PRAMANA
— journal of
physics

© Printed in India

Vol. 44, No. 4,
April 1995
pp. 357–374

S-wave baryons in an equally mixed scalar-vector square root potential model of independent quarks

S N JENA and M R BEHERA
Department of Physics, Berhampur University, Berhampur 760007, India

MS received 18 January 1994; revised 28 November 1994

Abstract. A relativistic model of independent quarks based on Dirac equation with an equally mixed scalar-vector square root confining potential is used to compute the quark core contributions to the static properties like magnetic moments, charge radii and axial vector coupling constant ratios of the baryon octet. The results obtained with appropriate corrections due to centre-of-mass motion agree fairly well with experimental values. The model is also extended to the study of magnetic moments of the quark core of baryons in the charmed and b-flavoured sectors and the overall predictions so obtained compare well with other model predictions.

Keywords. Dirac equation; linear potential; baryon octet; magnetic moments; charge radii; axial vector coupling constant; charmed and b-flavoured baryons; centre-of-mass correction; Qq mesons.

PACS No. 13·40

1. Introduction

Most of the theoretical and phenomenological models constructed in connection with the study of the heavy mesons of Ψ and Υ-families are based on the assumption of the non-relativistic motion of quarks inside hadrons. The two body interquark force believed to be flavour independent is generally described by the traditional coulomb plus linear potential [1–3] obtained by a simple superposition of both the asymptotic limits. Although one expects QCD to be the theory underlying this description, one cannot as yet derive such a potential completely from first principles. Therefore, various kinds of phenomenological potentials [1] have been suggested in the process, each having its own theoretical or phenomenological motivations and all of them have been found to be more or less successful in describing the mass spectra and decay widths etc. of the heavy mesons. These non-relativistic quark models when extended to ordinary light hadrons have also shown remarkable qualitative success.

However, tackling baryons, unlike mesons, in these non-relativistic quark models, have not been always quite straight forward and simple due to their three body (qqq) bound state character for which they have not been so extensively investigated. Nevertheless, well within the limitations involved in the process, some aspects of the ground and excited states of hadrons have been studied [4–7] in such models with reasonable success. But Stanley and Robson [8] and Bhaduri *et al* [9] have pointed out some drawbacks in non-relativistic quark-model studies of the hadronic hyperfine levels in the usual perturbative approach. They have shown that the conventional coulomb potential as expected from QCD and the resulting spin–spin hyperfine interaction term taken to have the usual contact form proportional to $S_1 \cdot S_2 \delta(r)/$

$m_{q_1} \cdot m_{q_2}$ invalidates the non-relativistic approximation in addition to making inaccurate the perturbation estimates of the mass splittings for lighter hadrons. The required remedy suggested by Bhaduri *et al* [9] is to take an effective potential with a noncoulombic part instead of coulomb one in addition to replacing $\delta(r)$ in the spin–spin hyperfine interaction by some suitable form factor $f(r, r_0)$. With harmonic or linear potential, in place of coulomb one, they have tried various forms of $f(r, r_0)$ in order to validate the use of the non-relativistic approach for study of light hadrons.

However, instead of such hand-waving techniques it would be more justified, in principle, to treat the motion of the quarks inside light hadrons relativistically. Moreover, besides the spectroscopic implications, many physical quantities such as the magnetic moments, the axial vector coupling constants and the electromagnetic form factors in baryonic sector are closely related to the relativistic motion of the quarks.

The relativistic bound states of quarks composing hadrons have been studied by various quark models [10–12]. Still no unique relativistic quark model for all ranges has emerged so far either from the first principle theoretical derivations or from simple phenomenological studies. In the common areas of investigation belonging to mesonic and baryonic sectors the existing data do not seem to prefer uniquely any one of the various potential models available so far in the literature. Only a wider range of more accurate data involving certain aspects, more sensitive to the specific features of the models, may enable one to select out some in preference over the others. The bag model [13–15] with its different versions is essentially the only phenomenological model which has been found very extensive in its applications and reasonably successful in predictions. Yet this model is believed to be neither unique nor totally free from difficulties and objections. Therefore, there have been, in recent past, several attempts to formulate alternative schemes with confining potentials of appropriate Lorentz structures. For example, a scalar potential in linear form was used by Critchfield [16] to confine the relativistic individual quarks in nucleons. Potentials of a different type of Lorentz structure with equally mixed scalar-vector parts in linear [17–19], harmonic [20–23], logarithmic [24–27], as well as non-coulombic power law [28–31] form have also been investigated in this context.

The effective potential of individual quarks in such models which is basically due to the interaction of quarks with the gluon field, may be thought of as being mediated in a self-consistent manner through Nambu-Jona-Lasino (NJL) type models [32–33] by some kind of instanton induced effective quark–quark contact interaction with position dependent coupling strength. The position dependent coupling strength supposed to be determined by the multigluon mechanism cannot be calculated theoretically from first principles, although it is believed to be small at the origin and increases rapidly towards the hadronic surface. Therefore, one needs to introduce the effective potential for individual quarks in a phenomenological manner to seek a posteriori justification in finding its conformity with the supposed qualitative behaviour of the position dependent coupling strength in the contact interaction. With such arguments Tegen-Brockmann and Weise group [34–36] have introduced the phenomenological scalar harmonic and cubic potentials for the study of electromagnetic properties of baryons. All these potential models, used in a limited baryonic sector, only meet with a success more or less identical to the bag model. However, this does not establish the non-uniqueness of bag-like models, unless the potential models of such kind are pursued extensively over a much wider range of mesonic and baryonic phenomena. The present work is only a step in that direction where a phenomenological model with an independent quark square-root potential taken as a combination of a scalar and the fourth component of a vector in equal parts in the form

$$U_q(r) = \tfrac{1}{2}(1 + \gamma^0)(ar^{1/2} + V_0), \quad a > 0, \tag{1}$$

has been used in Dirac equation to study various aspects of the mesonic and baryonic phenomena. This potential is assumed to represent the average non-perturbative multigluon interaction responsible for the confinement of individual constituent quarks.

