the neutrino emissivity rate by quark matter on the basis of β-equilibrium and charge neutrality condition has been given by Iwamoto [3]. The basic β-decay processes are

$$\mathrm{s} \to u + e^- + \bar{\nu}_e$$

$$\mathrm{d} \to u + e^- + \bar{\nu}_e \tag{1.1}$$

along with their inverse reactions

$$u + e^- \to \mathrm{s} + \nu_e$$

$$u + e^- \to \mathrm{d} + \nu_e. \tag{1.2}$$

Iwamoto has given a simple formula for neutrino emissivity rate from β-decay of quarks (see eq. (3.12) of ref. [3]) viz.

$$E_\nu(i) = 6V^3 \left(\prod_{j=1}^{4} \int \frac{\mathrm{d}^3 p_j}{(2\pi)^3} \right) E_2 \; W_{\mathrm{fi}} n(E_1)(1 - n(E_3))(1 - E_4)) \tag{1.3}$$

where $j = 1$ to 4 refers to s(or d), ν_e, u, e^- respectively and $i = \mathrm{s}$ or d as the case may be, $n(E_j)$ is the distribution function of jth particle species and W_{fi} is the weak decay transition rate. Using the Weinberg–Salaam model, as extended to semi-leptonic processes to describe weak interactions, the transition rate for β-decay of s- or d-quark is given by

$$W_{\mathrm{fi}}=64G_i^2(2\pi)^4\delta^4(p_1-p_2-p_3-p_4)(p_1\cdot p_2)(p_3\cdot p_4)\Big/\prod_{j=1}^{4}(2E_j V) \tag{1.4}$$

where $G_i^2=G^2\sin^2\theta_c$ for $i=$ s-quark and $G_i^2=G^2\cos^2\theta_c$ for $i=$ d-quark. Neglecting the neutrino momentum p_2 in the δ-function and by replacing the particle energies and momenta by their respective chemical potentials and fermi-momenta, Iwamoto has given the following formulas for neutrino emissivity rate (see eq. (3.21) and (4.11) of ref. [3])

$$E_{\nu d}=\frac{914}{315}G^2\cos^2\theta_c\alpha_c p_f(\mathrm{d})p_f(u)p_f(e)\,T^6 \tag{1.5}$$

$$E_{\nu s}=\frac{457\pi}{840}G^2\sin^2\theta_c\mu_s p_f(u)p_f(e)(1-\cos\theta_{34})\,T^6. \tag{1.6}$$

Equation (1.5) is the emissivity rate for the massless d-quark decay while (1.6) is for the s-quark decay. In deriving these results, Iwamoto also assumes that electrons are completely degenerate and so their energies and momenta, like those of the quarks, are replaced by their chemical potentials and fermi-momenta. Burrows [4] improved upon this calculation by including the effects of neutrino momentum and obtained a T^7 behaviour for the emissivity rate instead of T^6. Duncan *et al* [5] (1983, paper I; 1984, paper II) studied in detail the dependence of emissivity on the QCD coupling α_c. They also show that the electron fraction in quark matter vanishes at a finite baryon density, $n_B=n_{ex}$ (extinction density) and so the emissivity becomes zero at $n_B\geqslant n_{ex}$. Also, recently Ghosh *et al* [6] have reported a calculation of the emissivity rate without making the assumptions of Iwamoto. They could, however, reduce (1.3) to a five-dimensional integral which they had to do numerically.

We, in this paper, have reanalyzed the emissivity rate and mean free path. Our starting point is, of course, again the same as (1.3). We have demonstrated that the angular integrals can be done exactly without requiring any assumptions to be made and leading to an expression for emissivity rate in terms of a three-dimensional integral. Further simplification requires one to assume electrons to be degenerate. However such an assumption is not valid at high temperatures, therefore we perform our calculations both with and without this assumption. For the thermodynamics of quark matter we retain terms to first order in α_c and include the effects of finite temperature and finite mass of the s-quark.

In § 2 we indicate the main steps involved in the derivation of emissivity rate and mean free path. The technical details are relegated to the Appendix. In § 3, we describe our results for emissivity rate and mean free path as functions of α_c, T, m_s and n_B. Finally, we compare these results with those of Iwamoto and Duncan *et al* and discuss their significance.

2. Formalism

We consider three-component quark matter (u, d, s) in β-equilibrium with a small fraction of electrons. The thermal equilibrium condition leads to the following relation among the chemical potentials.

$$\mu_d = \mu_s = \mu_u + \mu_e \tag{2.1}$$

If one assumes that neutrinos escape freely from quark matter and hence do not take part in thermal equilibrium, the neutrino chemical potential, μ_ν, can be put equal to zero. Charge neutrality condition sets an additional constraint on the number densities $n_i (i = \mathrm{u, d, s, e})$ viz.

$$\frac{Q}{e} = \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0 \tag{2.2}$$

