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Static and dynamic properties of pseudoscalar mesons

D K CHOUDHURY*+, PRATIBHA DAS**, D D GOSWAMI*** and J K SARMA*
*Department of Physics, Gauhati University, Guwahati 781 014, India
**Department of Physics, Nalbari College, Nalbari 781 335, India
***Department of Physics, North Lakhimpur College, North Lakhimpur 787001, India
+ Present address: International Centre for Theoretical Physics, P.O. Box 586, 34100 Trieste, Italy

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Abstract. We report the results of form factors, charge radii and decay constants of both light and heavy flavoured pseudoscalar mesons in a QCD inspired quark model. We use the quantum mechanical perturbation theory and discuss its limitations in the present problem. Several predictions are also made for bottom and top flavours.

Keywords. Meson wave function; form factors; decay constants; charged radii.

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1. Introduction

Study of hadron wave functions and their phenomenology is an important topic in quantum chromodynamics [1]. Constraints on the baryon and meson distribution amplitudes have been obtained in recent years within the framework of QCD sum rules [2] and lattice QCD [3].

In the more phenomenological approach of non-relativistic potentials however, attention is paid for enumerating the spectra of hadrons rather than the wave function structures since the classical work of De Rujula *et al* [4]. Even in most recent analysis [5], harmonic oscillator basis for the trial wave functions are considered rather than the exact wave functions themselves. The reason is easily understandable; the spin dependent terms of the Fermi–Breit Hamiltonian [4, 6] are more singular than r^{-2} and are therefore illegal operators in the Schrödinger equation.

The application of QCD potentials to light quark systems is rather handicapped due to their relativistic modifications. In the works referred to earlier [5], ad hoc smearing functions as well as momentum dependent strengths are considered as such possible relativistic modifications.

However, it is impossible to solve the eigen functions and eigen energies of the Hamiltonian with such terms directly from the Schrödinger equation. This has resulted in the use of the variational method [5] rather than the perturbation theory.

The aim of the present paper is to report an analysis based on perturbation theory which uses the variational method as well to extract the modification due to spin effect. Specifically, we use the two-body Schrödinger equation and obtain the first order perturbed wave function of the potential using Dalgarno's method [7]. We then incorporate relativistic effects at the wave function level by introducing standard Dirac modifications [8, 9] rather than full covariantization as in Bethe–Salpeter approach [10–12].

In order to test the wave function, one needs to calculate the properties of mesons such as form factors, decay constants and charge radii. This paper attempts to derive them within the present formalism. In § 2 we include a discussion on QCD inspired quark model, formulate the wave function and analyse the properties of pseudoscalar mesons. Section 3 contains the summary and conclusions.

2. Theory

2.1 *The non-relativistic QCD inspired quark model*

The non-relativistic QCD inspired quark model has its origin in the work of ref. [4]. Since then, it has been applied to explain schematically the vast body of information available on mesons and baryons [11–13]. However, the non-relativistic treatment of light quark systems is clearly inadequate and needs improvement.

On the other hand, the Fermi–Breit Hamiltonian [4] which is the basis of the QCD inspired quark model has terms which are more singular than γ^2 and hence is not exactly soluble.

Godfrey and Isgur [5] improved upon the model by postulating a relativistic potential $V(\mathbf{p}, \mathbf{r})$ which differs from its non-relativistic limit in two ways (i) the coordinate ($\mathbf{r}$) becomes smeared at over distances of the order of the inverse quark mass and (ii) the coefficients of the various potentials become dependent on the momentum of the interacting quarks. The smearing of the potentials results in taming of all their singularities. Specifically, the smearing function between two quarks of masses m_i, m_j is taken as

$$\rho_{ij}(r_i - r_j) = \frac{\sigma_{ij}^3}{\pi^{3/2}} \exp[-(\sigma_{ij}^2 (r_i - r_j)^2)] \tag{1}$$

with the prescription

$$\sigma_{ij}^2 = \sigma_0^2 \left[\frac{1}{2} + \frac{1}{2}\left(\frac{4m_i m_j}{(m_i + m_j)^2}\right)^4\right] + S^2 \left(\frac{2m_i m_j}{m_i + m_j}\right)^2, \tag{2}$$

where σ_0 and S are the relativistic parameters determined from data. Similarly, the momentum dependence introduced in relativistic treatment transforms mass function $(1/m_i)$ of the potentials into energies $1/(m_i^2 + p_i^2)^{1/2}$.

Such a method although claimed to be phenomenologically successful, is crude enough to warrant alternate route of improving the non-relativistic QCD inspired quark model. In the present paper, we improve upon the model without additional free parameters, unlike [5]. To that end we use perturbation theory [7] and parameter-free Dirac modification.

2.2 *Fermi–Breit Hamiltonian*

We take the non-relativistic two body Schrödinger equation as our basis, i.e.

$$H|\psi\rangle = (\hat{H}_0 + \hat{H}')|\psi\rangle = E|\psi\rangle, \tag{3}$$

where H_0 is the free Hamiltonian for two quarks of masses m_i and m_j and three-momenta $\mathbf{P}_i$ and $\mathbf{P}_j$. H_0 is defined as

$$H_0 = \frac{p_i^2}{2m_i} + \frac{p_j^2}{2m_j}, \tag{4}$$

and H is the Fermi–Breit Hamiltonian with confinement which is defined as

$$H(r) = H^{\text{conf}}(r) + H^{\text{hyp}}(r) + H^{\text{S.O.}}(r). \tag{5}$$

Here,

$$H^{\text{conf}}(r) = +\left(-\frac{\alpha_s(r)}{r} + \frac{3}{4}br + \frac{3}{4}c\right)(F_i \cdot F_j) \tag{6}$$