The important feature of the scalar plus vector potential taken here is that it provides a consistent picture for both mesons and baryons. On the other hand if one takes a pure vector potential it would produce only $q\bar{q}$ bound states where as the choice of a pure scalar potential provides an attractive force for both the $q\bar{q}$ and qq states [37]. Since there are no diquark states, the q–q interaction must be weaker, which can possibly be provided by the repulsive nature of the vector potential. Thus for the confinement of quarks a mixed scalar and vector potential is a more appropriate choice. In fact, the choice of such a potential has been immensely successful in the predictions of the hadronic properties. Additionally, the Lorentz scalar plus vector potential does not have the Klein-paradox. Moreover, if one chooses an effective potential with an equal mixture of scalar and vector parts as taken in (1), then both the scalar and vector parts in equal proportion at every point would render the solvability of Dirac equation for independent quarks by reducing it to the form of Schrödinger-like equation. This has also an additional advantage of generating no spin-orbit splitting, as observed in the experimental baryon spectrum. The implications of such potential forms in the Dirac framework of independent quarks have been discussed by Smith and Tassie [38]. Bell and Ruegg [39] have also shown that the spin-orbit interaction is absent in such a scheme due to exact cancellation of such terms coming from vector and scalar parts of the potential if taken in equal proportion. This is clearly a welcome aspect of the model in case of baryons since the contribution of the spin-orbit interaction term to the baryon mass splittings is already known [40] to be negligible. Eichten and Feinberg [41] provide further support to the above Lorentz structure from a gauge invariant formalism where confinement mechanism is assumed to be purely colour electric in character. Furthermore, such a Lorentz structure of the two body confining potential have been observed [21, 42, 43] phenomenologically in the study of fine hyperfine splittings of heavy meson spectra.

The choice of the effective square root confining potential in the present study is motivated by the fact that qualitatively the confining potential would be strongly weakened by the screening effects due to the quark pair creation above threshold. On the other hand evidences for a confinement potential weaker than the linear one also exist [44, 45] because the linear potential has discrepancies with the experimental data only in the domains where either spin-dependent effects cannot be explained (such as the hyperfine splittings of p-wave heavy quarkonia) or decay processes (such as leptonic decays) where the derivative of confining potential are involved. In fact, the increasing of linear potential with r seems too fast so that the calculated leptonicwidths of higher $c\bar{c}$ and $b\bar{b}$ excited states exceed the experimental data. Olsson and Suchyta III [46] discussed the spin orbit contribution of the confining interaction and found that the confining potential must fall below the linear potential. More recently, the calculation of lattice QCD for baryon spectrum leads to an effective potential [47] $V(r) = Ar^{\varepsilon}$ ($A = 0{\cdot}508$, $\varepsilon = 0{\cdot}671$) which also shows a potential weaker than linear might be more suitable for describing effective quark confinement in hadrons. Therefore, a phenomenological square-root potential of the form $(ar^{1/2} + V_0)$ which is inspired by the fact that it is weaker than the linear potential has been chosen in this work for an unified study of mesons and baryons. In fact, recently a square-root potential of the form $ar^{1/2}$ has been used by Xiatong Song [48] as a Lorentz scalar part along with a Lorentz vector part of the form $br^{-1/2}$ for the study of heavy quarkonium. However in the present work the potential $ar^{1/2}$ has been used with equally mixed scalar and vector parts in the form $\frac{1}{2}(1 + \gamma^0)(ar^{1/2} + V_0)$ because of its simplifying feature in converting the single quark Dirac equation into an effective Schrödinger equation for the upper component of the Dirac spinor which can be solved easily.

In the baryonic dimensions, the long range behaviour of the quark–quark interaction which arises out of non-perturbative multi-gluon mechanism including gluon self-couplings, is believed to be dominant enough in comparison to its short-range coulomb like behaviour arising out of one-gluon exchange. In absence of any detailed knowledge about this long-range confining part of the quark–quark interaction from QCD, one usually relies on phenomenological approaches providing confinement of individual constituent quarks through some average potential, to determine the zeroth order quark dynamics inside the baryon core. With this motivation we have introduced here a phenomenological square-root potential with equally mixed scalar-vector Lorentz structure to study the quark-core contributions to the static properties of baryons. The residual interactions like the one arising out of short distance one gluon exchange and also that due to possible quark-pion coupling which are believed to be weak are left out here to be treated in our subsequent work as low order perturbations. It must be pointed out here that in such a perturbative study solvability of the zeroth order equation in a straight forward and simple manner can render sufficient tractability and enough insight into the whole mechanism involved in the process of the investigation. With the present choice of $V(r)$ in non-coulombic square-root form, analytic solutions to the zeroth order equation exists in the WKB limit. On the other hand for exactness, the model predictions of the physical quantities to be investigated in baryon sector can be expressed in closed form in terms of some appropriate expectation values which can be evaluated numerically in a straight forward manner. In the present work we intend to understand the static baryon properties in the framework of a relativistic independent quark-model based on the Dirac equation with the confining square-root potential taken in the form (1). Here we study the quark core contributions to the static properties like magnetic moments μ_B, r.m.s. charge radii $\langle r^2 \rangle_B^{1/2}$ and axial vector coupling constant ratio (g_A/g_V) of octet baryons taking into account the possible centre-of-mass corrections; but for the moment we leave aside the correction due to pion cloud effects on the static baryon properties. This model is also extended to study the magnetic moments of charmed and b-flavoured baryons. Finally, in order to justify the further applicability of the model in the mesonic sector the mass levels of atom-like light quark systems $Q\bar{q}$ have been investigated.