The total baryon density of quark matter is given by

$$n_B = \frac{1}{3}(n_u + n_d + n_s) \tag{2.3}$$

The above equations imply that only one parameter, say μ_u, is left free. Now consider the thermodynamics of the system. The pressure and energy are given by [2, 7]

$$P = -\sum_i \Omega_i - B \tag{2.4a}$$

$$E = -P + \sum_i \mu_i n_i - T \sum_i \frac{\partial \Omega_i}{\partial T} \tag{2.4b}$$

where the thermodynamic potentials are given by

$$\Omega_i = -(\mu_i^4/4\pi^2)[(1 - 2\alpha_c/\pi) + 2\pi^2 (T/\mu_i)^2 (1 - 2\alpha_c/3\pi)] \tag{2.4c}$$

$$\begin{aligned}\Omega_s = {} & -(\mu_s^4/4\pi^2)[(1 - 2{\cdot}5\lambda^2)(1-\lambda^2)^{0{\cdot}5} + 1{\cdot}5\lambda^4 \ln(1 + (1-\lambda^2)^{0{\cdot}5}/\lambda) \\ & + 2\pi^2 (1-\lambda^2)^{0{\cdot}5} (T/\mu_s)^2 - (2\alpha_c/\pi)(3((1-\lambda^2)^{0{\cdot}5} - \lambda^2 \ln(1 + (1-\lambda^2)/\lambda))^2 \\ & - 2(1-\lambda^2)^2 + 3\lambda^4 (\ln(\lambda))^2 + \pi^2 (T/\mu_s)^2 ((1 - 2/3\lambda^2) - \lambda^2 \\ & \quad \times \ln(1 + (1-\lambda^2)^{0{\cdot}5}/\lambda)/(1-\lambda^2)^{0{\cdot}5}) - 6(\lambda^2(1-\lambda^2)^{0{\cdot}5} \\ & - \lambda^4 (\ln(1 + (1-\lambda^2)^{0{\cdot}5})/\lambda) + (\pi^2/3)(T/\mu_s)^2 \lambda^2/(1-\lambda^2)^{0{\cdot}5}) \ln(\mu_0/\mu_s))].\end{aligned} \tag{2.4d}$$

Number densities of the particle species are obtained as

$$n_u = (\mu_u^3/\pi^2)(1 - 2\alpha_c/\pi) + \mu_u T^2 (1 - 2\alpha_c/3\pi) \tag{2.4e}$$

$$n_d = n_u (u \leftrightarrow d) \tag{2.4f}$$

$$\begin{aligned}n_s = {} & [4(\mu^2 - m_s^2)^{1{\cdot}5} + 2\pi^2 T^2 (2\mu^2 - m_s^2)/(\mu^2 - m_s^2)^{0{\cdot}5} - (8\alpha_c/\pi)\mu_s(\mu^2 - m_s^2) \\ & - (2\alpha_c/\pi)(-12 m_s^2 (\mu^2 - m_s^2)^{0{\cdot}5} \ln[(\mu_s + (\mu^2 - m_s^2)^{0{\cdot}5})/m_s] \\ & + 6(m_s^4/\mu_s)\ln(m_s/\mu_s)) + (4\pi^2/3) T^2 \mu_s - 6(m_s^2(\mu^2 - m_s^2)^{0{\cdot}5} \\ & + (\pi^2/3) T^2 (m_s^2/(\mu^2 - m_s^2)^{0{\cdot}5}) - (m_s^2/\mu_s)\ln[(\mu_s + (\mu^2 - m_s^2)^{0{\cdot}5})/m_s]) \\ & + 6 m_s^2 (2(\mu^2 - m_s^2)^{0{\cdot}5} - (\pi^2/3) m_s^2 T^2/(\mu^2 - m_s^2)^{1{\cdot}5})\end{aligned}$$

$$+\pi^2 T^2/(\mu^2 - m_s^2)((2\mu^2 - 5m_s^2)\mu/3$$
$$+ (m_s^4/(\mu^2 - m_s^2)^{0.5}) \ln[(\mu_s + (\mu^2 - m_s^2)^{0.5}/m_s)]])(1/4\pi^2) \tag{2.4g}$$
$$n_e = (\mu_e^3/3\pi^2)(1 + \pi^2 T^2/\mu_e^2) \tag{2.4h}$$

where $i = \mathrm{u,d,s}$ in (2.4a), (2.4b) and $i = \mathrm{u,d}$ in (2.4c), $\alpha_c = g^2/4\pi$ and $\lambda = m_s/\mu_s$. Note that Iwamoto and Duncan *et al* define QCD coupling constant through $\alpha_c = g^2/16\pi$. In the above expressions the terms up to first order in α_c and to second order in temperature T have been retained.

The relation between fermi-momenta and the particle densities is given by

$$p_{fi} = (\pi^2 n_i)^{1/3} \tag{2.5}$$

for $i = \mathrm{u,d,s},e^-$. For eqs (2.4e–2.4h) and (2.5) we then obtain relations between the chemical potentials and fermi-momenta of ith particle species. In the $T = 0$ limit these equations reduce to

$$\mu_i \simeq a p_{fi} \tag{2.6}$$

where $a = 1 + (2\alpha_c/3\pi)$ and $i = \mathrm{u,d}$. For the s-quarks, we obtain

$$\mu_s \cong m_s((1 + x^2)^{0.5} + (2\alpha_c/3\pi)[x - 3\ln\{(x + (1 + x^2)^{0.5})/(1 + x^2)^{0.5}\}]) \tag{2.7}$$

where $x = p_{fs}/m_s$. Equations (2.6) and (2.7) are identical to (8, 9) and (17a) of Duncan *et al* [5].

The stability of quark matter at finite temperature is determined by the value of energy per baryon, E/n_B. We consider the quark matter as stable so long as E/n_B stays below 930 MeV at zero pressure. For choosing the various parameters viz. α_c, B, μ_0 and m_s, the criterion is to determine the stability region discussed above. Once these values are chosen we are then left with one parameter, say $\mu_d = \mu_s = \mu$. This can be determined by fixing the baryon density n_B.

