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# Two-loop Majorana neutrino mass and magnetic moment in a gauge model

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**Abstract.** Two-loop contributions to Majorana mass and transition magnetic moment in a gauge model not in conflict with decaying neutrino dark matter (DDM) hypothesis have been studied. Another variant of an earlier model [J Dhar and S Dev, *Pramana – J. Phys.* **39** 541 (1992)] consistent with the DDM hypothesis is proposed and is shown to lead to large enough neutrino magnetic moment and consistent with the phenomenological constraints on neutrino mass.

**Keywords.** Majorana transition magnetic moment; horizontal symmetry; VVO mechanism; LMA mechanism; BFZ mechanism; decaying dark matter; pseudo Dirac neutrino.

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## 1. Introduction

The neutrino electromagnetic properties were first mooted by Bethe [1]. Cisneros [2] sought to explain the solar neutrino deficit by attributing a sizeable magnetic moment to the electronic neutrino. However, the idea was abandoned after the realization that the neutrino magnetic moment predicted by the theories in vogue at that time was not large enough for this to work in the solar interior. Neutrino magnetic moment hypothesis was again revived in view of the apparent anticorrelation of the solar neutrino flux with the solar magnetic activity suggested by Homestake experiment [3]. Although, Kamiokande [4] and SAGE [5] experiments have not detected any anticorrelation, the statistics is not reliable enough to conclusively rule it out either. However, if further experiments confirm the anticorrelation, magnetic moment solution would be overwhelmingly favoured since no other proposed solution can explain the anticorrelation.

In the minimal standard model, the absence of the right-handed neutrinos in the fermionic spectrum coupled with the conservation of lepton number imply that the neutrino has no mass and no magnetic moment. The addition of one right-handed neutrino for each generation generates a magnetic moment for the neutrino via $W$-loops. However, since $\nu_R$ has no gauge interactions the chirality flip occurs on the external leg and, consequently, the neutrino magnetic moment is proportional to the neutrino mass. In view of the experimental constraint on neutrino mass from triton decay [6]: $m_{\nu_e} < 9{\cdot}5\,\mathrm{eV}$, the resulting magnetic moment is many orders of magnitude smaller than required. In fact, one always encounters a problem with small neutrino mass and a large neutrino magnetic moment when both are loop induced effects [7]. Since both the mass and the magnetic moment operators flip chirality, any mechanism for a large neutrino magnetic moment tends to induce large neutrino mass. The neutrino magnetic moment $\mu_\nu$ arises only at one-loop level from the diagrams of the generic type (figure 1) with charged fermions $l$ and charged scalars $\Phi$ with mass $M$. Since Majorana neutrinos cannot possess a static magnetic moment, $\mu_\nu$ is necessarily a transition magnetic moment $\mu_{\nu_e\nu_\mu}$ or $\mu_{\nu_e\nu_\tau}$. More precisely, the transition is of the form $\nu_{eL} \to (\nu_{\mu L}, \nu_{\tau L})^C$ to account for a change of chirality. Since dipole moments have dimensions $[\text{mass}]^{-1}$ one expects the dipole moments generated by the loop diagram to be of the order $eG/M$, where $G$ is a combination of dimensionless parameters appearing in the graph such as coupling constants, mixing angles, mass ratios and the loop factors. The photonless counterpart of the same diagram is a higher order correction to the neutrino propagator and represents a contribution to neutrino mass. One expects the mass diagram to involve the same combination $G$ of the dimensionless parameters as in the diagram with an electromagnetic vertex. Thus, one estimated $m_\nu \sim GM$. One expects the neutrino mass $m_\nu$ and magnetic moment $\mu_\nu$ to be related as $em_\nu/M^2 \sim \mu_\nu$ where the scalar mass $M$ is expected to be of the order of vector boson masses or even larger. Therefore, the existing strong constraint on neutrino mass translates into a stringent upper bound on the magnetic moment. If $\mu_\nu \sim 10^{-11}\mu_B$ then the neutrino mass constraint $m_{\nu_e} < 10\,\text{eV}$ implies $M < 1\,\text{GeV}$. Some of the scalars in the graph must be charged and enough is known about charged particles of mass less than 1 GeV and their interactions to rule out the possibility of such a large neutrino magnetic moment arising from that quarter.

Two different scenarios have been invoked to avoid this serious conflict. In the first [8], a symmetry larger than the standard $SU(2) \times U(1)$ group is imposed on such models of large magnetic moment for light neutrinos. As a result, contributions to both the mass and magnetic moment come from pairs of diagrams related by this additional symmetry and the two contributions add for the magnetic moment whereas they cancel in the case of the induced mass. In the second scenario advocated by Georgi and collaborators [9], one can find diagrams which contribute only to the mass but not to the magnetic moment since all the intermediate particles are neutral. The mass contribution from these more exotic diagrams can be arranged to be cancelled by the mass contribution from the usual diagrams. However, this scenario sets too low an intermediate mass scale to be acceptable.

