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Gravitational and electromagnetic fields of a charged tachyon

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Abstract. An axially symmetric exact solution of the Einstein–Maxwell equations is obtained and is interpreted to give the gravitational and electromagnetic fields of a charged tachyon. Switching off the charge parameter yields the solution for the uncharged tachyon which was earlier obtained by Vaidya. The null surfaces for the charged tachyon are discussed.

Keywords. Einstein–Maxwell equations; tachyon.

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1. Introduction

The tardyons, the luxons, and the tachyons are the names given to particles which move, respectively, with velocities smaller than, equal to, and greater than the speed of light in vacuum [1–2]. Among these three classes of objects, tachyons have remained undetected. However, the subject of the superluminary objects have been fascinating many physicists' minds. Lucretius (50 B.C.) was probably the first scientist who mentioned such objects [3]. Few years before the special theory of relativity was given, Thomson, Heavyside, and Sommerfeld had investigated the questions arising from the assumptions that the superluminary objects exist in the nature [4]. However, with the outset of the special theory of relativity (STR), due to a misinterpretation by Einstein himself, a wrong idea prevailed that the non-existence of tachyons is a direct consequence of the STR [1, 4]. To this end, Bilaumik *et al* (BDS) [4] reexamined this subject and found that it is rather the special theory of relativity that suggests a possibility for existence of superluminary objects. In order to have the energy and momentum (which are measurable quantities) real, BDS [4] hypothesized the tachyons to have their proper masses imaginary. Likewise, the proper lengths and the proper times of tachyons are also imaginary quantities. The velocity of a tachyon increases on the loss of its energy.

The causality problems (raised by Tolman, Schmidt, and Terletski) due to the assumption that the tachyons exist were resolved by Sudarshan and his co-workers [2, 3]. BDS [4] suggested that the Cerenkov effect is likely to be an avenue for the detection of the faster-than-light objects if they carry electric charge. Many experimentalists [5–8] have put considerable effort to detect tachyons. Their experiments could not confirm the existence of tachyons. However, inspired by the Gell-Mann's totalitarian principle (which states that in physics anything which is not forbidden is compulsory) [2, 3], the subject of tachyons continued to be of interest to many researchers. Recami [3] presented a review on this subject.

Narlikar and Sudarshan [9] studied the propagation of tachyons in an expanding universe. They showed that the pre-mordial tachyons in a big-bang universe could not survive unless it had very large energy in the beginning. The trajectories of tachyons in the Schwarzschild background were studied by Narlikar and Dhurandhar [10]. These investigations were further extended by Dhurandhar [11] in the Kerr geometry. Vaidya [12] obtained a static, axially symmetric vacuum solution of the Einstein equations and interpreted it as describing the gravitational field of a tachyon. He found that there is a gravitational repulsion between a tachyon and a tardyon. Raychaudhuri [13] showed that there is an attraction between tachyons themselves. We [14] obtained the gravitational field of a tachyon (uncharged) in a de Sitter universe. The electromagnetic interactions between a charged tachyon and a charged tardyon, and between charged tachyons themselves have not been investigated. Therefore, it is of interest to obtain the gravitational as well as the electromagnetic fields of a charged tachyon. In the present paper, we obtain an axially symmetric exact solution of the Einstein–Maxwell equations and interpret it to give the gravitational and electromagnetic fields of a charged tachyon. We use the geometrized units ($G = 1, c = 1$) and follow the convention that the Latin indices run from 0 to 3.

2. Einstein–Maxwell equations

The Einstein–Maxwell equations are

$$R_i^k - \tfrac{1}{2} g_i^k R = 8\pi (T_i^k + E_i^k), \tag{1}$$

where

$$E_i^k = \frac{1}{4\pi}\left[- F_{im} F^{km} + \tfrac{1}{4} g_i^k F_{mn} F^{mn} \right], \tag{2}$$

$$\frac{1}{\sqrt{-g}} (\sqrt{-g} F^{ik})_{,k} = 4\pi J^i, \tag{3}$$

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0. \tag{4}$$

R_i^k is the Ricci tensor. T_i^k and E_i^k are the energy-momentum tensors due to the matter and the electromagnetic field, respectively. J^i stands for the electric current density vector.

3. Solution for charged tachyon

By transforming the Reissner–Nordström solution, we have obtained a static and axially symmetric exact solution of the Einstein–Maxwell equations which is given by the line element,

$$d\tau^2 = B dt^2 - B^{-1} d\rho^2 - \frac{\rho^2}{(1 - v\cos\theta)^2} (d\theta^2 + \sin^2\theta \, d\phi^2), \tag{5}$$

where

$$B = 1 - v^2 + \frac{m}{\rho} + \frac{Q^2}{\rho^2} (1 - v^2)^2 \tag{6}$$

and the only component of the electromagnetic field tensor

$$F_{\rho t} = -\frac{Q(1-v^2)}{\rho^2}. \tag{7}$$

The non-vanishing components of the energy-momentum tensor of the electromagnetic field are

$$E^t_t = E^\rho_\rho = -E^\theta_\theta = -E^\phi_\phi = \frac{Q^2}{8\pi\rho^4}(1-v^2)^2. \tag{8}$$

The current density vector and the energy-momentum tensor of matter are given by

$$J^i = 0, \quad T^k_i = 0. \tag{9}$$

We now proceed to show that the above solution gives the gravitational and electromagnetic fields of a charged tachyon.