Substituting expression (1.4) in (1.3), the emissivity rate for the ith quark is given by

$$E_\nu(i) = \frac{24}{(2\pi)^8} G_i^2 \left(\prod_j^4 \int p_j \mathrm{d}E_j\right) E_2 n(E_1)(1 - n(E_3))(1 - n(E_4))$$
$$\delta(E_1 - E_2 - E_3 - E_4)\left(\prod_j^4 \int \mathrm{d}\Omega_j\right)(p_1 \cdot p_2)(p_3 \cdot p_4)\delta^3(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4). \tag{2.8}$$

The distribution function $n(E_j)$ of the jth particle species is given by (for $j = \mathrm{u,d,s},e^-$)

$$n(E_j) = (\exp[(E_j - \mu_j)/T] + 1)^{-1}. \tag{2.9}$$

Defining the angular integrals in (2.8) by

$$I(E_1, E_2, E_3, E_4) = (p_1 p_2 p_3 p_4/128\pi^2)\left(\prod_j^4 \int \mathrm{d}\Omega_j\right)(p_1 \cdot p_2)(p_3 \cdot p_4)$$
$$\times \delta^3(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4).$$

The angular integrations involved in the above expression can be performed without recourse to any approximations. The rather complicated expression for $I(E_1, E_2, E_3, E_4)$ and some of the steps involved in the evaluation of angular integrals are given in the

Appendix. We can rewrite (2.8) as

$$E_\nu(i) = \frac{12G_i^2}{\pi^6}\int_{m_1}^{\infty} dE_1 \int_{m_2}^{\infty} dE_2 \int_{m_3}^{\infty} dE_3 \int_{m_4}^{\infty} dE_4 E_2 n(E_1)(1-n(E_3))$$
$$\times (1-n(E_4))\delta(E_1-E_2-E_3-E_4) I(E_1,E_2,E_3,E_4) \tag{2.10}$$

At this stage this eq. (2.10) can be compared with (3.13), (3.15) and (3.18) of ref. [3]. Instead of the factor $p_f(d)p_f(u)p_f(e)E_2^2$ of Iwamoto, we obtain the function I, which is a function of all the four variables E_1, E_2, E_3 and E_4. The most crucial difference is that the function I is non-zero only for a certain range of values of these variables. We now introduce the following dimensionless parameters in expressions (2.9) and (2.10)

$$x_1 = (E_1 - \mu_1)/T,$$
$$x = E_2/T,$$
$$x_3 = (\mu_3 - E_3)/T,$$
$$x_4 = (\mu_4 - E_4)/T.$$

On substituting for the distribution function $n(E_j)$ from (2.9) into (2.10), the emissivity rate can be rewritten as

$$E_\nu(i) = \frac{12}{\pi^6} G_i^2\, T^4 \int_0^{\infty} x dx \int_{l_1}^{\infty} dx_1 \int_{l_2}^{-\infty} dx_3 \int_{l_3}^{-\infty} dx_4 \delta(x_1+x_3+x_4-x)$$
$$\times I(E_1,E_2,E_3,E_4)/(1+\exp(x_1))(1+\exp(x_3))(1+\exp(x_4)) \tag{2.11}$$

where

$$l_1 = (m_1 - \mu_1)/T,$$
$$l_2 = (\mu_3 - m_3)/T,$$
$$l_3 = (\mu_4 - m_4)/T.$$

Carrying out a similar analysis for the mean free path, we arrive at the following formula

$$\lambda^{-1} = \frac{24G_i^2}{\pi^4 x^2}\int_{l_1}^{\infty} dx_1 \int_{l_2}^{-\infty} dx_3 \int_{l_3}^{-\infty} dx_4 \delta(x_1+x_3+x_4+x)$$
$$\times I'(E_1,E_2,E_3,E_4)/(1+\exp(x_1))(1+\exp(x_3))(1+\exp(x_4)) \tag{2.12}$$

where l_1, l_2, l_3 are the same as defined before for (2.11). The expression for I', which differs from that for I only in the sign of certain terms, is given in the Appendix.

Let us now look at the temperature dependence of emissivity rate. Since the chemical potentials of the quarks are a few hundred MeV for temperatures up to a few MeV, we can treat u, d and s quarks as completely degenerate. The limits of x_1 and x_3 integrals now go from $(-\infty, \infty)$ and two of the integrations can be performed exactly. The results are

$$E_\nu(i) = (3/2\pi^5)G_i^2\, T^4 \int_0^{\infty} x dx \int_{-\infty}^{(\mu_4 - m_4)/T} dx_4 I(\mu_1, xT, \mu_3, \mu_4 - x_4 T)$$
$$\times (x - x_4)/(1+\exp(x_4))(\exp(x-x_4)-1) \tag{2.13}$$

Similarly for the mean free path, λ, we obtain

$$\lambda_i^{-1} = (3/\pi^3) G_i^2 (T/x)^2 \int_{-\infty}^{(\mu_4 - m_4)/T} \mathrm{d}x_4 I'(\mu_1, xT, \mu_3, \mu_4 - x_4 T)$$
$$\times (x + x_4)/(1 + \exp(x_4))(1 - \exp(-x - x_4)) \tag{2.14}$$

So far we have not assumed electrons to be degenerate as indeed they are not, particularly at higher densities. However, if we assume the degeneracy of electrons, the x_4 integral can also be performed leading to the following simple results

$$E_\nu(i) = (3/4\pi^5) G_i^2 T^4 \int_0^\infty \mathrm{d}x \frac{x(x^2 + \pi^2)}{1 + \exp(x)} I(\mu_1, xT, \mu_3, \mu_4) \tag{2.15}$$