Recently, Barr *et al* (BFZ) [10] have discovered a very elegant mechanism, based on Lorentz symmetry, to suppress the contribution to the mass which at the same time allows a sizeable magnetic moment large enough to explain the solar neutrino deficit and, in particular, the anticorrelation of solar neutrino flux with solar activity. Here, the magnetic transition moment and the transition mass are induced at the two-loop level but the contribution to the transition magnetic moment does not involve a chirality flip in the internal lepton line. Due to spin considerations, the mass contribution has a strong-suppression proportional to the charged lepton mass squared in comparison with the dimensional estimate $m_\nu \sim M^2/e\,\mu_\nu$. It is interesting to note that this kind of scenario is especially useful for decaying dark matter (DDM)

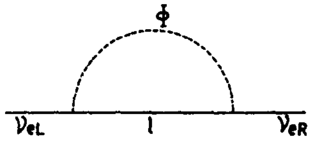


**Figure 1.** A generic one loop diagram for induced mass in the standard model.

hypothesis [11] where dark matter (DM) could consist of a massive neutrino which decays into hydrogen ionizing photons with a life time $10^{23}$ s. The DDM hypothesis could solve, besides the DM problem, several ionization problems in astrophysics and cosmology if the heavy neutrino mass lies between 27·8 eV and 29 eV and the residual light neutrino in the decay has a mass less than 3 eV. In the DDM scenario, the magnetic moment interactions are responsible for the decay of the heavy neutrino which, however, generate large neutrino oscillations whenever the induced transition mass and the transition magnetic moment are generated at the same loop level and the necessary chirality flip is realized on the internal fermion line. However, this is precisely the situation in the most viable models for large neutrino magnetic moment proposed so far and this would invalidate the DDM scenario from the viewpoint of particle physics in spite of its tremendous potential in astrophysics and cosmology. Thus DDM hypothesis would especially favour models for large neutrino magnetic moment based on BFZ mechanism. The implementation of this mechanism allows to realize naturally the DDM scenario without dangerous neutrino oscillations since the mixing angle becomes suppressed by a factor $(m_l|M_{\text{loop}})^2$.

In fact, BFZ examined two-loop contributions to neutrino mass and magnetic moment in a Zee-type model containing an additional scalar doublet. However, BFZ realized that the 'spin polarization mechanism' operating in their model is quiet general and could be implemented in a general class of models with or without lepton flavor symmetry. BFZ also emphasized the need to examine the two-loop moments in the original Zee model with just two Higgs doublets. The realization that the two-loop neutrino magnetic moment could be large enough to solve the solar neutrino puzzle actually motivated us to undertake an investigation of two loop contribution to mass and magnetic moment of Majorana neutrinos in a variant of a model proposed recently [12] with an explicit lepton flavor symmetry.

## 2. Laboratory, astrophysical and cosmological constraints on neutrino magnetic moment

Two different types of magnetic moments are possible for the neutrino: a static Dirac-type moment would transform the neutrino into a sterile right-handed neutrino whereas the Majorana moment would be essentially a transition moment that would connect a left-handed neutrino of one generation to an active right-handed antineutrino of another generation via magnetic moment interactions of the type $\bar{\nu}_e\sigma_{\mu\nu}\nu^c_\mu F^{\mu\nu}$. However, both types of electromagnetic moments are severely constrained by reactor experiments as well as from astrophysics and cosmology. Neutrino magnetic moments of both types would contribute to the scattering of reactor neutrinos on electrons via photon exchange and the resulting amplitude does not interfere with the standard weak amplitude as a result of the difference in the final state helicities so that the consistency with the standard model [13] implies a constraint $\mu_\nu < 4 \times 10^{-10}\mu_B$ which is still large enough for a solution of the solar neutrino puzzle. A large neutrino magnetic moment either or Dirac of Majorana type would lead to plasmon decay $\gamma \to \bar{\nu}\nu'$ inside a star which would drastically alter stellar evolution which severely constrains neutrino magnetic moment [14]. This implies a bound $\mu_\nu \leqslant 1{\cdot}3 \times 10^{-10}\mu_B$ from the sun: $\mu_\nu \leqslant 0{\cdot}4 \times 10^{-10}\mu_B$ from white dwarf cooling and a little more stringent bound $\mu_\nu \leqslant 0{\cdot}14 \times 10^{-10}\mu_B$ from red giants. Also Dirac magnetic moments for the neutrinos are severely constrained from big bang nucleosynthesis scenario which puts an upper limit of 3·3 on the number of neutrino species in equilibrium with the plasma during the nucleosynthesis epoch. If the Dirac magnetic moment is larger than $2 \times 10^{-11}\mu_B$, then the right-handed neutrinos would achieve equilibrium via their magnetic moment interactions with electrons which would add another neutrino species in conflict with the nucleosynthesis bound [15]. The observed neutrino pulse from the supernova SN 1987A severely constrains Dirac moment since the magnetic moment interactions of the photon could mediate the reaction $\nu_L e \to \nu_R e$ in the supernova core and $\nu_R$s thus produced would escape freely thus draining energy. The magnetic moment interactions themselves could trap the right-handed neutrinos provided $\mu_\nu \geqslant 10^{-9}\mu_B$ which, however, violates the laboratory bound. The requirement that the energy drained by right-handed neutrinos be smaller than the total energy released in the supernova collapse imposes a very stringent bound $\mu_\nu \leqslant 10^{-12}\mu_B$ on the Dirac type moments [16]. One could postulate new interactions for $\nu_R$ which would trap $\nu_R$s inside the supernova core but these very interactions would also keep $\nu_R$s in equilibrium during nucleosynthesis epoch thus contributing an extra neutrino species and violating the nucleosynthesis bound. This would rule out the relevance of Dirac-type magnetic moment for the solar neutrinos. However, the supernova and the nucleosynthesis bounds do not apply to Majorana transition moment as the right-handed component is active and would not drain energy away. We propose to enlarge this list of already known constraints by including the DDM constraint discussed in detail in § 1.