Keeping the Lagrangian mass parameters of the quarks as current quark masses within the limits of broken $SU(3)$ $(m_u = m_d \neq m_s)$, we present in § 2 a brief outline of the potential model and its solution leading to a complete description of the relativistic bound states of the individual confined quarks of the hadron core. Then with the

Dirac wave-function for the ground state in hand, the core contribution to the static properties of the baryon $(1/2)^+$ octet in terms of magnetic moments μ_B, charge radii $\langle r^2 \rangle_B^{1/2}$ and axial vector constant ratio (g_A/g_V) for β-decay processes are calculated in the usual manner. We also present here the modified expressions for the static properties due to center-of-mass correction. In § 3, we estimate the potential parameters and the quark masses suitably in order to yield an appropriate ground state energy for the non-strange quarks which gives the average nuclear and Δ (1232) mass approximately, keeping in mind the correction due to center-of-mass motion and the pionic cloud. The static properties of octet baryons calculated after c.m. corrections turn out to be in reasonable agreement with experimental results. The quark core contributions to the magnetic moments of charmed and b-flavoured baryons are also predicted in § 3. Using the same set of parameters we also compute the mass spectra of atom-like $Q\bar{q}$ mesons. Lastly, § 4 is devoted to brief discussion of the results and the conclusions.

2. Basic theory

In this section we briefly outline the framework of the model adopted here to study the quark-core contribution to the static properties of baryons and the corrections due to the centre-of-mass motion.

2.1 *The relativistic potential model*

We start with the assumption that the quarks in a hadron core move independently in an average flavour-independent potential taken in the form

$$U_q(r) = \tfrac{1}{2}(1 + \gamma^0)\, V(r) \tag{2.1}$$

where $V(r) = ar^{1/2} + V_0$, with $a > 0$, and the quarks obey the Dirac equation derivable from a Lagrangian density

$$\mathscr{L}_q(x) = \bar{\Psi}_q(x)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - m_q - U_q(r)\right]\Psi_q(x). \tag{2.2}$$

Hence the Lagrangian mass parameter m_q for the quarks must be regarded as the current quark mass. Then the independent quark wave function $\Psi_q(\mathbf{r})$ satisfies the Dirac equation ($\hbar = c = 1$)

$$[\gamma^0 E_q - \boldsymbol{\gamma}\cdot\mathbf{p} - m_q - U_q(r)]\Psi_q(\mathbf{r}) = 0 \tag{2.3}$$

The solution to the independent-quark wave function $\Psi_q(\mathbf{r})$ can be obtained in the two-component form as

$$\Psi_q(\mathbf{r}) = N_q \begin{pmatrix} ig_q(r)/r \\ \boldsymbol{\sigma}\cdot\hat{r}\, f(r)/r \end{pmatrix} y_{ljm}(\hat{r}) \tag{2.4}$$

where the normalized spin angular part

$$y_{ljm}(\hat{r}) = \sum_{m_l, m_s} \langle l, m_l, \tfrac{1}{2}, m_s | j, m_j \rangle\, Y_l^{m_l}(r)\chi_{1/2}^{m_s} \tag{2.5}$$

and the reduced radial part of the upper component of the Dirac spinor $\Psi_q(\mathbf{r})$ satisfies the equation

$$\frac{d^2 g_q(r)}{dr^2} + \left[\lambda_q(E_q - m_q - V(r)) - \frac{l(l+1)}{r^2} \right] g_q(r) = 0 \tag{2.6}$$

which can be transformed into a convenient dimensionless form

$$\frac{d^2 g_q(\rho)}{d\rho^2} + \left[\varepsilon_q - \rho^{1/2} - \frac{l(l+1)}{\rho^2} \right] g_q(\rho) = 0 \tag{2.7}$$

where $\rho = (r/r_{0q})$ is a dimensionless variable with

$$r_{0q} = [a(E_q + m_q)]^{-2/5} \tag{2.8}$$

and

$$\varepsilon_q = (E_q - m_q - V_0)[(E_q + m_q)/a^4]^{1/5}. \tag{2.9}$$

The solution of basic eigen-value equation either by a standard numerical method or by WKB method [49–50] would yield a positive definite value of $\varepsilon_q = \varepsilon_{nl}$ corresponding to binding energy $E_q = E_{nl}$ for the independent-quarks. Taking

$$(E_{nl} - m_q - V_0) = a x_{nl} \tag{2.10}$$

and

$$2m_q + V_0 = ab \tag{2.11}$$

the eqn. (2.9) can be converted to the form

$$a^2 x_{nl}^5 (x_{nl} + b) = \varepsilon_{nl}^5 \tag{2.12}$$

Then using ε_{nl} values from the solution of (2.7) and b from (2.11) with the known potential parameters V_0, a and m_q, eqn. (2.12) can be solved for a unique positive root x_{nl} which would yield the independent-quark binding energy E_{nl} as

$$E_{nl} = m_q + V_0 + a x_{nl} \tag{2.13}$$

The independent-quark wave-function $\Psi_q(\mathbf{r})$ given by (2.4) and the corresponding binding energy E_{nl} leads to a complete description of the relativistic bound states of the confined constituent quarks inside the hadron core.