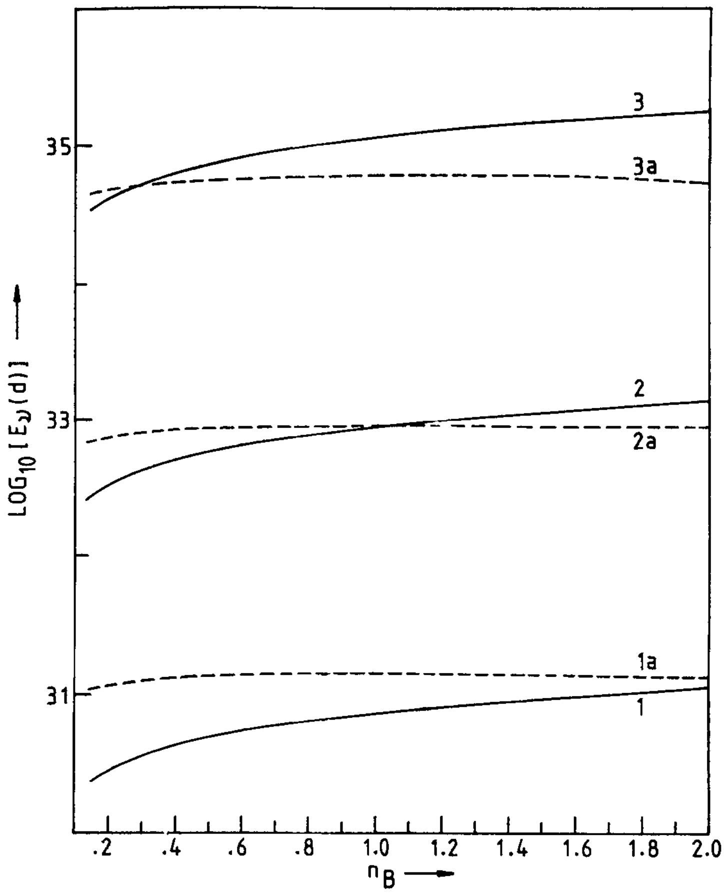

Figure 1a. Emissivity rate $E_\nu(d)$ (in erg/cm^{3}sec), for d-quark as a function of baryon density n_B (in fm^{-3}) at different temperatures for $\alpha_c = 0{\cdot}025$. Curves 1, 2, 3 correspond to $T = 1, 2, 4$ MeV respectively. Curves 1a, 2a, 3a refer to Duncan *et al* results for the same temperatures.

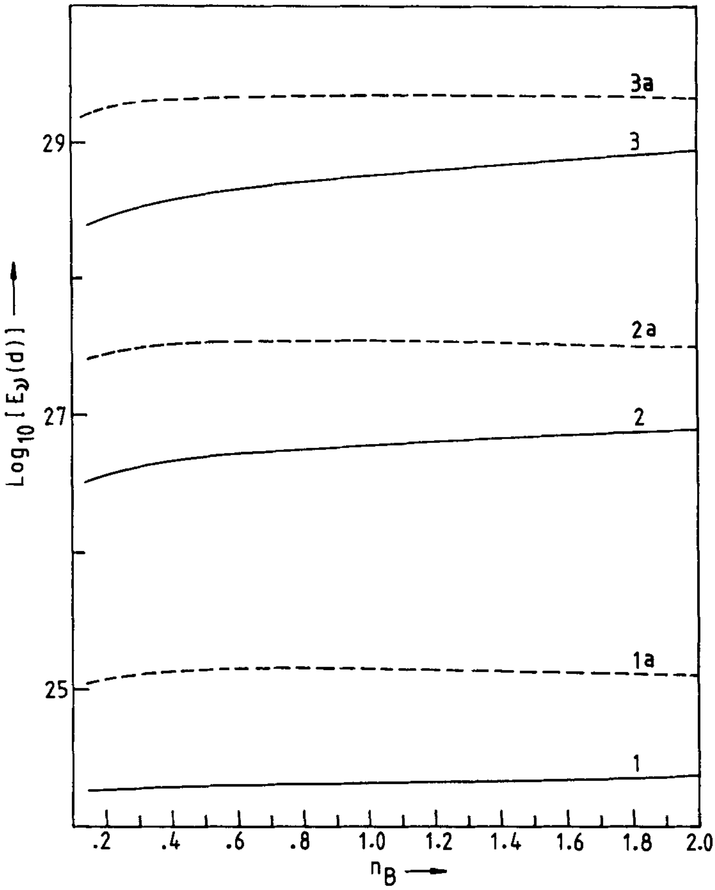

Figure 1b. Same as in figure (1a) but for temperatures $T = 0{\cdot}1$, $0{\cdot}25$ and $0{\cdot}5$ MeV. Curves 1, 2, 3 are our results and 1a, 2a, 3a refer to Duncan *et al* results.

$$\lambda_i^{-1} = (3/2\pi^3)G_i^2(T/x)^2\left(\frac{x^2+\pi^2}{1+\exp(-x)}\right)I'(\mu_1, xT, \mu_3, \mu_4) \tag{2.16}$$

3. Results and discussion

For the actual numerical evaluation of emissivity rate and mean free path from eqs (2.13–2.16), one requires the numerical values of various parameters involved. For the chemical potentials, we took a simple first order QCD model [2, 7] and expressed the number densities in terms of chemical potentials, temperature and the QCD coupling constant α_c, (see eqs (2.4e) to (2.4h) of § 2). The values of constants α_c, B, m_s and μ_0 are chosen by demanding that the uds-quark matter be stable, that is to have its energy per baryon less than 930 MeV at some density while the ud-quark matter be unstable with its energy per baryon greater than 940 MeV at the same density. One such choice of these constants, consistent with this requirement is [8]

$$\alpha_c = 0{\cdot}025, \quad B = 70\,\mathrm{MeV/fm}^3, \quad m_s = 150\,\mathrm{MeV}, \quad \mu_0 = 310\,\mathrm{MeV}.$$

