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Quantal behaviour of magnetically kicked electron

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Abstract. The problem of a free electron periodically kicked by a magnetic field has been solved. The system shows a transition from quantum recurrence to instability at $\omega T = 2$ where ω is the Larmor frequency and T is the period of the kick. The existence of recurrent behaviour amounts to the confinement of the electron by magnetic kicking. Since the theory holds for all types of charged particles, it has many practical applications.

Keywords. Quantum recurrence; magnetic confinement; Quantum instability wave function.

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1. Introduction

It is now well-known that the behaviour of a quantum system subject to time periodic potential is dependent on the nature of its quasienergy spectrum. If the quasienergy spectrum is discrete, the system is quantum mechanically recurrent whereas if the spectrum is continuous the quantal behaviour of system is unstable characterized by unbounded growth of energy [1–2]. There exist systems whose quasienergy spectrum makes a transition from discrete to continuous at a certain value of system parameter. Such systems are characterized by a transition from quantum mechanically recurrent to non-recurrent behaviour [3–5]. The non-recurrent behaviour is particularly interesting since it is here one may expect quantum instability. Some criteria have already been discussed according to which one may distinguish between quantum irregularity and quantum chaos. In the former the quasienergy spectrum is absolutely continuous whereas in the latter it is singularly continuous [6–7]. However most of the studies reported has so far been confined to one-dimensional systems [8–9].

It is obvious that more interesting possibilities are expected in systems having more than one degree of freedom. The different modes of such systems can be recurrent or unstable individually or collectively depending on the separability of the problem which in turn would depend on the choice of appropriate coordinate system. If the problem is not separable then one could imagine mixing between unstable and recurrent modes leading to newer types of behaviour. This has sufficient motivation for the study of more than one-dimensional problems. There are a very few such problems which find discussion in literature [10]. In the following sections we consider the two-dimensional problem of a magnetically kicked free electron. It has been shown that the quasienergy spectrum is determined by the product ωT where ω is Larmor frequency and T is the time period of the kick. The solution results in the magnetic confinement of the electron through appropriate kicking.

2. Wave function of electron after N kicks

Consider an electron of charge e and mass m subject to steady magnetic field $\mathbf{H}$. It is described by the Hamiltonian

$$\hat{H} = \frac{p_x^2 + p_y^2}{2m} + \left[\frac{e^2 A^2}{2mc^2} - \frac{e}{2mc}(\mathbf{A}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{A})\right] \tag{1}$$

where $\mathbf{p}$ is the linear momentum of free electron, $\mathbf{A}$ is the vector potential corresponding to magnetic field $\mathbf{H}$.

The first term represents the free particle Hamiltonian whereas the second term is the contribution of the magnetic field. We consider an electron in which the second term appears in the form of δ function kicks so that

$$\hat{H} = \frac{p_x^2 + p_y^2}{2m} + \left[\frac{e^2 A^2}{2mc^2} - \frac{e}{2mc}(\mathbf{A}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{A})\right]\sum_n \delta(t - nT) \tag{2}$$

where T is the time period of the kick. Assuming that the magnetic field is in z direction, (2) can be written in Coulomb gauge as

$$\hat{H} = \frac{p_x^2 + p_y^2}{2m} + \left[\frac{1}{2}m\omega^2(x^2 + y^2) - \omega L_z\right]\sum_n \delta(t - nT) \tag{3}$$

where $\omega = eH/2mc$ is the Larmor frequency of electron and L_z is the z component of orbital angular momentum.

The time evolution of the system is governed by one step Floquet operator

$$U = \exp\left[-\frac{i(p_x^2 + p_y^2)T}{2m\hbar}\right]\exp -\frac{iT}{\hbar}\left[\frac{1}{2}m\omega^2(x^2 + y^2) - \omega L_z\right]. \tag{4}$$

Using the technique developed by Blumel *et al* [3], it can be shown that

$$U = \exp -\left(\frac{iG_x T}{\hbar}\right)\exp -\left(\frac{iG_y T}{\hbar}\right)\exp\left(\frac{i\omega L_z T}{\hbar}\right) \tag{5}$$

where

$$G_x = \lambda\left[\frac{p_x^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \omega^2 T(xp_x + p_x x)\right] \tag{6}$$

$$\lambda = \sin^{-1}(r\omega T)/r\omega T \tag{7}$$

$$r^2 = 1 - \omega^2 T^2/4. \tag{8}$$

The wave function $|\psi_N\rangle$ of the system after N kicks is given by

$$|\psi_N\rangle = U^N|\psi_0\rangle \tag{9}$$

where $|\psi_0\rangle$ is the wave function of the electron at time $t = 0$. Since G_x, G_y and L_z are commuting operators, we may write (9) as

$$|\psi_N\rangle = \exp\left(\frac{iN\omega TL_z}{\hbar}\right)\exp\left(-\frac{iNG_x T}{\hbar}\right)\exp\left(-\frac{iNG_y T}{\hbar}\right)|\psi_0\rangle. \tag{10}$$

It can be further written as

$$|\psi_f\rangle = \exp\left(-\frac{iNG_xT}{\hbar}\right)\exp\left(-\frac{iNG_yT}{\hbar}\right)|\psi_0\rangle \tag{11}$$

where

$$|\psi_f\rangle = \exp\left(-\frac{iN\omega TL_z}{\hbar}\right)|\psi_N\rangle \tag{12}$$

In the position representation (11) may be written as

$$\psi_f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K(x,y;x',y')\psi_0(x',y')\mathrm{d}x'\mathrm{d}y' \tag{13}$$

where

$$K(x,y;x',y') = \left\langle x,y\left|\exp\left(-\frac{iNG_xT}{\hbar}\right)\exp\left(-\frac{iNG_yT}{\hbar}\right)\right|x',y'\right\rangle \tag{14}$$

is the propagator for the system and contains all information about the system.