Following the usual bag model approach we further assume that all the three quarks in the baryon core are in their ground states with $J^p = \frac{1}{2}^+$ and $J_z = \frac{1}{2}$. Therefore, for baryons the independent-quark wave-function can be written as

$$\Psi_q(\mathbf{r}) = N_q \begin{bmatrix} \Phi_q(\mathbf{r}) \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\lambda_q} \Phi_q(\mathbf{r}) \end{bmatrix} \chi\uparrow \tag{2.14}$$

where

$$\lambda_q = E_q + m_q$$

and

$$\phi_q(\mathbf{r}) = A_q \frac{g_q(r)}{r} y_0^0(\theta, \phi) \tag{2.15}$$

is the normalised radial angular part of $\Psi_q(\mathbf{r})$ with normalization constant A_q. The overall normalization constant N_q of $\Psi_q(\mathbf{r})$ can be easily obtained as

$$N_q^2 = [1 + (E_q - m_q - V_0 - a\langle r^{1/2}\rangle_q)/\lambda_q]^{-1}. \tag{2.16}$$

The value of $\langle r^{1/2}\rangle_q$ in (2.16) can be easily obtained through WKB method [49, 50] as

$$\langle r^{1/2}\rangle_q = 4(E_q - m_q - V_0)/5a \tag{2.17}$$

with which the expression (2.16) simplifies to

$$N_q^2 = 5(E_q + m_q)/(6E_q + 4m_q - V_0). \tag{2.18}$$

For the ground state, the eqn. (2.12) can be solved for a unique positive root x_{1S} which would yield from (2.13) the individual quark binding energy $E_q = E_{1S}$ for quarks in the baryon.

Now with the ground state wavefunction $\Psi_q(\mathbf{r})$ as given in equation (2.14) it is straight forward to derive the expressions for the quark-core contributions to certain measurable quantities of the S-wave baryons which are obtained simply by appropriately adding the contributions of each individual quark.

2.2 *Static properties of the S-wave baryon core*

(i) *Magnetic moments of baryon core*: The magnetic moment of a $\frac{1}{2}^+$ baryon primarily consists of contributions from its quark-core in terms of the corresponding constituent quark moments which are defined as

$$\mu_q = \frac{1}{2}\int d^3r[\mathbf{r} \times \mathbf{J}(\mathbf{r})]_z, \tag{2.19}$$

where

$$\mathbf{J}(\mathbf{r}) = e_q\bar{\Psi}_q(r)\gamma\Psi_q(\mathbf{r}).$$

Here e_q is the charge of the quark in units of proton charge. Using (2.14) for $\Psi_q(\mathbf{r})$, (2.19) simplifies to

$$\mu_q = (2M_pN_q^2/\lambda_q)e_q \text{n.m}, \tag{2.20}$$

where M_p is the mass of the proton and n.m stands for nuclear magneton. The quark-core contribution to the magnetic moment of a baryon B is given by

$$\mu_B = \text{£}B\uparrow\left|\sum_q \mu_q\sigma_z^q\right|B\uparrow^\circ \tag{2.21}$$

where $|B\uparrow\rangle$ represents the state vectors of the baryons. In the case of octet baryons $|B\uparrow\rangle$ represents the regular $SU(6)$ state vectors. For the charmed and b-flavoured baryons the corresponding state vectors are obtained by the straightforward extensions as given by Singh [51]. The relations for the magnetic moments, ordinary, charmed and b-flavoured baryons in terms of constituent-quark moments are well-known [52–57] and can be used to compute the quark-core contributions to the magnetic moments of baryons with the present model.

(ii) *Mean squared charge radii of the baryon core*: The distribution of electric charge within a baryon core is determined by a convenient parameter called the mean square charge radius $\langle r^2 \rangle_B$ which is usually defined as

$$\langle r^2 \rangle_B = \sum_q e_q \int \Psi_q^+(\mathbf{r}) r^2 \Psi_q(\mathbf{r}) d^3 r \tag{2.22}$$

where e_q is the quark electric charge.

Using the expression for $\Psi_q(\mathbf{r})$ from (2.13), eqn. (2.21) can be easily simplified to

$$\langle r^2 \rangle_B = \sum_q e_q \frac{N_q^2}{\lambda_q} \left[(2E_q - V_0) \langle\langle r^2 \rangle\rangle_q - a \langle\langle r^{5/2} \rangle\rangle_q + \frac{3}{\lambda_q} \right] \tag{2.23}$$

The double angular brackets appearing in (2.23) are the expectation values with respect to $\phi_q(r)$ which can be evaluated numerically or by the WKB method [49, 50] through the general expression

$$\langle\langle r^\alpha \rangle\rangle = 2\alpha(2\alpha + 1) r_{0q}^\alpha \varepsilon_q^{2\alpha} \beta(2\alpha, 5/2) \tag{2.24}$$

(iii) *Center-of-mass corrections*: In this model there would be a sizeable spurious contribution to the energy E_q from the motion of the centre-of-mass of the three quark system. Unless this aspect is duly accounted for, the concept of the independent motion of quarks inside the core will not lead to a physical baryon state of definite momentum, since our shell type relativistic quark model is not translationally invariant. The problem appears in the same way in nuclear physics in case of He and also in the MIT bag model [58] and has to be resolved accordingly [59–61].

Here following the technique adopted by Bartelski and Eich [61, 62], the expressions for static core quantities μ_B, $\langle r^2 \rangle_B$ and (g_A/g_V) as obtained above can be modified taking the c.m. motion

$$\mu_B' = [3\mu_B + Q_B(M_P/M_B)(1 - \delta_B)]/(1 + \delta_B + \delta_B^2), \tag{2.25}$$

$$\langle r^2 \rangle_B' = \left[3\langle r^2 \rangle_B - 3\sum_q \left\{ 2e_q \left(\frac{E_q}{E_B}\right) - Q_B \left(\frac{E_q}{E_B}\right)^2 \right\} \langle r^2 \rangle_q \right] \Big/ (2 + \delta_B^2) \tag{2.26}$$

and

$$(g_A/g_V)' = 3(g_A/g_V)/(1 + 2\delta_B), \tag{2.27}$$

where

$$\delta_B = (M_B/E_B) \simeq (1 + \delta_B^2)/2, \tag{2.28}$$

$$\delta_B^2 = (M_B^2/E_B^2) = 1 - \langle \mathbf{p}_B^2 \rangle / E_B^2. \tag{2.29}$$