Using the retarded time coordinate u given by

$$u = t - \int [B(\rho)]^{-1} \mathrm{d}\rho \tag{10}$$

in place of t, the above line element can be written as

$$\mathrm{d}\tau^2 = B\mathrm{d}u^2 + 2\mathrm{d}u\mathrm{d}\rho - \frac{\rho^2}{(1-v\cos\theta)^2}(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2) \tag{11}$$

and the non-vanishing component of the electromagnetic field tensor is given by

$$F_{\rho u} = -\frac{Q(1-v^2)}{\rho^2}. \tag{12}$$

Now by a coordinate transformation ρ going to r, where $\rho = r(1-v\cos\theta)$, the line element is

$$\mathrm{d}\tau^2 = D\mathrm{d}u^2 + 2\mathrm{d}u\,\mathrm{d}[r(1-v\cos\theta)] - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \tag{13}$$

where

$$D = 1 - v^2 + \frac{m}{r(1-v\cos\theta)} + \frac{Q^2(1-v^2)^2}{r^2(1-v\cos\theta)^2} \tag{14}$$

and the surviving components of the electromagnetic field tensor are

$$F_{ru} = -\frac{Q(1-v^2)}{r^2(1-v\cos\theta)}$$

$$F_{\theta u} = -\frac{Q(1-v^2)v\sin\theta}{r(1-v\cos\theta)^2}. \tag{15}$$

For $|v| < 1$, the solution can be transformed to the Reissner–Nordström solution given by the line element,

$$\mathrm{d}\tau^2 = B'\mathrm{d}u'^2 + 2\mathrm{d}u'\mathrm{d}r' - r'^2(\mathrm{d}\theta'^2 + \sin^2\theta'\mathrm{d}\phi^2), \tag{16}$$

where

$$B' = 1 - \frac{2M}{r'} + \frac{Q^2}{r'^2} \tag{17}$$

and

$$M = -\frac{m(1 - v^2)^{-3/2}}{2}, \tag{18}$$

and the non-vanishing component of the electromagnetic field tensor

$$F_{r'u'} = -\frac{Q}{r'^2}, \tag{19}$$

by the coordinate transformations

$$\begin{aligned} u' &= u\sqrt{1 - v^2}, \\ r' &= \frac{r(1 - v\cos\theta)}{\sqrt{1 - v^2}}, \\ \cos\theta' &= \frac{\cos\theta - v}{1 - v\cos\theta}. \end{aligned} \tag{20}$$

We obtained the axially symmetric solution by following the reverse process given here, i.e., we began with the Reissner–Nordström solution and obtained the axially symmetric solution by a complex transformation ($|v| > 1$). However, for convenience in presentation we have first written the axially symmetric solution (the line element in diagonal form) and then have shown that it transforms to the Reissner–Nordström solution for $|v| < 1$. The coordinate transformations expressed by (20) show that the origin of the frame S' moves with respect to the frame S with a uniform velocity v along the common Z-axis in the flat spacetime background [12]. The solution of the Einstein–Maxwell equations obtained by us has the following characteristics: (a) the gravitational field given by the line element (5) has all the geometrical characteristics of the Reissner–Nordström field, (b) the solution is axially symmetric which is required as there cannot be a tardyonic observer which can find a tachyon at rest, and (c) it contains a velocity parameter v, which is not possible to be transformed away for $|v| > 1$. However, for $|v| < 1$, the solution transforms to the Reissner–Nordström solution. Therefore, we interpret the solution to give the gravitational and the electromagnetic fields of a non-rotating charged tachyon. Switching off the charge parameter Q, one gets the solution for the uncharged (non-rotating) tachyon which was obtained by Vaidya [12].

4. Discussion

The solution given by us is singular at $r = 0$ and $\sec\theta = v$. The latter gives a right circular cone of semi-vertical angle arcsec(v) as the singularity surface. Again the null surfaces are given by the equation

$$(v^2 - 1)\rho^2 - m\rho - Q^2(v^2 - 1)^2 = 0. \tag{21}$$

This equation has two roots given by

$$\rho = \rho_{\pm}, \tag{22}$$

where

$$\rho_+ = \frac{m + \sqrt{m^2 + 4(v^2-1)^3 Q^2}}{2(v^2-1)},$$

$$\rho_- = \frac{m - \sqrt{m^2 + 4(v^2-1)^3 Q^2}}{2(v^2-1)}. \tag{23}$$

Using $\rho = r(1 - v\cos\theta)$, $r = (x^2+y^2+z^2)^{1/2}$, and $r\cos\theta = z$, the equations of null surfaces given by (22) can be written as

$$\left(z\sqrt{v^2-1} + \frac{v\rho_\pm}{\sqrt{v^2-1}}\right)^2 - (x^2+y^2) = \frac{\rho_\pm^2}{v^2-1} \tag{24}$$

which give two hyperboloids of revolution of two sheets in the three (spatial) space corresponding to ρ equals to ρ_+ and ρ equals to ρ_-. It is clear from (22)–(23) that the null surfaces for the charged tachyon are different from that of the uncharged tachyon. It is of interest to study the trajectories of the charged tardyons in the field of a charged tachyon.

After the completion of this work, a paper [15] came to our notice wherein the authors, using the complex tetrads, obtained a general Kerr–Schild class of solutions. They mentioned that a particular solution gives the charged rotating tachyon. The solution given by us has a clear physical interpretation. Dadhich brought to our notice his paper on charged tachyon [16]. Our method of obtaining the gravitational and electromagnetic fields of a charged tachyon is different from his.

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