$$H^{\text{hyp}}(r) = \frac{-\alpha_s(r)}{m_i m_j}\left[\frac{8\pi}{3}\mathbf{S}_i \cdot \mathbf{S}_j \delta^3(r) + \frac{1}{r^3}\left\{\frac{3(\mathbf{S}_i \cdot \mathbf{r})(\mathbf{S}_j \cdot \mathbf{r})}{r^2} - \mathbf{S}_i \cdot \mathbf{S}_j\right\}\right](\mathbf{F}_i \cdot \mathbf{F}_j) \tag{7}$$

$$H^{\text{S.O.}}(r) = H^{\text{S.O.(c.m.)}} + H^{\text{S.O.(t.p.)}} \tag{8}$$

$$H^{\text{S.O.(c.m.)}} = \frac{\alpha_s(r)}{r^3}\left(\frac{1}{m_i} + \frac{1}{m_j}\right)\left(\frac{\mathbf{S}_i}{m_i} + \frac{\mathbf{S}_j}{m_j}\right)\mathbf{L}(\mathbf{F}_i \cdot \mathbf{F}_j) \tag{9}$$

$$H^{\text{S.O.(t.p.)}}(r) = -\frac{1}{2r}\frac{\partial H^{\text{conf}}}{\partial r}\left(\frac{\mathbf{S}_i}{m_i^2} + \frac{\mathbf{S}_j}{m_j^2}\right)\mathbf{L}. \tag{10}$$

Here, $\mathbf{S}_i$ and $\mathbf{S}_j$ are the spins of the ith and jth quarks separated by a distance r. For ground state ($l = 0$), only the contact term $\propto \delta^3(r)$ contributes and the Hamiltonian takes the simpler form

$$H = \frac{4\alpha_s}{3}\left[-\frac{1}{|r|} - \frac{8\pi}{3}\delta^3(\mathbf{r})\frac{\mathbf{S}_i \cdot \mathbf{S}_j}{m_i m_j}\right] + br + c. \tag{11}$$

2.3 *Formulation of wave function*

Neglecting the δ-function in (11) the spin-independent Hamiltonian is

$$H = -\frac{4\alpha_s}{3r} + br + c \tag{12}$$

so that

$$H'(r) = br + c \tag{13}$$

can be treated as the perturbation to the unperturbed Hamiltonian

$$H_0 = \frac{p_i^2}{2m_i} + \frac{p_j^2}{2m_j} - \frac{4\alpha_s}{3r}. \tag{14}$$

Let us now calculate the first order perturbed eigen function $\psi^{(1)}$ and eigen energy $W^{(1)}$ using the relation [7]

$$H_0\psi^{(1)} + H\psi^{(0)} = W^{(0)}\psi^{(1)} + W^{(1)}\psi^{(0)} \tag{15}$$

where

$$W^{(1)} = \langle \psi^{(0)} | H' | \psi^{(0)} \rangle. \tag{16}$$

From (15), we calculate $\psi^{(1)}$ by Dalgarno's method and obtain

$$\psi^{(1)}(r) = -\frac{1}{2\sqrt{\pi a_0^3}}\mu b a_0 r^2 \exp(-r/a_0), \tag{17}$$

where μ is the reduced mass defined as

$$\mu = \frac{m_i m_j}{m_i + m_j} \tag{18}$$

and

$$a_0 = (4/3\mu\alpha_s)^{-1}, \tag{19}$$

α_s being the strong coupling constant.

The normalized wave function with coulomb plus linear potential will be

$$\psi(r) = \psi^{(0)}(r) + \psi^{(1)}(r)$$
$$= \frac{1}{[\pi a_0^3(1 - 3\mu b a_0^3 + (45/8)\mu^2 b^2 a_0^6)]^{1/2}}\left(1 - \frac{1}{2}\mu b a_0 r^2\right)\exp(-r/a_0). \tag{20}$$

Its momentum transform is

$$\psi(Q) = \left[\frac{2^3}{\pi^3(1 - 3\mu b a_0^3 + (45/8)\mu^2 b^2 a_0^6)}\right]^{1/2} \sum_i e_i \left(1 - \frac{6\mu b a_0^3(1 - a_0^2 Q_i^2)}{(1 + a_0^2 Q_i^2)^2}\right) \tag{21}$$

where Q_i's are defined as

$$Q_i = \sum_{j \neq i} m_j Q \Big/ \sum_i m_i. \tag{22}$$

2.4 *Status of confinement as perturbation*

The question naturally arises about the validity and consequences of treating linear confining potential as perturbation. The reason is understandable: linear confining potential being strong, is expected to be more dominant than the coulombic piece. However the confinement potential is operative within the characteristic size of a hadron. The characteristic distance of heavy flavour mesons is taken to be of the order of $\frac{1}{2}m_Q, m_Q$ being the mass of heavy quark [16]. Even for heavy light meson, the corresponding distance is of the order of quarkonium size, smaller than the hadronic scale Λ_{QCD}^{-1}. A similar observation has been done in QCD sum rules approach as well [17, 18]. If such an observation is taken seriously, treating confinement potential as perturbation appears to be justified for mesons containing at least one heavy flavour. For light mesons however, such argument does not hold good. We therefore assume it to be an ansatz to obtain explicit mesonic wave functions for light quark systems. The validity or otherwise of it will then be tested through available data.