Here $E_B = \sum_q E_q$ is the relativistic energy of the quark core and Q_B is the charge of the baryon. The physical mass of the bare baryon core is $M_B = (E_B^2 - \langle \mathbf{P}_B^2 \rangle)^{1/2}$ and P_B is the centre of mass momentum and $\langle \mathbf{P}_B^2 \rangle$ is evaluated with the usual approximation as

$$\langle \mathbf{P}_B^2 \rangle = \sum_q \langle \mathbf{P}^2 \rangle_q,$$

where $\langle \mathbf{P}^2 \rangle_q$ are the mean-square individual quark momenta with respect to $\Psi_q(r)$. In the present model we find that

$$\langle \mathbf{P}^2 \rangle_q = N_q^2 [2E_q(E_q - m_q) - (3E_q - m_q - V_0) V_0 - (3E_q - m_q - 2V_0) a \langle\langle r^{1/2} \rangle\rangle + a^2 \langle\langle r \rangle\rangle] \tag{2.30}$$

The double angular brackets appearing in (2.30) are the expectation values with respect to $\Phi_q(r)$ and the evaluation of $\langle \mathbf{P}^2 \rangle_u$ and $\langle \mathbf{P}^2 \rangle_s$ would lead to a correct estimate of the static properties of the octet baryons.

(iv) *Axial vector coupling constant ratio* (g_A/g_V): Interpreting the weak β-decays of the baryons $B \to B' + e^- + \bar{\nu}_e$ as quark β-decays like $q_j - u + e^- + \bar{\nu}_e$ occurring inside the baryon core (where q_j could be a d or s quark), one can obtain in lines of bag model calculations [63, 58] the axial vector coupling constant ratio g_A/g_V as

$$(g_A/g_V) = (g_A/g_V)^{\mathrm{NR}}(1 - 2\Delta_{ju}) \tag{2.31}$$

where g_A and g_V are the axial-vector and vector coupling constants for the baryons. $(g_A/g_V)^{\mathrm{NR}}$ is the non-relativistic $SU(6)$ value [64] which for various β-decays can be computed by using $SU(6)$ wave function for $|B\uparrow\rangle$ states [50]. Δ_{ju} is the relativistic correction in the axial coupling for the β-decay of quarks 'j' into the quark 'u' and is given by

$$\Delta_{ju} = \int \psi_u^+ L_z \psi_j \mathrm{d}^3 r \tag{2.32}$$

which on putting the expression for $\psi_q(r)$ from (2.14) for quarks in the $1S_{1/2}$ state becomes

$$\Delta_{ju} = \left(\frac{2N_u N_j}{3\lambda_u \lambda_j}\right) I_{uj} \tag{2.33}$$

with

$$I_{uj} = \int_0^\infty r^2 \mathrm{d}r f'_u(r) f'_j(r). \tag{2.34}$$

Integrating by parts and using (2.6) with $\frac{g_q(r)}{r} = f_q(r)$, we get

$$I_{uj} = \frac{1}{r_{0j}^2} \int_0^\infty \mathrm{d}r [\varepsilon_q - (r/r_{0j})^{1/2}] g_j(r) g_u(r) \tag{2.35}$$

Here we use for simplicity the WKB solution for $g_q(r)$ [49] to obtain I_{uj}

$$I_{uj} = \frac{36}{231} \frac{\varepsilon_q}{r_{0j}^2} \left(\frac{r_{0j}}{r_{0u}}\right)^{1/2} \quad F\left(\frac{1}{4}, 2, \frac{15}{4}; k^2\right) \tag{2.36}$$

where $k^2 = \left(\frac{r_{0j}}{r_{0u}}\right)^{1/2}$ and $F\left(\frac{1}{4}, 2, \frac{15}{4}; k^2\right)$ is the hypergeometric function. Thus g_A/g_V values for various β-decays can be computed using (2.31), (2.33), and (2.36). For neutron beta-decay

$n \to pe\bar{\nu}$(or $d - u + e + \nu_e^-$) eq. (2.26) reduces to

$$(g_A/g_V)_{n\text{-}pe\bar{\nu}_e} = \frac{5}{3}\left[1 - \frac{4}{3} \frac{N_u^2}{\lambda_u^2} \frac{\varepsilon_q}{r_{0u}^2} (1/5)\right], \tag{2.37}$$

which alternately simplifies to

$$(g_A/g_V)_{n\text{-}pe\bar{\nu}_e} = \frac{5}{9}(4N_u^2 - 1). \tag{2.38}$$

3. Model parameters and results

The computation of the static properties of octet baryons in the present model depends on the choice of the Lagrangian mass parameters (m_u, m_d, m_s) and the potential parameters (a, V_0) which would lead to the individual quark binding energies (E_u, E_d, E_s) and the corresponding quark wave function through the eigen value equations (2.4) to (2.13).

