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Fringe formation theory for real-time holographic interferometry using $Bi_{12}SiO_{20}$

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Abstract. A simple theory has been developed to explain the fringe formation in real-time holographic interferometry using BSO (bismuthsiliconoxide) crystal for interferometric applications. Detailed analysis has been given to choose the optimum size of the crystals.

Keywords. Photorefractive crystals; real-time holography.

PACS No. 42·40

1. Introduction

Photorefractive crystals are increasingly used as a recording medium for real-time holographic interferometry [1–7] due to their several advantages like non-requirement of developing and fixing. Also, the information can be processed in real-time by coupling the output with a computer and above all the medium is reusable unlike the conventional holographic recording media. The popular PR crystals used for real-time holographic interferometry are $Bi_{12}SiO_{20}$ (bismuthsiliconoxide), $Bi_{12}TiO_{20}$ (bismuthtitaniumoxide) and $Bi_{12}GeO_{20}$. Among these both BSO and BTO are used widely for real-time holographic interferometry since the former is very sensitive to bluish green (typically argon-ion laser wavelength) region and the latter to popular red wavelength region. These two PR crystals utilise anisotropic self-diffraction phenomena in which the diffracted beam polarization is rotated with respect to the transmitted beams. Many geometries have been proposed for recording real-time holograms in these crystals based on four-wave mixing and two-beam coupling. The four-wave mixing technique needs read out beam and some times with a different frequency and more optics but the two-beam coupling requires only simple geometry. The most efficient real-time holographic interferometry geometry was proposed by Kamshalin *et al* using BTO crystal sandwiched between two polarizers in a two-beam coupling geometry [4]. This geometry utilizes the anisotropic self-diffraction phenomena and later this was successfully implemented by Troth and Dainty using a BSO crystal for real-time holographic interferometry [7]. The theoretical explanation for hologram formation inside PR crystals was developed by many authors using Kogelnik's coupled mode theory utilizing volume hologram phenomena [2]. However no theory has been proposed so far for the fringe formation for real-time holographic interferometry in a PR crystal. This paper reports a fringe formation theory for real-time holographic interferometry utilizing anisotropicself-diffraction phenomena in BSO. This theory also predicts the optimum size of the crystals to be used for real-time holographic interferometric applications.

2. Fringe formation theory

The proposed fringe formation theory is derived in two ways. First by considering optical activity and secondly by ignoring it. For both the analyses a BSO crystal sandwiched between two polarizers as shown in figure 1 is considered. The BSO crystal is cut in the (110) crystal orientation with its $[1\bar{1}0]$ crystal direction parallel to the plane of incidence illuminating radiation.

2.1 *Consideration of optical activity of the crystal*

Let $E_0 e^{i\phi_o}$ be the object wave and $E_r e^{i\phi_r}$ be the reference wave forming the writing beams which incident on the BSO crystal. These two writing beams interfere inside the crystal to form the necessary phase grating which in turn produces the necessary refractive index change through diffusion mechanism. The transfer matrix for such a photorefractive crystal can be written as [3],

$$M_{tr}^1 = |\tilde{E}_0 + \tilde{E}_r| \begin{vmatrix} \cos At - i\sin\psi_0 \sin At & -\sin At \cos\psi_0 \\ \cos\psi_0 \sin At & \cos At + i\sin\psi_0 \sin At \end{vmatrix} \tag{2.1}$$

where, t is the thickness of the crystal; $A = [\rho^2 + \{(\Delta n)^2/4\}]^{1/2}$; $\psi = \tan^{-1}(\Delta n/2\rho)$; ρ is the optical activity per unit length and Δn is the birefringence per unit length.

The value of Δn for BSO crystal used in this analysis is $1{\cdot}49 \times 10^{-5}$ (calculation is shown in appendix).