The matrix element on the right side of (14) can be easily evaluated using well-known techniques and it turns out to be

$$K(x,y;x',y') = \left(\frac{mr\omega}{2\pi i\hbar\sin\eta}\right)\exp\left[\frac{-imr\omega}{2\hbar\sin\eta}(x^2+y^2+x'^2+y'^2)\cos\eta - 2(xx'+yy')\right]\exp[ib(x^2+y^2-x'^2-y'^2)] \tag{15}$$

where

$$b = m\omega^2T/4\hbar, \tag{16}$$

$$\eta = N\sin^{-1}(r\omega T). \tag{17}$$

We represent the free electron by two-dimensional wave packet of width σ at $t = 0$ by

$$\psi_0(x',y') = \frac{1}{(2\pi\sigma^2)^{1/2}}\exp\left[-\frac{(x'^2+y'^2)}{4\sigma^2}\right]\exp[i(k_xx'+k_yy')] \tag{18}$$

Using (18) and (15) in (13) and performing the integration in plane polar coordinates (ρ,ϕ) we obtain

$$\psi_f(\rho,\phi) = \left(-\frac{v}{2\pi\sigma^2}\right)^{\frac{1}{2}}\exp\left[-\frac{4\rho^2}{4\sigma^2}\right]\exp\left[-\frac{4\rho\sin\eta}{4br\sigma^2}(k_\rho\cos\phi + k_\phi\sin\phi)\right] \times\exp\left[-\frac{k^2v\sin^2\eta}{16b^2r^2\sigma^2}\right]\exp[-if(\rho,\phi)] \tag{19}$$

where

$$v = r^2\left[\frac{\sin^2\eta}{16b^2\sigma^4} + \left(r\operatorname{con}\eta + \frac{\omega T}{2}\sin\eta\right)^2\right]^{-1} \tag{20}$$

$$f(\rho,\phi) = bd\rho^2 - \frac{\rho va\sin\eta}{r}(k_\rho\cos\phi + k_\phi\sin\phi) + \tan^{-1}(4\sigma^2ba) - k^2va\sin^2\eta/4br^2 \tag{21}$$

Table 1. Variation of position vector of electron with kicks.

η	0	$\pi/3$	$2\pi/3$	π
ρ	$1{\cdot}54 \times 10^{-12}$	$10{\cdot}36 \times 10^{-31}$	$10{\cdot}36 \times 10^{-31}$	$1{\cdot}54 \times 10^{-12}$

$$d = a(1 - v) - \omega T \tag{22}$$

$$a = r \cot \eta + \omega T/2. \tag{23}$$

3. Energy and position of electron after N kicks

The energy of electron after N kicks is given by

$$E_N = \langle \psi_f | \hat{H}_0 | \psi_f \rangle \tag{24}$$

where

$$\hat{H}_0 = \frac{p_x^2 + p_y^2}{2m} \tag{25}$$

is free electron Hamiltonian.

Substituting the value of $\hat{H}_0$ in (24) we obtain

$$E_N = \frac{\hbar^2}{2m}\left[\frac{v}{2\sigma^2} + \frac{4b^2 d^2 \sigma^2}{v} + \frac{2k^2 \sin^2 \eta}{r^2}(av + d)^2\right]. \tag{26}$$

For $\omega T < 2$ the energy oscillates showing quantum recurrence whereas for $\omega T > 2$ the energy grows exponentially with a time scale proportional to the kicking strength.

The position of the electron after N kicks is given by

$$\langle \rho \rangle = \langle \psi_f | \rho | \psi_f \rangle. \tag{27}$$

Using (19) it takes the form

$$\langle \rho \rangle = \frac{\sqrt{\pi\sigma}}{(2v)^{1/2}} \exp\left(-\frac{k^2 v \sin^2 \eta}{8b^2 r^2 \sigma^2}\right) {}_1F_1\left(\frac{3}{2}, 1, -\frac{k^2 v \sin^2 \eta}{8b^2 r^2 \sigma^2}\right) \tag{28}$$

where ${}_1F_1$ is the degenerate hypergeometric function.

Table 1 shows the numerical values of ρ corresponding to different values of η for $\omega T = 1$. It is clear that ρ is periodic with period π.

4. Conclusions

The behaviour of the electron kicked magnetically has been investigated using Floquet theory. The propagator of electron after N kicks has been explicitly evaluated and it has been utilized to calculate the wave function after N kicks in plane polar coordinates. This wave function has been further used to find the energy and position vector of the electron.

It has been found that time evolution of electron is determined by the product of Larmor frequency ω and time period T of the kick. The energy of electron oscillates for $\omega T < 2$. This is a manifestation of phenomenon of quantum recurrence. This phenomenon of quantum recurrence is equivalent to magnetic confinement of electron.

The position vector of electron has been evaluated numerically for $\omega T = 1$ and $H = 10^4$ gauss. It has been found to be periodic function of η with period π. It deviates significantly from its original position during the first kick and then returns to its original after third kick. Similiar behaviour occurs for other values of $\omega T < 2$. This implies that electron can be localized magnetically by proper kicking.

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