If confinement is to be treated as perturbation, then from (21)

$$\left|\sum_i e_i \frac{6\mu b a_0^3(1 - a_0^2 Q_i^2)}{(1 + a_0^2 Q_i^2)^2}\right| < 1. \tag{23}$$

Equation (23) does not hold good for very low Q^2 with $b = 0{\cdot}183\,\text{GeV}^2$ [19]. As a result, the present perturbation method breaks down for low $Q^2, Q^2 \leqslant Q_0^2$ where Q_0^2 is determined from the condition

$$\sum e_i \frac{6\mu b a_0^3(1 - a_0^2 Q_0^2)}{(1 + a_0^2 Q_0^2)^2} = 1. \tag{24}$$

Values of Q_0^2 are different for different mesons due to the quark mass dependence occurring in Q_i. In table 1 we record the values of Q_0^2 for several light and heavy flavoured mesons. As $b \to 0$, the value of Q_0^2 will be decreased so that the formalism works for lower Q^2 range. It is thus clear that for fixed Q^2, perturbation works for smaller b.

We note that α_s as occurred in Q_0 [eq. 19] is Q^2 dependent. However as the dependence is only logarithmic, for our numerical analysis (table 1), we have assumed it to be approximately constant. For the Q^2 range under study, such variation is indeed found to be quantitatively small [5]. We also note that justification for neglecting the running of α_s for qualitative and numerical analysis has recently been advocated in other areas of QCD as well [20, 21].

2.5 *Relativistic effects*

The relativized version of (20) is [8, 9]

$$\psi^{\text{Rel}}(r) = \frac{N}{\sqrt{\pi a_0^3}}\left(1 - \frac{1}{2}\mu b a_0 r^2\right)\left(\frac{r}{a_0}\right)^{-\varepsilon} \exp(-r/a_0) \tag{25}$$

Table 1. Values of Q_0^2 (in GeV^2) for different mesons above which linear potential can be treated as perturbation for $b = 0{\cdot}183\,\text{GeV}^2$.

	Non-relativistic version (eq. (23))		Relativistic version (eq. (31))		
Particles	$\alpha_s = 0{\cdot}5$	α_s determined from meson mass	$\alpha_s = 0{\cdot}65$	$\alpha_s = 0{\cdot}6$	$\alpha_s = 0{\cdot}5$
$\pi^\pm$	6·4	0·3	3·6	4·2	5·6
$K^\pm$	5·5	0·3	3·1	3·6	4·8
k^0	28·3	15·0	13·0	18·0	23·8
$D^\pm$	6·2	0·2	3·5	4·1	5·5
D^0	4·0	3·0	2·2	2·6	3·3
$D_s^\pm$	6·3	4·0	3·5	4·2	5·8
$B^\pm$	2·6	2·0	1·4	1·7	2·3
B_d^0	6·0	5·0	3·4	3·9	5·0
B_s^0	6·9		3·9	4·4	5·5
$B_c^\pm$	0·5		0	0·4	2·0
T_u^0	2·6		1·5	1·7	2·1
$T_d^\pm$	4·5		2·5	3·0	4·0
$T_s^\pm$	4		3·3	2·9	27·0
T_c^0	5·9		0	0	0

where N is the normalization constant and ε is defined by

$$\varepsilon = 1 - \sqrt{1 - (4\alpha_s/3)}. \tag{26}$$

The corresponding momentum transform is

$$\psi^{\mathrm{Rel}}(Q) = \frac{N\sqrt{2a_0}\Gamma(2-\varepsilon)}{\pi Q_i} \times \left[\frac{\sin(2-\varepsilon)\theta_i}{(1+a_0^2Q_i^2)^{(2-\varepsilon)/2}} - \frac{\mu b a_0^3(3-\varepsilon)(2-\varepsilon)(4-\varepsilon)\theta_i}{2(1+a_0^2Q_i^2)^{(4-\varepsilon)/2}}\right], \tag{27}$$

where

$$\theta_i = \sin^{-1}\frac{Q_i}{\left(\frac{1}{a_0^2} + Q_i^2\right)^{1/2}} \tag{28}$$

and Q_i is defined in (22)

For small Q^2, (i.e. for small Q_i)

$$\sin^{-1}\frac{Q_i}{\left(\frac{1}{a_0^2 + Q_i^2}\right)^{1/2}} \cong \frac{Q_i}{\left(\frac{1}{a_0^2} + Q_i^2\right)^{1/2}}, \tag{29}$$

implying that θ_i defined in (28) will be small for small Q_i. Hence, for low Q^2, corresponding to small value of Q_i, $\psi^{\mathrm{Rel}}(Q)$ reduces to

$$\psi^{\mathrm{Rel}}(Q) = \sum \frac{N a_0^{3/2}\sqrt{2}(2-\varepsilon)\Gamma(2-\varepsilon)}{\pi(1+a_0^2Q_i^2)^{(3-\varepsilon)/2}}\left[1 - \frac{(4-\varepsilon)(3-\varepsilon)\mu b a_0^3}{2(1+a_0^2Q_i^2)}\right]. \tag{30}$$

In this case, confinement will be perturbation only if

$$\sum_i e_i \frac{(4-\varepsilon)(3-\varepsilon)\mu b a_0^3}{2(1+a_0^2Q_i^2)} \ll 1. \tag{31}$$

The value of Q_0^2 is now dependent on ε besides quark flavour as recorded in table 1.

2.6 *Spin effect*

It is observed that the masses of the hadrons with same quark content but different in spin are different. For instance the vector mesons (ρ, K^* etc.) are heavier than the pseudoscalar mesons (π, K etc.) despite having the same quark content. The difference in their masses must be attributed to a spin–spin interaction, the QCD analog to hyperfine splitting in the ground state of hydrogen. In our approach, we attempt to incorporate spin effect from the observed meson masses in the spirit of variation method [22].