The anisotropic self-diffraction phenomenon occurs in BSO/BTO type of crystals of sillenite family whenever the writing beams interfere in the crystal. Figure 2 shows both the diffraction phenomena namely isotropic and anisotropic self-diffraction. In BSO crystal the relative intensity of two writing beams creates a space charge field which in turn produces a refractive index grating $\pi/2$ phase shifted relative to the intensity. This also induces axes of linear birefringence with refractive indices $n_0 + \Delta n$ and $n_0 - \Delta n$. After the formation of hologram inside the crystal to read out, one of the writing beams itself is used by assuming that the other writing beam i.e. the signal beam is turned off. This assumption can be considered since the phase grating will exist for a time due to inherent inertia of the photo refractive effect. Figure 3 shows that the reference wave polarization (assuming it is vertical at the centre of the crystal) is decomposed into two components polarized in the c_1 and c_2 directions. These orthogonal components will diffract off the grating polarized in the same directions, but with a phase delay between them. Consider the phase delays of the c_1 and c_2 relative to the reading wave. The refractive index that the c_1 and c_2 components of the read out wave see is different, due to induced birefringence, and are equal in magnitude but opposite in sign. Thus the phase difference between the two orthogonal components of diffraction is $+\pi/2 - (-\pi/2) = \pi$. Referring to figure 3, supposing that the polarization component of the reference wave in the c_1 direction $E_r(c_1)$ diffracts from the $n_0 - \Delta n$ grating with the same sense of polarization $E_r(c_1)$ at a particular instant. But, the polarization component in the c_2 direction $E_r(c_2)$ which is out of phase by π with respect to $E_r(c_1)$. The net resultant polarization of the diffracted reference wave is orthogonal to the input polarization. This is not so in the case of conventional holographic reconstruction. The advantage of such diffraction phenomenon is that it eliminates completely the background noise while forming the hologram. The above phenomenon can be derived from eq. (1) in the following way. Let the reference wave $E_r e^{i\phi_r}$ reads out the recorded

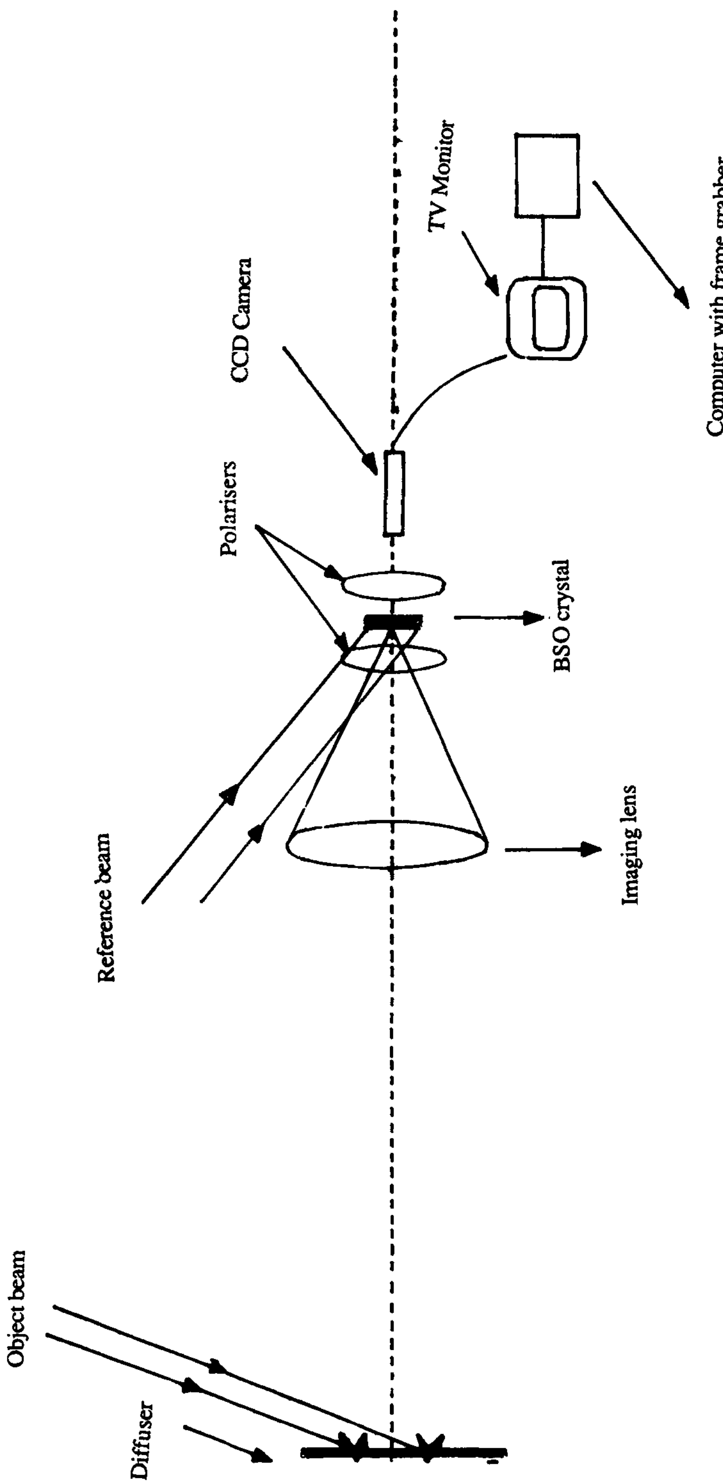

Figure 1. Shows the schematic of the experimental set-up used for this analysis.