In the Lagrangian formulation adopted here we first of all choose to fix the quark mass parameters

$$(m_u = m_d, m_s) = (5, 210)\,\text{MeV} \tag{3.1a}$$

in the current quark limit. To estimate the potential parameters a and V_0, we prefer to take $N-\Delta$ spin-isospin average mass $\bar{M}_N = 1173\,\text{MeV}$ and the Ω^- mass $M_{\Omega^-} = 1670\,\text{MeV}$ as the guiding inputs to suggest the individual quark binding energies $E_u = E_d \simeq \frac{1}{3}\bar{M}_N$ and $E_s \simeq \frac{1}{3}M_{\Omega^-}$. However keeping in view the possible corrections like those due to centre-of-mass motion and the pion-cloud effect etc to be accounted for in subsequent work, to yield the correct N, Δ and Ω^- masses, we prefer these binding energies to be somewhat greater than the above suggested values and we find that with binding energies as $E_u = E_d = 560\,\text{MeV}$ and $E_s = 700\,\text{MeV}$ and with $\varepsilon_q = 1{\cdot}8418$ which is provided by the numerical solution of the dimensionless eq. (2.7), the eqs (2.9) to (2.12) for the non-strange and the strange quarks separately yield the parameters

$$(a,\ V_0) = (0{\cdot}2698\,\text{GeV}^{1\cdot5},\ -0{\cdot}1679\,\text{GeV}) \tag{3.1b}$$

which subsequently generate the contributions of the quark cores to the static baryon properties as

$$(\mu_P, \mu_A) = (2{\cdot}62,\ -0{\cdot}5956)\text{nm}, \tag{3.2}$$

$$\langle r^2 \rangle_p^{1/2} = 1{\cdot}1255\,\text{fm} \tag{3.3}$$

and

$$(g_A/g_V)_{n\to pe\bar{\nu}} = 1{\cdot}2142 \tag{3.4}$$

These results seem to be quite reasonable in view of the possible c.m motion corrections involved.

With the parameters a, V_0 and m_q along with the corresponding binding energy E_q being known, all the relevant quantities leading to the predictions of the magnetic moments of the octet baryons can be calculated. First of all we obtain the quark wave function normalizations from (2.18) as

$$(N_u^2 = N_d^2, N_s^2) = (0{\cdot}7963, 0{\cdot}8736) \tag{3.5}$$

which in turn yield from eq. (2.20) the constituent quark magnetic moments

$$\mu_u = -2\mu_d = 1{\cdot}7466\,\text{nm} \tag{3.6}$$

and

$$\mu_s = -0{\cdot}5956\,\mathrm{nm}$$

With these values it is straightforward to compute the uncorrected magnetic moments of baryon cores in the nucleon octet using expression (2.21). The results so obtained are presented in table 1.

In the framework of the present model we calculate the expression (2.24) for u and s quark as

$$\begin{aligned}\langle\langle r^2\rangle\rangle_u &= 1{\cdot}1164\,\mathrm{fm}^2, \quad \langle\langle r^2\rangle\rangle_s = 0{\cdot}7632\,\mathrm{fm}^2\\ \langle\langle r^{5/2}\rangle\rangle_u &= 1{\cdot}2277\,\mathrm{fm}^{5/2}, \quad \langle\langle r^{5/2}\rangle\rangle_s = 0{\cdot}7633\,\mathrm{fm}^{5/2}\end{aligned} \tag{3.7}$$

which lead to the evaluation of r.m.s charge radii $\langle r^2\rangle_B^{1/2}$ for the octet baryons from (2.23). The results thus calculated without c.m correction are given in table 2. Then computing the expression I_{uj} in eq. (2.36) to obtain the values as

$$(I_{uu}, I_{us}) = (0{\cdot}0818, 0{\cdot}1034)\,\mathrm{GeV}^2. \tag{3.8}$$

We can obtain the axial-vector coupling constant g_A for the β-decay processes corresponding to the members of the nucleon octet with appropriate values [64] of $(g_A/g_V)^{\mathrm{NR}}$. The results obtained here without c.m. corrections are presented in table 3.

For introducing centre-of-mass corrections, we estimate the factor δ_B from expressions (2.28) and (2.29), which certainly depends on the flavour combinations of the quark core in baryons. Accordingly we find $\delta_B = 0{\cdot}9449$, 0·9475 and 0·9499 respectively for the baryon cores with three nonstrange quarks, two nonstrange

Table 1. Magnetic moments of the nucleon octet calculated by the present model with and without c.m correction as compared with the results of cloudy-bag-model (CBM) and the experimental data (all numbers in nuclear magnetons).

Baryon	Present calculation: Uncorrected	Present calculation: After cm correction	CBM calculation [13, 71]	Experimental (Particle data group [69])
p	2·6200	2·7809	2·60	2·7928444 $\pm$ 0·0000011
n	$-$ 1·7466	$-$ 1·8463	$-$ 2·01	$-$ 1·91304308 $\pm$ 0·00000054
Λ^0	$-$ 0·5956	$-$ 0·6280	$-$ 0·58	$-$ 0·613 $\pm$ 0·004
Σ^+	2·5274	2·6747	2·34	2·379 $\pm$ 0·020
Σ^0	0·7807	0·8232	—	0·61 $\pm$ 0·08*
Σ^-	$-$ 0·9658	$-$ 1·0282	$-$ 1·08	$-$ 1·14 $\pm$ 0·05
Ξ^0	$-$ 1·3764	$-$ 1·4476	$-$ 1·27	$-$ 1·250 $\pm$ 0·014
Ξ^-	$-$ 0·5031	$-$ 0·5379	$-$ 0·51	$-$ 0·69 $\pm$ 0·04
(Λ, Σ^0)	$-$ 1·5126	$-$ 1·5990	—	$-$ 1·82 $+$ 0·18** $-$ 0·25

*Values computed according to the definition $\mu_0 = \frac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-})_{\mathrm{exp}}$ which is quite often referred to as the measured value.
**Dydak *et al* [70].

Table 2. $\langle r^2 \rangle_B^{1/2}$ rms charge radii of octet baryons calculated in the present model with and without cm corrections as compared to the experiment and Ferreira–Helayel–Zagury (FHZ) model values (all numbers in fm).