We have computed the emissivity rate and the mean free path at different temperatures and densities. In figures 1a and 1b we plot $\log_{10}(E_\nu(d))$ as a function of the baryon density n_B, at temperatures $T = 1, 2, 4$ MeV and $T = 0{\cdot}1, 0{\cdot}25, 0{\cdot}5$ MeV respectively. Also drawn in these figures are the curves obtained by using the formalism of Duncan *et al* [5] and Iwamoto [3] at the same values of temperatures. We observe a reasonably close agreement in the two results at $T = 1, 2$ and 4 MeV while at lower temperatures there is a difference of up to one order of magnitude, our results being lower than those of Duncan *et al.*

In figures 2a and 2b we show the results for s-decay at temperatures $T = 1, 2, 4$ MeV and $T = 0{\cdot}1, 0{\cdot}25, 0{\cdot}5$ MeV respectively and compare with those of Duncan *et al.* We see that the two results agree broadly at higher densities for all temperatures. At lower densities, however, there is a marked difference in the two results. In our case the emissivity remains appreciable for all values of density, whereas in the case of Duncan *et al* there is a threshold

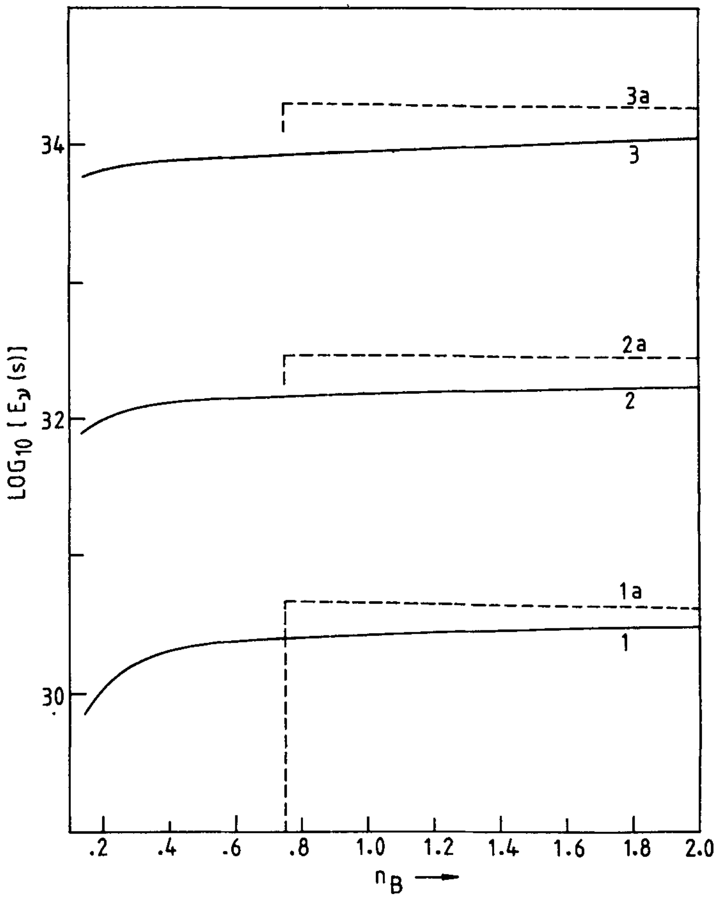

Figure 2a. Emissivity rate $E_\nu(s)$ (in erg/cm^3 sec), for s-quark as a function of baryon density n_B (in fm^{-3}) at different temperatures for $\alpha_c = 0{\cdot}025$. Curves 1, 2, 3 correspond to $T = 1, 2, 4$ MeV respectively. Curves 1a, 2a, 3a refer to Duncan *et al* results for the same values of temperatures.

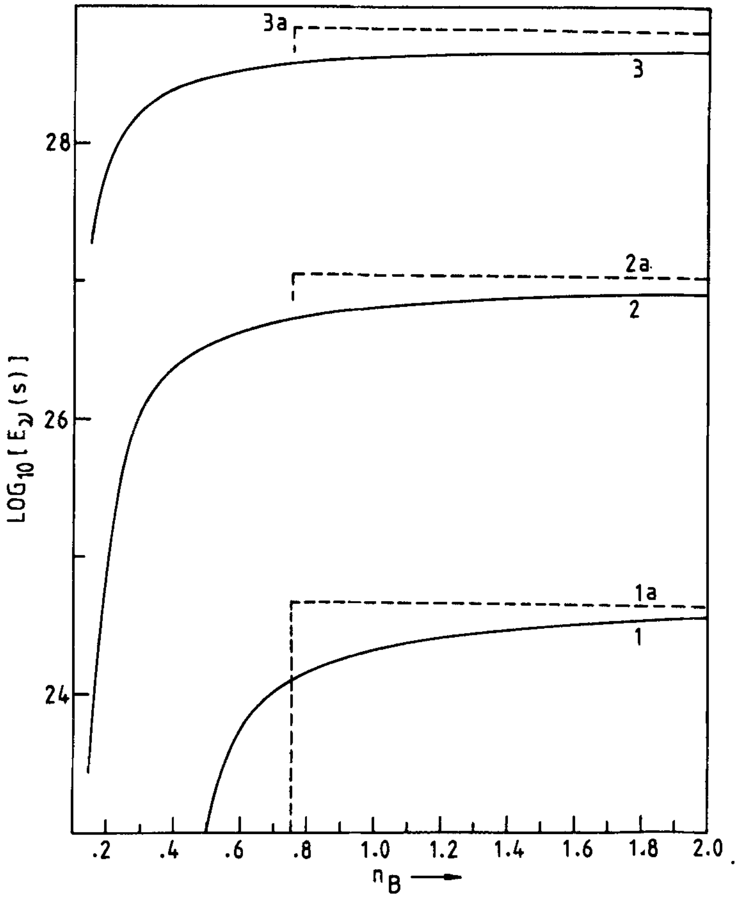

Figure 2b. Same as in figure 2a but for temperatures $T = 0{\cdot}1$, $0{\cdot}25$ and $0{\cdot}5$ MeV. Curves 1, 2, 3 are our results and 1a, 2a, 3a refer to Duncan *et al* results.

density below which the emissivity rate is zero. This threshold appears due to the kinematic restriction coming from an average over the angular integrations. In our case, the angular integrations are done exactly, as a result of which this threshold either disappears or is very low depending upon the temperature.