The energy shift of mass splitting due to spin interaction in the perturbation theory reads [23],

$$\Delta E = \int \psi^* \left\{\frac{32\pi\alpha_s}{9}\delta^3(r)\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}\right\}\psi \mathrm{d}^3 r \tag{32}$$

leading to

$$\Delta E = \frac{32\pi\alpha_s}{9m_i m_j}(\mathbf{S}_i\cdot\mathbf{S}_j)|\psi(0)|^2. \tag{33}$$

Taking this energy shift into account, the pseudoscalar meson mass is expressed as [24]

$$M_P = m_i + m_j - \frac{8\pi\alpha_s}{3m_i m_j}|\psi(0)|^2 \tag{34}$$

$$\text{since } (\mathbf{S}_i\cdot\mathbf{S}_j) = -\tfrac{3}{4} \text{ for pseudoscalar mesons} \tag{35}$$

Let us now estimate the effect of spin in $\psi(0)$ and construct $\psi(r)$ using the variation method. The wave function (20) can be rewritten as

$$\psi(r) = \psi(0)\left(1 - \frac{1}{2}\mu b a_0 r^2\right)\exp(-r/a_0) \tag{36}$$

where

$$\psi(0) = \frac{1}{[\pi a_0^3(1 - 3\mu b a_0^3 + (45/8)\mu^2 b^2 a_0^6)]^{1/2}}. \tag{37}$$

The spin effect can then be incorporated in (35) by modifying the value of α_s through a_0,

$$a_0^P = \frac{1}{4/3\mu\alpha_s^P} \tag{38}$$

so that

$$\psi(0) = \frac{1}{[\pi a_0^{P^3}(1 - 3\mu b a_0^{P^3} + (45/8)\mu^2 b^2 a_0^{P^6}]^{1/2}}. \tag{39}$$

α_s^P is then obtained from the fitting of $\psi(0)$ to the pseudoscalar meson mass equation (34), which is different for different mesons.

The spin modified wave function therefore takes the form

$$\psi^P(r) = \frac{1}{[\pi a_0^{P^3}(1 - 3\mu b a_0^{P^3} + (45/8)\mu^2 b^2 a_0^{P^6}]^{1/2}} \times \left(1 - \frac{1}{2}\mu b a_0^P r^2\right)\cdot\exp(-r/a_0^P). \tag{40}$$

2.7 *Spin-cum-relativistic effects*

The spin effect depends on the wave function at the origin, $\psi(0)$. But as $r \to 0$, relativistic wave function (25) develops a $r^{-\varepsilon}$ behaviour for fixed α_s. As a result, in the fixed α_s approximation used in the present work, $\psi^{\mathrm{Rel}}(0)$ is undefined and hence spin effect cannot be introduced together with its relativistic counterpart. We consider it as an inherent limitation of the present formalism.

In order to circumvent the problem, we use the additional ansatz that due to asymptotic freedom $\varepsilon \to 0$ as $r \to 0$, so that relativity does not modify the wave function at the origin and $\psi(0)$ is finite. To see the effect of spin in such a constrained wave function, we again use the variation method discussed earlier. If the spin modified value a_0^P in (38) is such that $\alpha_s^P > \frac{3}{4}$, then ε [eq. (26)] becomes complex and the formalism might even break down.

We will study such possibilities in our subsequent analysis and discuss critically its limitations.

2.8 *Properties of mesons*

2.8.1 *Form factors*: The elastic charge form factor for a charged system of point quarks [25] is defined as

$$F(Q^2)=\frac{1}{e}\int\psi^+(\tau)\sum_{i=1}^{n}e_i\exp(i\mathbf{Q}_i\mathbf{r}_i)\psi(\tau)\mathrm{d}\tau \tag{41}$$

in which e is the total charge, e_i and r_i are the charge and position vector of the ith quark, and $\psi(\tau)$ is the model eigenvector solution, collectively denoting all internal coordinates. Equation (41) can be recast in a simplified form [26]:

$$eF(Q^2)=\sum_i\frac{e_i}{Q_i}\int_0^\infty r|\psi(r)|^2\sin Q_i r\,\mathrm{d}r. \tag{42}$$

With eq. (20), (42) becomes

$$eF(Q^2)=\frac{4N^2}{a_0^3}\sum_i\frac{e_i}{Q_i}\int_0^\infty\left(1-\frac{1}{2}\mu b a_0 r^2\right)\exp(-2r/a_0)r\sin(Q_i r)\mathrm{d}r \tag{43}$$

which after normalization, becomes

$$\begin{aligned}eF(Q^2)=\sum_i&\frac{e_i}{(1-3\mu b a_0^3+(45/8)\mu^2b^2a_0^6)}\\&\times\left[\frac{1}{(1+(a_0^2Q_i^2/4))^2}-\frac{3\mu b a_0^3(1-(a_0^2Q_i^2/4))}{(1+(a_0^2Q_i^2/4))^4}\right.\\&\left.+\frac{15\mu^2b^2a_0^6}{8(1+(a_0^2Q_i^2/4))^4}\left(3-\frac{4a_0^2Q_i^2}{(1+(a_0^2Q_i^2/4))}-\frac{a_0^4Q_i^4}{(1+(a_0^2Q_i^2/4))^2}\right)\right]\end{aligned} \tag{44}$$

Spin effect will modify a_0 to a_0^P as noted earlier.