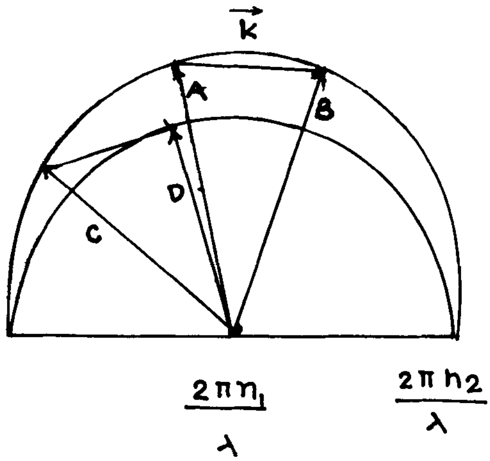

Figure 2. A, B shows the diagram of the isotropic diffraction and C, D shows the anisotropic self-diffraction phenomena.

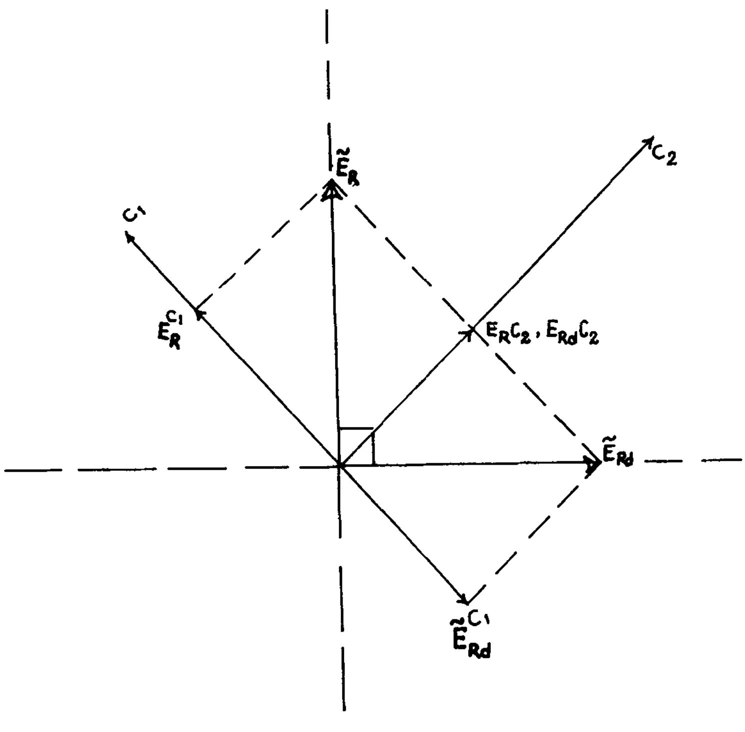

Figure 3. Shows the read out mechanism of anisotropic self-diffraction.

hologram (phase grating) with its vertical polarization at the centre of the crystal and assuming the signal beam is turned off momentarily. The output beam immediately after the crystal and just before the output polarizer is,

$$E^1_{Rd} = |\tilde{E}_0 + \tilde{E}_r| \begin{vmatrix} \cos At - i\sin\psi_0 \sin At & -\sin At\cos\psi_0 \\ \cos\psi_0 \sin At & \cos At + i\sin\psi_0 \sin At \end{vmatrix} \begin{vmatrix} 0 \\ \tilde{E}_r \end{vmatrix}. \tag{2.2}$$

The amplitude of the transmitted residual beam just outside the crystal is,

$$E^1_{Rd} = \begin{vmatrix} 0 & 0 \\ 0 & \cos At + i\sin\psi_0 \sin At \end{vmatrix} (E_0^2 + E_r^2)E_r e^{i\phi_r}$$
$$+ \begin{vmatrix} -\sin At \cos\psi_0 (E_0 e^{i\phi_0} + E_0 e^{-i\phi_0}) & 0 \\ 0 & 0 \end{vmatrix}. \quad (2.3)$$