In figure 3a we show the effect of α_c on the emissivity rate $E_{\nu_l}(d)$, as a function of the baryon density n_B, at $T = 0{\cdot}1$ MeV and $m_s = 150$ MeV. The behaviour of emissivity with n_B changes altogether as α_c is increased. For small $\alpha_c(\leqslant 0{\cdot}05)$ the emissivity keeps on increasing with n_B (up to $2{\cdot}0\,\mathrm{fm}^{-3}$). For higher values of α_c there is, however, an extinction density n_{ex}, beyond which the emissivity vanishes. This n_{ex} decreases with increasing α_c and is around $1{\cdot}25\,\mathrm{fm}^{-3}$ at $\alpha_c = 0{\cdot}065$. Also reproduced, in the same figure, are the results obtained by Duncan *et al* for the same values of α_c. It can be seen that the two results are qualitatively similar, however, in the case of Duncan *et al*, the extinction density appears at higher values of α_c. To see the effect of strange quark mass on the extinction density we have shown in figure 3b, the emissivity rate, $E_\nu(\mathrm{d})$, as a function of baryon density n_B, at different values of α_c for $m_s = 300$ MeV and $T = 0{\cdot}1$ MeV. Figures 3a and 3b show that the extinction

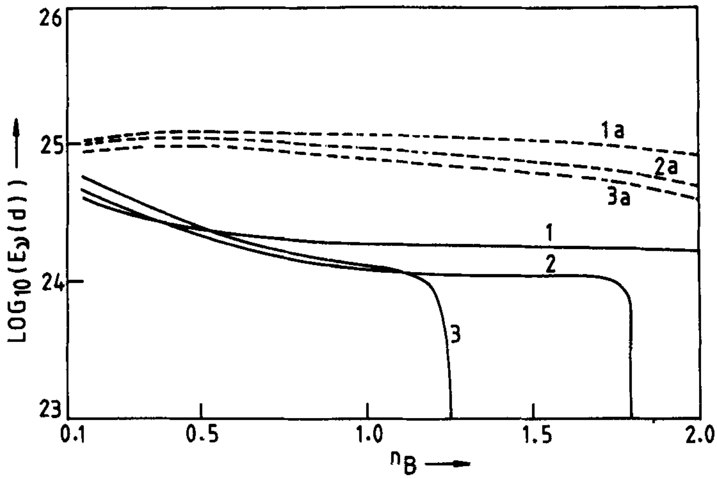

Figure 3a. Emissivity rate $E_\nu(d)$ (in erg/cm^{3}sec), for d-quark as a function of baryon density n_B(in fm^{-3}) at $T = 0{\cdot}1$ MeV for different values of α_c. Curves 1, 2, 3 refer to $\alpha_c = 0{\cdot}05$, $0{\cdot}6$ and $0{\cdot}065$ respectively. Curves 1a, 2a, 3a refer to the Duncan *et al* results for the same values of α_c.

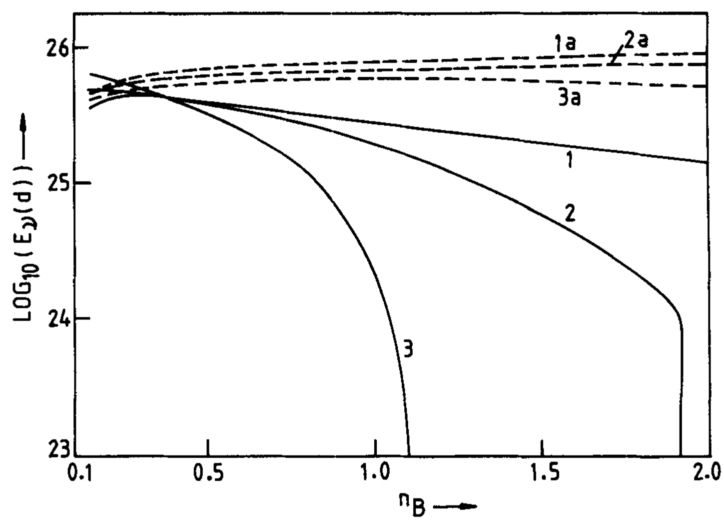

Figure 3b. Same as in figure 3a but for $m_s = 300$ MeV. Curves 1, 2, 3 correspond to our results at $\alpha_c = 0{\cdot}065$, $0{\cdot}08$ and $0{\cdot}1$ respectively. Curves 1a, 2a, 3a refer to Duncan *et al* results for the same values of α_c.

density remains much lower than that of Duncan *et al* as we increase α_c. This difference is due to the fact that we have taken first order QCD thermodynamics [2, 7] with subtraction included. And if the subtraction term is ignored, the difference in the values of n_{ex}, obtained by us and Duncan *et al* becomes negligible. For s-decay the behaviour of extinction density with an increase of α_c and m_s is similar to that for d-decay.