Incorporating relativistic effect, (43) becomes

$$eF(Q^2)=\sum\frac{4N'^2e_i}{Q_i a_0^{3-2\varepsilon}}\int_0^\infty\left(1-\frac{1}{2}\mu b a_0 r^2\right)^2 r^{1-2\varepsilon}\sin(Q_i r)\exp(-2r/a_0)\mathrm{d}r \tag{45}$$

where N' is the normalization constant. After performing the integration, it has the form

$$\begin{aligned}eF(Q^2)=\frac{N'^2}{a_0 2^{-2\varepsilon}}\sum_i\frac{e_i}{Q_i}&\left[\frac{\Gamma(2-2\varepsilon)\sin(2-2\varepsilon)\theta_i}{(1+(a_0^2Q_i^2/4))^{(1-\varepsilon)}}\right.\\&-\frac{1}{4}\mu b a_0^3\frac{\Gamma(4-2\varepsilon)\sin(4-2\varepsilon)\theta_i}{(1+(a_0^2Q_i^2/4))^{(2-\varepsilon)}}\\&\left.+\frac{1}{64}\mu^2b^2a_0^6\frac{\Gamma(6-2\varepsilon)\sin(6-2\varepsilon)\theta_i}{(1+(a_0^2Q_i^2/4))^{(3-\varepsilon)}}\right].\end{aligned} \tag{46}$$

As noted in (29) small θ_i corresponds to low Q^2. In this limit, (46) becomes

$$eF(Q^2) \simeq \sum \frac{N^2 e_i}{2^{-2\varepsilon}} \left[\frac{\Gamma(2-2\varepsilon)(2-2\varepsilon)}{2(1+a_0^2 Q_i^2/4))^{(3/2-\varepsilon)}} - \frac{1}{4}\mu b a_0^3 \frac{\Gamma(4-2\varepsilon)(4-2\varepsilon)}{2(1+(a_0^2 Q_i^2/4))^{(5/2-\varepsilon)}} + \frac{1}{64}\mu^2 b^2 a_0^6 \frac{\Gamma(6-2\varepsilon)(6-2\varepsilon)}{2(1+(a_0^2 Q_i^2/4))^{(7/2-\varepsilon)}} \right]. \tag{47}$$

In perturbative QCD, the running coupling constant $\alpha_s(Q^2)$ is defined as

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2N_f)\ln Q^2/\Lambda^2} \tag{48}$$

where N_f = number of flavours.

For very low $Q^2, Q^2 \to 0$, (48) diverges and hence several extrapolated forms have been proposed in recent literature. In the work of Godfrey and Isgur [5], $\alpha_s(Q^2)$ is defined as,

$$\alpha_s(Q^2) = 0{\cdot}25 e^{-Q^2} + 0{\cdot}15 e^{-Q^2/10} + 0{\cdot}2 e^{-Q^2/1000} \tag{49}$$

so that $\alpha_s \leqslant 0{\cdot}6$ yielding $\varepsilon \leqslant 0{\cdot}4$.

There is another hypothesis for such extrapolation of $\alpha_s(Q^2)$ for low Q^2. With the idea of "frozen coupling constant" Ji and Amiri [27] as well as Cornwall [28] assume

$$\alpha_s(Q^2) = \frac{4\pi}{(33-2N_f)\ln[(Q^2+4m_g^2)/\Lambda^2]} \tag{50}$$

where m_g is interpreted as an effective dynamical gluon mass. With a value of m_g about 0·5 GeV/C and Λ of the order of 0·2 GeV [29], $\alpha_s(0) \sim 0{\cdot}5$ and $\varepsilon \sim 0{\cdot}25$.

On the other hand, if $(Q^2)^{-1}$ fall is expected [30, 31] one needs $\alpha_s \sim 0{\cdot}65$ which leads to $\varepsilon \sim 0{\cdot}5$.

Equation (47) shows that confinement effect invariably introduces higher power of $1/Q^2$ in the form factor and modify the nonasymptotic behaviour. In figure 1 we show the prediction of (44) for pion form factor with $b = 0, 0{\cdot}07, 0{\cdot}1$ and $0{\cdot}183\ \mathrm{GeV}^2$ and using $a_0^\pi = 3{\cdot}03\ \mathrm{GeV}^{-1}$. Data are taken from Amendolia *et al* [32]. It shows that the present formalism falls short of accommodating reasonably large confinement effect in form factor. In figure 2 we show similar analysis for kaon using $a_0^K = 2{\cdot}91\ \mathrm{GeV}^{-1}$ and compare with more recent data of Amendalia *et al* [33]. The results are similar to figure 1.

2.8.2 *Charge radii.* The average charge radii square for mesons can be obtained using the relation

$$\langle r^2 \rangle = -6 \left. \frac{\mathrm{d}F(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0} \tag{51}$$

which yields for masses m_i and m_j

$$\langle r^2 \rangle = \frac{3n}{2} a_0^2 g_{\mathrm{conf}} \left(\frac{e_i}{(1+(m_i/m_j))^2} + \frac{e_j}{(1+(m_j/m_i))^2} \right) \tag{52}$$