Where the first term is the diffracted beam and the second term is the transmitted residual amplitude part. To obtain the transmitted residual amplitude the input and output polarizers are to be crossed and mathematically this can be written from (2.3) as,

$$E^1_{Rd} = \left(\begin{vmatrix} 0 & 0 \\ 0 & \cos At + i\sin\psi_0 \sin At \end{vmatrix} (E_0^2 \tilde{E}_r + E_r^2 \tilde{E}_r) \right) \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$$
$$+ \left(\begin{vmatrix} -\sin At \cos\psi_0 [(E_0 e^{i\phi_0}) + (E_0 e^{-i\phi_0})] & 0 \\ 0 & 0 \end{vmatrix} \right) \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}. \quad (2.4)$$

The residual transmitted amplitude of the object beam can be written from the above matrix as,

$$|E^1_{Rd}| = (\sin At \cos\psi_0) E_0 e^{i\phi_0}. \quad (2.5)$$

Equation (2.5) represents the holographic image of the object alone and in real-time holographic interferometry, there will be a change in the phase of the object beam due to deformations/displacements of the test object as soon as the holographic image is formed then, in such a case eq. (2.5) will become,

$$|E^{11}_{Rd}| = (\sin At \cos\psi_0) E_0 e^{i\phi'_0} \quad (2.6)$$

Adding (2.5) and (2.6) the resultant transmitted residual intensity after complete cancellation of the diffracted beams is,

$$I_{Rd} = 2E_0^2 \sin^2 At \cos^2\psi_0 (1 + \cos^2\delta_0) \quad (2.7)$$

where $\delta_0 = (\phi_0 - \phi'_0)$.

The maximum intensity for the holographic interferometric fringes in conventional holographic interferometry depends upon the values of δ_0, that is the phase changes occurring in the object beam before and after in the ambient state of the object. In dynamic holographic interferometric case, it is clear from (2.7) that in addition to the value of δ_0 the intensity of the interferometric fringes depends upon the values of optical activity (ρ) and the birefringence value of the crystal. Figure 4 shows the plot between the thickness of the crystal and the visibility of the holographic interferometric fringes. Visibility is defined as $V = (I_{rd\,max} - I_{rd\,min})/(I_{rd\,max} + I_{rd\,min})$ for holographic interferometric fringes and the maximum value is one. It is clear from figure 4 that for obtaining maximum visibility the optimum thickness of the PR crystal (here BSO) should be between 1·5 mm and 3·5 mm for different values of optical activity in addition to the value of δ_0 where it should be $m\pi$ (m is an integer).

2.2 *Ignoring optical activity of the crystal*

The fringe formation theory can be derived by ignoring the optical activity of the

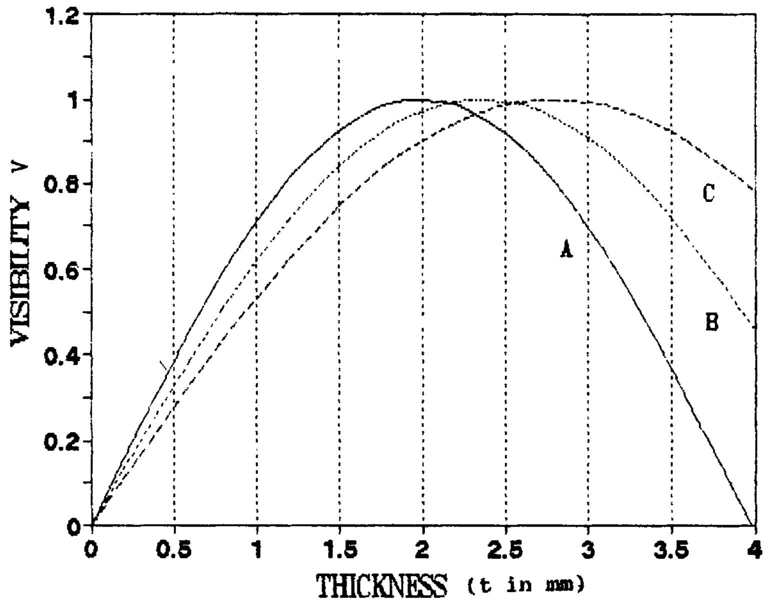

Figure 4. Shows the plot between visibility V and the thickness t of the BSO crystal for a constant value of $\lambda = 514$ nm. Curve A, $\rho = 45°/\text{mm}$; curve B, $\rho = 38°/\text{mm}$ and curve C, $\rho = 32°/\text{mm}$.