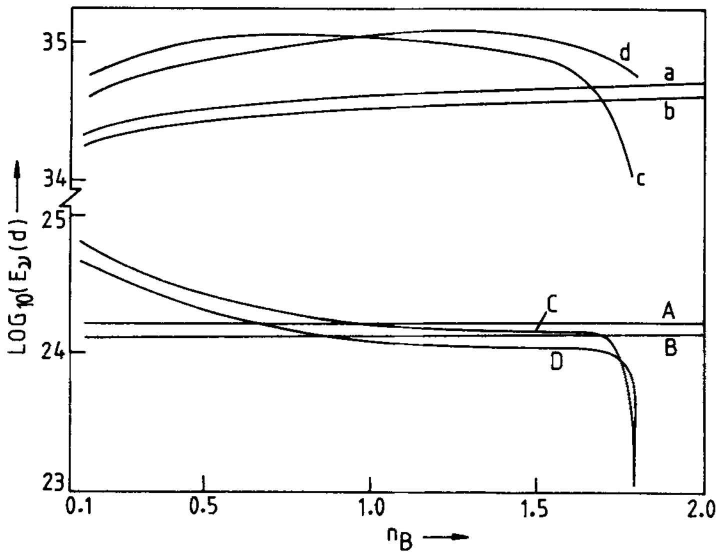

Figure 4. Emissivity rate $E_\nu(d)$ (in erg/cm^3 sec), for d-decay as a function of baryon density n_B (in fm^{-3}) at $T = 0{\cdot}1$ and 4 MeV. Curves A, B, C, D correspond to $T = 0{\cdot}1$ MeV and a, b, c, d to $T = 4$ MeV. Curves A, a and C, c refer to 1-dimensional integral results for $\alpha_c = 0{\cdot}025$ and $0{\cdot}06$ respectively. Curves B, b and D, d refer to the 2-dimensional integral results for $\alpha_c = 0{\cdot}025$ and $0{\cdot}06$ respectively.

In figure 4 we show the comparison of 1-dimensional (electrons degenerate case) and 2-dimensional (electrons non-degenerate case) results at $T = 0{\cdot}1$ and 4 MeV for d-decay. We observe a fairly close agreement of 1-dim. with the 2-dim. results for $\alpha_c = 0{\cdot}025$, whereas for $\alpha_c = 0{\cdot}065$ differences start appearing only at high densities for both $T = 0{\cdot}1$ and 4 MeV. Thus electron degeneracy has significant effect only at densities very close to the extinction density.

Finally, in figure 5 we show the mean free path λ, as a function of the baryon density n_B, at $T = 1$ and 4 MeV and for neutrino energies, $E_2 = T$. We find that λ drops below the 10 km limit (approximate radius of a neutron star) at $T > 4$ MeV.

To summarize, in this paper we have calculated the emissivity and mean free path for the u, d, s-quark matter. We have followed the basic formalism of Iwamoto and others but we have done the various angular integrals involved exactly. As a consequence, the results obtained are not in terms of analytic expressions but in terms of certain integrals which are evaluated numerically. For d-decay, we find a reasonably close agreement with Duncan *et al* at higher temperatures while at lower temperatures there is a difference of up to one order of magnitude in the emissivity rate. For s-decay, our results agree broadly with those of Duncan *et al* particularly at higher densities. However in our case there is no threshold density below which the emissivity vanishes, unlike in their case where such threshold exists. The effect of QCD coupling constant α_c, and the strange quark mass m_s, on the emissivity rate has been studied in detail. Apart from the magnitude of emissivity rate, we find that the extinction density also depends critically on these parameters. As for the mean free path we have shown that it is less than the typical dimension of a quark star for temperatures, $T > 4$ MeV. So the assumption that the neutrinos stream out freely from the star and do not take part in

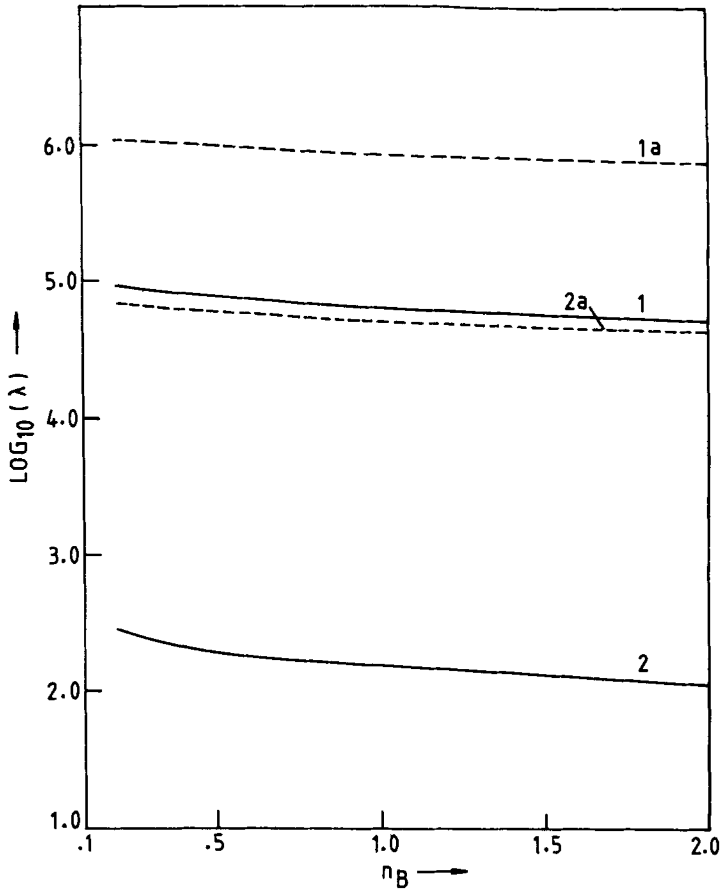

Figure 5. Mean free path, λ (in km), as a function of baryon density n_B (in fm^{-3}), at different temperatures and neutrino energies. Curves 1, 2 refer to $T = 1$ and 4 MeV respectively and $e_\nu = T$. Curves designated a refer to the results obtained using the Iwamoto expression for the same temperatures and neutrino energies.

chemical equilibrium is valid only for temperatures, $T \leqslant 4\,\text{MeV}$. Though we have shown results for one choice of the parameters, quite similar results follow for any other suitable choice. The basic formalism for many other physical processes like bulk viscosity and cooling rates is similar. Calculations in this direction are in progress.