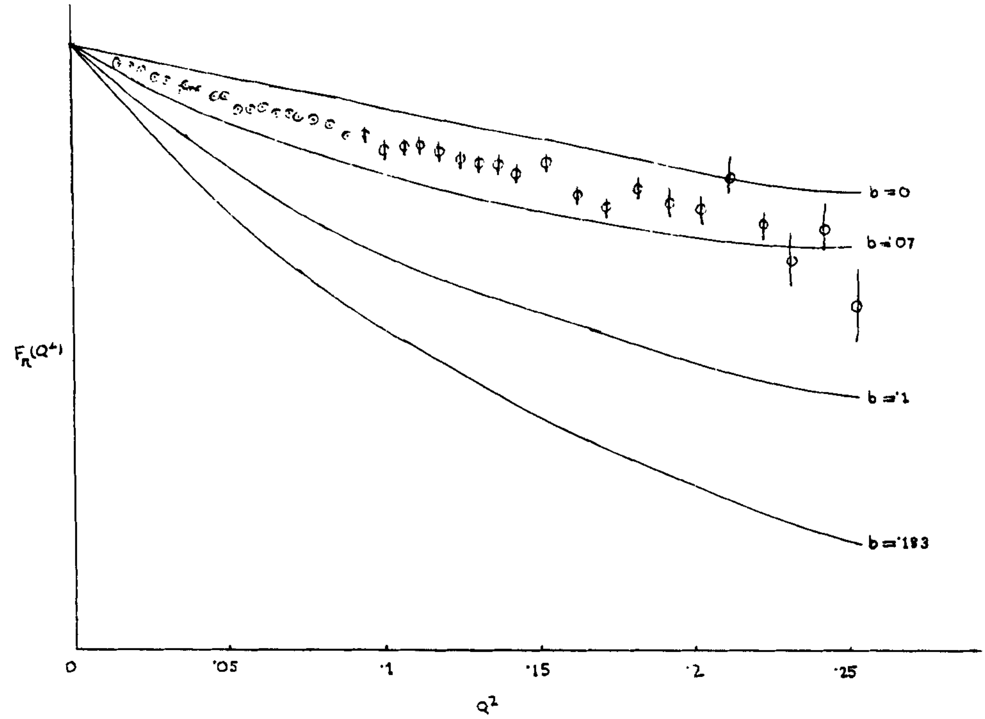

Figure 1. $F_\pi(Q^2)$ vs Q^2 for confinement parameter for $b = 0, 0{\cdot}07, 0{\cdot}1$ and $0{\cdot}183\,\mathrm{GeV}^2$ using (41) and $a_0^\pi = 3{\cdot}03\,\mathrm{GeV}^{-1}$. Data are taken from Amendolia *et al* [32].

when $n = 1$, $1{\cdot}1$ and $1{\cdot}25$ correspond to $\varepsilon = 0{\cdot}5$, $0{\cdot}4$ and $0{\cdot}25$ respectively. In non-relativistic case, $n = 2$ and $a_0 \to a_0^P$. In (52) g_{conf} is the confinement factor given by

$$g_{\mathrm{conf}} = \frac{1 - (15/2)\mu b a_0^3 + (105/4)\mu^2 b^2 a_0^6}{1 - 3\mu b a_0^3 + (45/8)\mu^2 b^2 a_0^6} \quad (n = 2) \tag{53a}$$

$$= \frac{1 - 3\mu b a_0^3 + (45/8)\mu^2 b^2 a_0^6}{1 - (3/2)\mu b a_0^3 + (15/8)\mu^2 b^2 a_0^6} \quad (n = 1) \tag{53b}$$

$$= \frac{1 - 3{\cdot}35\mu b a_0^3 + 6{\cdot}76\mu^2 b^2 a_0^6}{1 - 1{\cdot}76\mu b a_0^3 + 2{\cdot}4\mu^2 b^2 a_0^6} \quad (n = 1{\cdot}1) \tag{53c}$$

$$= \frac{1 - 3{\cdot}72\mu b a_0^3 + 8{\cdot}45\mu^2 b^2 a_0^6}{1 - 2{\cdot}19\mu b a_0^3 + 3{\cdot}38\mu^2 b^2 a_0^6} \quad (n = 1{\cdot}25) \tag{53d}$$

With spin effect, a_0 of (53a) changes to a_0^P [eq. (38)], using the variational method [20]. For (53b–d), however, such modification is not possible as the present formalism falls short of accommodating spin and relativity simultaneously, as discussed earlier.

In figure 3 we plot g_{conf} vs b for pion using eqs (53a–d). The dashed region correspond to the allowed range of g_{conf} using the data of [25], which suggests $b \leqslant 0{\cdot}07\,\mathrm{GeV}^2$. A similar analysis for kaon in figure 4 suggests $b \leqslant 0{\cdot}05\,\mathrm{GeV}^2$ using data of [33]. Hence as in our previous analysis of form factors, charge radii too severely constrain confinement parameter b.