Baryon	$\langle r^2 \rangle_B^{1/2}$ uncorrected	$\langle r^2 \rangle_B^{1/2}$ with cm correction	Experiment	FHZ model [21]
p	1·1255	0·9632	0·87 ± 0·02	0·932
n	0	0	− 0·341	0
Λ^0	0·3828	0·3421		0·346
Σ^+	1·1889	1·0260		0·994
Σ^0	0·3828	0·3421		0·346
Σ^-	1·0584	0·9048		0·86
Ξ^0	0·5414	0·4884		0·48
Ξ^-	0·9867	0·9097		0·79

Table 3. Axial vector coupling constant ratio (g_A/g_V) calculated in the present model with and without cm correction as compared to the experiment.

Decay mode	Axial vector coupling constant ratio: Uncorrected (g_A/g_V)	Axial vector coupling constant ratio: With cm corrected (g_A/g_V)	(g_A/g_V) Experiment Ref. [69] (Particle data group [69])
$n \to pe\bar{\nu}$	1·2142	1·2604	1·254 ± 0·006
$\Lambda \to pe\bar{\nu}$	0·7766	0·8048	0·694 ± 0·025
$\Sigma^- \to ne\bar{\nu}$	− 0·2588	− 0·2682	− 0·362 ± 0·043
$\Xi^- \to \Lambda e\bar{\nu}$	0·1725	0·1785	0·25 ± 0·05
$\Xi^- \to \Sigma^0 e\bar{\nu}$	1·2944	1·3391	—
$\Xi^0 \to \Sigma^+ e\bar{\nu}$	1·2944	1·3391	—

quarks with one strange quark and one nonstrange quark with two strange quarks respectively. Then it is straightforward to obtain the corrected values of the magnetic moments, the proton rms charge radius and vector coupling constant from expressions (2.25), (2.26) and (2.27). These quantities so obtained after the centre-of-mass corrections, are presented in the appropriate tables (1, 2 and 3) in comparison with the corresponding uncorrected values as well as the experimental ones.

Now with the assumed flavour-independence of the potential we use the same set of potential parameters a and V_0 as obtained in (3.1) along with the quark masses

$$(m_c, m_b) = (1.449, 4{\cdot}764)\,\text{GeV} \tag{3.9}$$

taken-well within the limits of current algebra predictions, to obtain the energy eigen values from (2.13) as

$$(E_c, E_b) = (1{\cdot}7917, 5{\cdot}0056)\,\text{GeV} \tag{3.10}$$

Now with the help of these parameters we obtain from (2.18) the quark-wave function normalizations

$$(N_c^2, N_b^2) = (0.9694, 0.9916) \tag{3.11}$$

which are then used in (2.20) to compute the constituent quark magnetic moments as

$$(\mu_c, \mu_b) = (0.3713, -0.0630)\,\text{n.m} \tag{3.12}$$

The magnetic moments of the baryons in charmed and b-flavoured sectors are calculated in a straightforward way by using the constituent quark magnetic moments obtained in (3.6) and (3.12). Since no experimental data are available for the magnetic moments of charmed and b-flavoured baryons we do not incorporate the c.m corrections in these cases and the uncorrected results are displayed in tables 4 and 5 in comparison with the predictions of some other models [53, 65, 66]. The symbols used in these tables are according to Pandita *et al* [53].

To test the further applicability of the present model in understanding the atom-like $Q\bar{q}$ systems we calculate the mass spectra for $D(c\bar{u})$, $F(c\bar{s})$, $B(b\bar{u})$ and $G(b\bar{s})$ mesons. Using same set of parameters as chosen in (3.1b) and the quark masses given in (3.1a) and (3.9), we obtain the ground state mass values for $D(c\bar{u})$, $F(c\bar{s})$ and $B(b\bar{u})$ mesons from (3.13) as

$$\begin{aligned} M_{1S}(c\bar{u}) &= 2.010(2.01 \pm 0.0007)\,\text{GeV} \\ M_{1S}(c\bar{s}) &= 2.1493(2.14 \pm 0.06)\,\text{GeV} \\ M_{1S}(b\bar{u}) &= 5.325(5.325)\,\text{GeV} \end{aligned} \tag{3.13}$$

Table 4. Magnetic moment of charmed baryons computed in the present model as compared to the values obtained in the bag model and de Rujula–Georgi–Glashow (DGG) model (all numbers in nuclear magneton).

Symbol	Quark content	Present calculation	Bag model [65, 66]	DGG model [53]
Σ_c^{++}	*cuu*	2.2051	1.955	2.36
Σ_c^{+}	*cud*	0.4584	0.363	0.43
Σ_c^{0}	*cdd*	−1.2882	−1.23	−1.43
Ω_c^{0}	*css*	−0.9180	−0.98	−0.89
Ξ_c^{+}	*csu*	0.6435	0.475	0.73
Ξ_c^{0}	*csd*	−1.1031	−1.09	−1.16
Ξ_{cc}^{++}	*ccu*	−0.0871	−0.167	−0.12
Ξ_{cc}^{+}	*ccd*	0.7862	0.865	0.82
Ω_{cc}^{+}	*ccs*	0.6936	0.838	0.69
Λ_c^{+}	*c*(*ud*)*a*	0.3713	0.503	0.37
$\Xi_c'^{+}$	*c*(*us*)*a*	0.3713	0.503	0.37
$\Xi_c'^{0}$	*c*(*ds*)*a*	0.3713	0.503	0.37

Table 5. Magnetic moments of b-flavoured baryons computed in the present model as compared to the values obtained in the bag model and DGG model (all numbers in nuclear magneton).