Appendix

In this we describe some of the steps involved in the evaluation of the angular integral $I(E_1, E_2, E_3, E_4)$, which is defined as

$$I(E_1, E_2, E_3, E_4) = (p_1 p_2 p_3 p_4 / 128\pi^2) \int \left[\prod_i \mathrm{d}\Omega_i \right] (p_1 \cdot p_2)(p_3 \cdot p_4) \times \delta(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4). \tag{A1}$$

Rewriting (A1) in the form

$$I(E_1, E_2, E_3, E_4) \equiv I$$

where

$$I = (p_1 p_2 p_3 p_4/128\pi^2) \int \left[\prod_i \mathrm{d}\Omega_i \right] [E_1 E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2][E_3 E_4 - \mathbf{p}_3 \cdot \mathbf{p}_4] \times \delta(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4). \tag{A2}$$

Separating the three-dimensional δ-function into a delta function over the magnitude and one over the solid angles:

$$\delta(\mathbf{r} - \mathbf{r}_0) = \delta(r - r_0)\,\delta(\Omega - \Omega_0)/r^2 \tag{A3}$$

and using the latter to get rid of the integration over Ω_2, we obtain

$$I = (p_1 p_2 p_3 p_4/128\pi^2)(1/p_2^2) \int \mathrm{d}\Omega_3 \mathrm{d}\Omega_4 (E_3 E_4 - \mathbf{p}_3 \cdot \mathbf{p}_4) \int \mathrm{d}\Omega_1 \times [E_1 E_2 - p_1^2 + \mathbf{p}_1 \cdot (\mathbf{p}_3 + \mathbf{p}_4)]\,\delta[p_2 - (p_1^2 + (\mathbf{p}_3 + \mathbf{p}_4)^2 - 2\mathbf{p}_1(\mathbf{p}_3 + \mathbf{p}_4))^{1/2}]. \tag{A4}$$

Next, putting

$$\mathbf{p}_3 + \mathbf{p}_4 = \mathbf{P}; \quad \mathbf{p}_1 \cdot (\mathbf{p}_3 + \mathbf{p}_4) = \mathbf{p}_1 \cdot \mathbf{P} = p_1 P = p_1 P \cos\theta = p_1 P x$$

and using the remaining delta functions to perform the integration over x, we obtain

$$I = (p_1 p_2 p_3 p_4/128\pi^2)(2\pi/p_1 p_2) \int \mathrm{d}\Omega_3 \mathrm{d}\Omega_4\, \theta(1 - |(p_1^2 + P^2 - p_2^2)/2p_1 P|) \times [E_3 E_4 + 0{\cdot}5(p_3^2 + p_4^2) - 0{\cdot}5P^2][E_1 E_2 - 0{\cdot}5(p_1^2 + p_2^2) + 0{\cdot}5P^2]/P. \tag{A5}$$

The theta function has made its appearance because the integration over x is performed only from -1 to $+1$. The remaining integrations over $\mathrm{d}\Omega_3 \mathrm{d}\Omega_4$ can now be performed easily because only the angle in $\mathbf{p}_3 \cdot \mathbf{p}_4$ is involved; one only has to be careful about the limits of integration which are determined by the theta function. This leads to the result

$$I = (\pi/8)[k_-^{12} k_+^{34} I_1(1,2;3,4) + (1/6)(k_+^{34} - k_-^{12}) I_2(1,2;3,4) - (1/20) \times I_3(1,2;3,4)] \tag{A6}$$

where

$$k_\pm^{ij} = E_i E_j \pm 0{\cdot}5(p_i^2 + p_j^2)$$

and

$$\begin{aligned} I_n(i,j;k,l) = {} & (P_{ij}^{2n-1} - p_{ij}^{2n-1})\theta(P_{kl} - P_{ij})\theta(p_{ij} - p_{kl}) \\ & + (P_{ij}^{2n-1} - p_{kl}^{2n-1})\theta(P_{kl} - P_{ij})\theta(p_{kl} - p_{ij})\theta(P_{ij} - p_{kl}) \\ & + (P_{kl}^{2n-1} - p_{kl}^{2n-1})\theta(P_{ij} - P_{kl})\theta(p_{kl} - p_{ij}) \\ & + (P_{kl}^{2n-1} - p_{ij}^{2n-1})\theta(P_{ij} - P_{kl})\theta(p_{ij} - p_{kl})\theta(P_{kl} - p_{ij}) \end{aligned}$$

where

$$P_{ij} = p_i + p_j; \quad p_{ij} = |p_i - p_j|.$$

Similarly by considering expression (A1) for the mean free path and performing the angular integrations in the same way as for emissivity (except for taking care that in this case the integration over neutrino momentum is absent) we obtain (2·12) for the mean free path with the function $I'(E_1, E_2, E_3. E_4)$ given by

$$I'(E_1, E_2, E_3, E_4) = (\pi/8)[k_-^{12} k_+^{34} I_1(1,2;3,4) + (1/6) \times (k_+^{34} + k_-^{12}) I_2(1,2;3,4) + (1/20) I_3(1,2;3,4)]. \quad \text{(A7)}$$

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