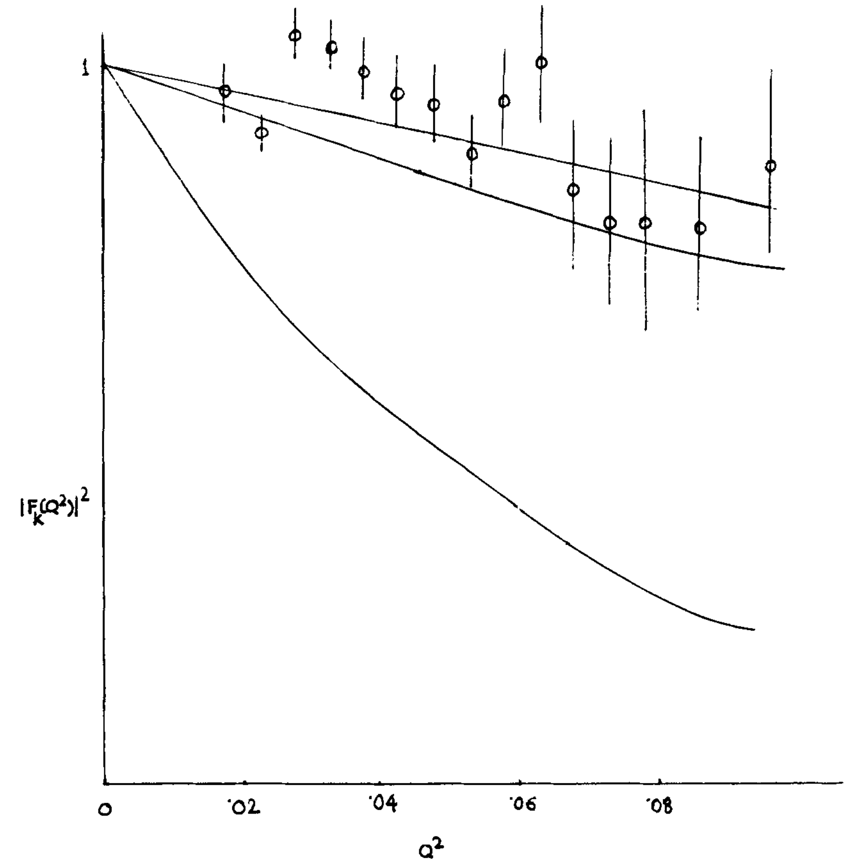

Figure 2. $|F_k(Q)|^2$ vs Q^2 for confinement parameter $b = 0$, 0·05 and 0·183 GeV^2 with $a_0^K = 2{\cdot}91\,\text{GeV}^{-1}$ compared to data of Amendolia *et al* [33].

2.8.3 *Decay constants*: The standard expression for pseudoscalar meson decay constant in non-relativistic quark model [34] is

$$f_P = \sqrt{\frac{12}{M_P}|\psi(0)|^2} \tag{54}$$

where $\psi(0)$ is the wave function at $r = 0$ and M_P is the mass of the pseudoscalar meson. The wave function at the origin is given by (39). Using spin averaged meson masses from [35, 36] in (54) and taking $0 \leqslant b \leqslant 0{\cdot}07\,\text{GeV}^2$, we obtain

$$151 \geqslant f_\pi \geqslant 109\,\text{MeV} \tag{55}$$

while for $0 \leqslant b \leqslant 0{\cdot}05\,\text{GeV}^2$, we get

$$119 \geqslant f_k \geqslant 99\,\text{MeV}. \tag{56}$$

Equations (55) and (56) show that confinement effect decreases the decay constants, the upperbound corresponding to $b = 0$.

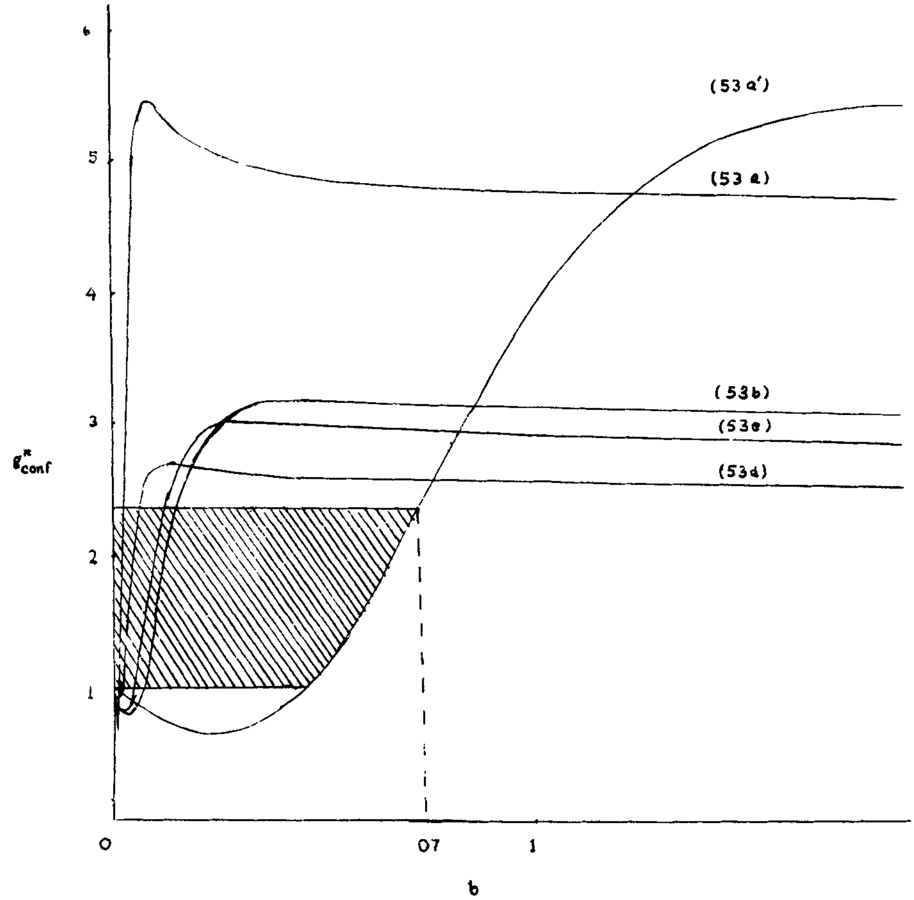

Figure 3. g^{π}_{conf} vs b for pion defined in (50a–d). (50a′) means a_0 changes to a_0^P. Data used are from ref. [25].

The weak decay constant f_{D_s} is presently a topical problem in heavy flavour physics [35, 37, 38]. Taking the physical mass $M_{D_s} \cong 1{\cdot}9688\,\text{GeV}$ in (54), we obtain $f_{D_s} \leqslant 237\,\text{MeV}$, upper limit again corresponding to $b = 0$ to be compared with the results of other authors [5, 34, 38–52] as given in table 2. Recent experiment reported average of f_{D_s} and $f^*_{D_s}$ as [35]

$$f_{D_s}(\cdot) \approx 267 \pm 28\,\text{MeV}. \tag{57}$$

However there are no data on $\langle r^2 \rangle_{D_s}$ to constrain b as in pion or kaon. The experimental limit $f_D \leqslant 310\,\text{MeV}$ [53] also cannot yield $f_D \leqslant 209\,\text{MeV}$, the upper limit corresponds to $b = 0$. It is to be compared with the results of other authors [27, 34, 38–41, 44, 52, 54, 55] as given in table 3.