crystal. The transfer matrix of eq. (1) after neglecting the value of optical activity will be,

$$M_{tr}^2 = |\tilde{E}_0 + \tilde{E}_r| \begin{vmatrix} \cos \mathrm{A}t & -\sin \mathrm{A}t \\ \sin \mathrm{A}t & \cos \mathrm{A}t \end{vmatrix} \tag{3.1}$$

where $\rho = 0$, $\mathrm{A} = \{(\Delta n)^2/4\}^{1/2}$ and $\Delta n =$ the birefringence per unit length. Now when the read out beam enters the crystal the output beam just before the output polarizer is,

$$E_{Rd}^2 = \begin{vmatrix} 0 & 0 \\ 0 & \cos \mathrm{A}t \end{vmatrix} (E_0^2 + E_r^2)\tilde{E}_r + \begin{vmatrix} -\sin \mathrm{A}t(E_0 e^{i\phi_0} + E_0 e^{-i\phi_0}) & 0 \\ 0 & 0 \end{vmatrix}. \tag{3.2}$$

The transmitted residual amplitude of the object beam after rotating the output polarizer will be

$$|E_{Rd}^2| = E_0 \sin \mathrm{A}t e^{i\phi_0}. \tag{3.3}$$

The final resultant transmitted intensity after the phase changes in the object beam is

$$I_{Rd}^2 = 2E_0^2 \sin^2 \mathrm{A}t(1 + \cos^2 \delta_0) \tag{3.4}$$

The maximum intensity of the residual transmitted intensity can be obtained only when $\mathrm{A}t = \pi/2$ in addition to the value of δ_0. Substituting the value of A and assuming $\delta_0 = \pi$, the thickness of the BSO crystal needed is high to get the maximum visibility of the holographic interferometric fringes. This contradicts our earlier theoretical prediction by considering the optical activity that the optimum thickness of the crystal required is about 1·5 mm to 3·5 mm. Also, Troth *et al* [9] have reported that the thickness of the

BSO crystal used in their experiment on holographic interferometry to study the noise and sensitivity is 2·25 mm for a value of optical activity $\rho = 38{\cdot}7^\circ\,\mathrm{mm}^{-1}$ for $\lambda = 514$ nm and this agrees with the theory. Another important point is that the maximum thickness of commercially available and good optical quality BSO and BTO crystals (which are commonly used for holographic interferometry) is between 1·5 mm and 3·5 mm. The above results show that the optical activity of the PR crystal is very important in BSO/BTO type of crystals to do real-time holographic interferometry. The results also show that even by considering the optical activity, the BSO crystal acts as a birefringent/half wave plate.

3. Conclusion

A simple theory to obtain real-time holographic interferometric fringes using BSO crystal sandwiched between two polarizers is given and this theory also predicts the optimum size of the crystal required. This theory also shows that by ignoring optical activity the BSO crystal will act as a birefringent plate only when the size of the crystal is large.

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Appendix

Calculation for finding Δn for the BSO crystal is used in this analysis. The formula for Δn following [2, 8] is,

$$\Delta n = n_0^3 r_{41}\left[\sqrt{\beta_0}\frac{\varepsilon_D \varepsilon_q}{(\varepsilon_q + \varepsilon_D)(1+\beta_0)}\right]$$

$$\varepsilon_D = \frac{K_B T K_g}{e}, \quad \varepsilon_q = \frac{e N_A}{\varepsilon \varepsilon_0 K_g} \tag{A1}$$

where K_B is the Boltzmann constant, T the temperature in Kelvin, e the electric charge of an electron, ε and ε_0 are the unperturbed permittivity of the material and the permittivity of the free space respectively and N_A is the trap number density in the crystal volume, r_{41} is the non-zero electro optic coefficient for crystals of sillenite family, β_0 is the signal to reference beam intensity ratio, n_0 is the unperturbed refractive index and K_g is grating spatial frequency.

The values of the constants are,

$$n_0 = 2{\cdot}165,\ r_{41} = 4{\cdot}51\,\mathrm{pm/V},\ K_B = 1{\cdot}380 \times 10^{-23}\,\mathrm{J/K},$$

$$T = 300\,\mathrm{K},\ N_A = 1{\cdot}27 \times 10^{22}/\mathrm{m}^3,\ e = 1{\cdot}602 \times 10^{-19}\,\mathrm{C},$$

$$K_g = 12{\cdot}6\,\mu\mathrm{m}^{-1},\ \varepsilon_0 = 8{\cdot}854 \times 10^{-12}\,\mathrm{F/m},\ \varepsilon = 56,$$

$$\beta_0 = 1.$$

Substituting the values of constants in (A1) the value of $\Delta n = 1{\cdot}49 \times 10^{-5}$.

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