Symbol	Quark content	Present calculation	Bag model [65, 66]	DGG model [53]
Σ_b^+	buu	2·3498	2·318	2·5
Σ_b^0	bud	0·6032	0·587	0·61
Σ_b^-	bdd	−1·1434	−1·117	−1·28
Ω_b^-	bss	−0·7732	−0·838	−0·55
Ξ_{cb}^0	bcd	−0·3136	−0·39	−0·38
Ξ_{cb}^+	bcu	1·4329	2·04	1·5
Ω_{ccb}^+	bcc	0·5161	0·894	0·51
Ξ_{bb}^0	bbu	−0·6662	−0·614	−0·7
Ξ_{bb}^-	bbd	0·2071	0·14	0·23
Ω_{bb}^-	bbs	0·1145	0·084	0·105
Ω_{cbb}^0	bbc	0·2077	−0·31	−0·21
Ξ_b^0	bsu	0·7883	0·73	0·87
Ξ_b^-	bsd	−0·9583	−0·977	−1·05
Ω_{cb}^0	bcs	−0·1285	−0·223	−0·11

which closely agree with the experimental values given in the parentheses. For this calculation we use the numerically calculated eigen values $\varepsilon_q = \varepsilon_{nl}$ corresponding to different bound states of these mesons. The results thus calculated from (3.13) for the mass spectra of $D(c\bar{u})$, $F(c\bar{s})$, $B(b\bar{u})$, $G(b\bar{s})$ mesons are displayed in table 6. All these predictions of our model may hopefully be tested in future experiments of CERN and Fermilab.

4. Discussion and conclusion

We observe that our results for the magnetic moments, rms charge radii and axial vector coupling constant ratios compare reasonably well with the existing experimental data. However, the proton charge radius $\langle r^2 \rangle_p^{1/2}$ which was 1·1255 fm before the centre-of-mass correction, becomes 0·9632 fm, as against the experimental value of $0{\cdot}87 \pm 0{\cdot}02$ fm. Furthermore we find $\langle r^2 \rangle_n^{1/2} = 0$ contrary to the experimental value of −0·341 fm. This is of course the case with most of the models of this kind including the bag model. In the absence of experimental data of $\langle r^2 \rangle_B^{1/2}$ for other members of the octet baryons we compare our results with those obtained by Ferreira *et al* [21].

The minor discrepancies of our results are common to most of the phenomenological models and can be partly accounted for by the suitable pionic corrections, as investigated in the cloudy bag model. However, these results might be improved by incorporating chiral symmetry so as to bring in the pionic effects which is being taken in our subsequent work. Nevertheless, in view of testing the applicability of the model in these areas of study, the present model has been found to be meaningful with its simple and straightforward approach yielding qualitatively encouraging and reasonable results.

Table 6. Mass spectrum for the atom-like mesons of $D(c\bar{u})$, $B(b\bar{u})$, $F(c\bar{s})$ and $G(b\bar{s})$ families calculated in the present model as compared with the experimental data.

n	l	ε_{nl}	$M_{nl}(D)$ GeV	Experimental value	$M_{nl}(B)$ GeV	Experimental value	$M_{nl}(F)$ GeV	Experimental value	$M_{nl}(G)$ GeV	Experimental value
1	S	1·8418	2·010	2·010 ± 0·007	5·325	5·325	2·1493	2·140 ± 0·60	5·4643	—
2	S	3·5686	2·2311		5·5461		2·3700		5·6850	
3	S	3·0743	2·3789		5·6939		2·5176		5·8326	
4	S	3·4726	2·4925		5·8075		2·6310		5·9460	
1	P	2·3069	2·1527		5·4677		2·2918		5·6068	
2	P	2·8691	2·3195		5·6345		2·4582		5·7732	
1	D	2·6629	2·2590		5·5740		2·3979		5·7129	
2	D	3·1336	2·3960		5·7110		2·5346		5·8496	

In our present investigation, we have not looked into the baryon mass spectrum which could strictly require taking into account the corrections due to (i) energy associated with centre-of-mass motion (ii) color-electric and magnetic energy arising out of the residual one gluon exchange interaction at short distance and (iii) the pionic self energy of the baryons arising out of the baryon-pion couplig at the vertex. The masses of baryons would therefore have contributions from the baryon core corrected by these contributions. Investigations along this line in similar schemes with different confinement potentials have been done [17–31, 67, 68] with success. Therefore, we believe that for the present model similar analysis can be done which would be taken up in our subsequent work. The present model can also be extended to study the phenomena like hyperfine splitting of P-wave quarkonia and the leptonic decays of mesons which would directly depend on the square-root nature of the confining potential, the investigations in this line which would certainly give new insight about the nature of a confining potential are being taken up with the present model and will be reported shortly in a separate communication.

In this work we find that a simple independent-quark model based on the Dirac equation with an equally mixed scalar-vector square-root confining potential of the form given by (1.1) provides an excellent fit to the static properties of baryon octet and mass spectra of $Q\bar{q}$ mesons in a flavour-indpendent manner. In view of the simplicity of the model in treating baryons and mesons in the same footing the results obtained are quite encouraging. This effective individual quark potential, phenomenologically representing the confining interaction due to a non-perturbative multigluon mechanism seems as an alternative scheme to the CBM in studying baryons and mesons. We find that our potential unlike the QCD based one is non-singular at the origin and it doe not possess the short distance coulombic part of the potential as given by QCD (i.e. r^{-1} or $r^{-1}|\ln(r/r_0)|)^{-1}$. Therefore, even if the non-singular behaviour of this potential does not conform to the expectations of QCD, we can successfully describe the static baryon properties as well as the mass spectra of the light $Q\bar{q}$ mesons in a flavour independent manner.

The purpose of this work is not so much as to claim that nature prefers such a square-root confinement scheme over other schemes advanced earlier in this correction but to present an analysis that complements studies based upon various potential models. Such studies with various quark models when extended to describe the hadronic properties in a much wider sector may provide an opportunity to resolve the question regarding the true nature of quark confinement which is not straightforward to conclude on purely theoretical grounds based on QCD.

Acknowledgement

The authors are thankful to Dr N Barik, Utkal University, India for his valuable suggestions.

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