There are no reliable data on f_B as well. In calculating $B_d^0 - \bar{B}_d^0$ mixing parameter, widely accepted value of f_B is [56]

$$100 < f_B < 200\,\text{MeV}. \tag{58}$$

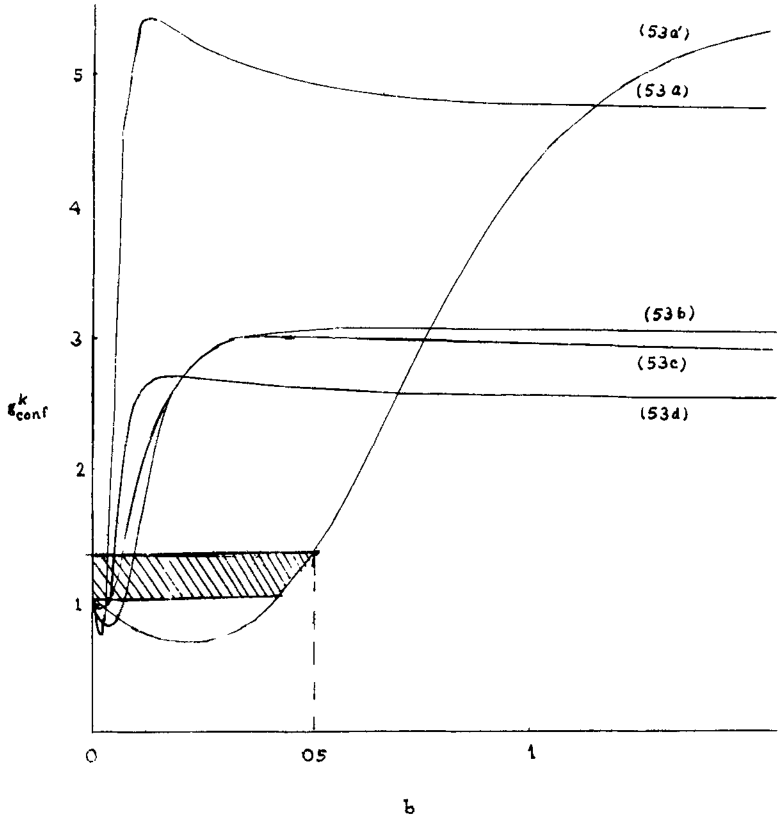

Figure 4. g^{K}_{conf} vs b for kaon using data of ref. [33].

Table 2. Compilation of some theoretical estimates for the weak decay constants f_{D_s} from potential models, bag models, sum rules and lattice calculations. The values are given in MeV.

Potential model	Bag model	Sum rules	Lattice
260 [34]	166 [43]	~232 [44]	215 ± 17 [48]
149 [39]		276 ± 13 [45]	157 ± 11 [49]
210 [40]		218 ± 20 [46]	234 ± 72 [50]
380–590 [38]		200 ± 15 [47]	280 [51]
356 [41]			209 ± 18 [52]
199 [42]			
290 ± 20 [5]			

However, several theoretical estimates of f_B have been made within potential models, lattice QCD, factorization and QCD sum rule approach as compiled in table 4. Taking $M_B \sim 5{\cdot}2786\,\text{GeV}$ in (54) we obtain $f_B \leqslant 107\,\text{MeV}$; upper limit again corresponds to $b = 0$.

Table 3. Compilation of some theoretical estimates for the weak decay constant f_D from potential models, sum rules and lattice calculations. The results are given in MeV.

Potential model	Sum rules	Lattice
112–141 [36]	290 [43]	198 ± 17 [52]
200 [34]	170 ± 30 [35]	
139 [39]		
112–137 [40]		
360–580 [38]		
281 [41]		
150 [54]		

Table 4. Compilation of some theoretical estimates for the weak decay constant f_B from potential models, QCD sum rules, factorization, lattice QCD calculations. The values are given in MeV.

Potential model	Sum rules	Factorization	Lattice
120 [34]	290 [44]	150 ± 50 [59]	366 ± 22 ± 55 [52]
93 [39]	190 ± 50 [55]		205 ± 40 [60]
75–114 [40]	200 ± 35 [57]		310 ± 25 ± 50 } [61]
260–300 [38]	170 ± 20 [58]		233 ± 42 } [61]
229 [41]	140 [45]		
< 100 [54]			

3. Summary and conclusion

In this paper, we have studied the form factors, charge radii and decay constants of pseudoscalar mesons within a chromodynamic potential model discussed in the text. We have also attempted to incorporate spin, relativity and confinement within our analysis. While, relativity and spin could not be introduced simultaneously, only mild confinement effects are found permissible within our formalism.

However, while confronting with experiments, we observe that two variants of the models still survive: the non-relativistic one with spin effect and the relativistic one with negligible confinement effect ($b \sim 0$) but having a large α_s, $\alpha_s \sim 0{\cdot}65$.

There are arguments against the dominance of perturbative QCD over soft, non-perturbative effects in exclusive processes at currently available range of Q^2 [32, 62, 63]. Within the present formalism, nonperturbative effects can be attributed to a large value of coupling constant α_s as Q^2 small or large value of string constant b. However, for large b the formalism itself breaks down. As a result, the low Q^2 limit of α_s is the only effective measure of nonperturbative effects in the present formalism. The analysis of the present paper indicates that invariably a large value of $\mathrm{Lt}_{Q^2 \to 0} \alpha_s(Q^2)$ is needed to make the relativistic version of the theory compatible with present experiments. It presumably hints the presence of such effect in the present analysis.

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