## 3. Model building for large neutrino magnetic moment

In the standard model based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_y$, the fermions are assigned to the following representation of the gauge group:

$$q_L(3,2,1/6) = \begin{bmatrix} u \\ d \end{bmatrix}_L, \quad \begin{bmatrix} c \\ s \end{bmatrix}_L, \quad \begin{bmatrix} t \\ b \end{bmatrix}_L; \quad u_R(3,1,2/3) \text{ etc,}$$

$$\psi_L(1,2,-1/2) = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L, \quad \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L, \quad \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix}_L \quad e_R(1,1,-1) \quad \mu_R, \tau_R \text{ etc.}$$

The model assumes the existence of a single doublet of Higgs bosons

$$\Phi(1,2,\tfrac{1}{2}) = \begin{bmatrix} \Phi^+ \\ \Phi^0 \end{bmatrix}.$$

There are no right-handed neutrinos in the minimal standard model and hence Dirac masses for neutrinos are not allowed. In the minimal version, lepton number is an exact global symmetry of the standard model lagrangian and therefore Majorana masses for neutrinos are also forbidden and also the neutrinos cannot have a magnetic moment.

A simple extension of the standard model is obtained if we add right-handed neutrinos into the leptonic sector which will not spoil any of the successes of the standard model. This will lead to non-zero Dirac as well as Majorana masses and the well-known see-saw mechanism [17]. However, it is not essential to introduce right-handed neutrinos into the fermionic spectrum of the standard model to generate non-zero neutrino masses. With a little more cumbersome scalar sector lepton number violation can occur which would lead to Majorana masses for left-handed neutrinos. In fact, the symmetries of the standard model allow the following scalars to couple to the leptons:

$$\Delta(1,3,1), h^{+}(1,1,1); k^{++}(1,1,2)$$

Introducing one or more of these scalar fields leads to different models of neutrino masses [18].

Here we choose to introduce a singly charged scalar field $h^{+}(1,1,1)$ in addition to the usual Higgs doublet following Zee [19]. This leads to trilinear coupling of the type $M_{\alpha\beta}\epsilon_{ij}\Phi^i_\alpha\Phi^j_\beta h$ where $\alpha, \beta$ distinguish between different scalar doublets. The gauge assignments of $h$ allow gauge invariant Yukawa couplings of the type.

$$f_{ab}(\psi^i_{aL} C \psi^j_{bL})\varepsilon_{ij} h$$

where $(a, b)$ denote the family indices, $(i, j)$ the isospin indices and $\varepsilon_{ij}$ the antisymmetric unit symbol. Fermi statistics requires $f_{ab} = -f_{ba}$, here $C$ is the charge conjugation matrix. The couplings $M_{\alpha\beta}\varepsilon_{ij}\Phi^i_\alpha\Phi^j_\beta h$ violate lepton number by two units and could generate Majorana masses for the neutrinos. In case $\Phi_\alpha = \Phi_\beta$, Bose symmetry would force these couplings to be zero. Thus, it is necessary to have at least two scalar doublets in this model. The trilinear scalar couplings would result in a mixing of the $h$ field with the two fields and result in a finite neutrino mass via diagrams of the type shown in figure 2. The same Yukawa couplings also generate neutrino magnetic moment via charged Higgs exchange where the photon can couple to the internal charged lepton or the charged Higgs line. However, the diagram generating the magnetic moment will also generate neutrino mass after the removal of the external photon line so that it is difficult to reconcile a small neutrino mass and a large neutrino magnetic moment when the two are loop induced effects. Of course, neutrino mass being a dimension 3 operator undergoes an infinite renormalization and one can always bring it down to the observed value by adding a mass counter-term whereas the magnetic moment being a dimension 5 operator it is always finite. This, however, does not provide any insight into the lightness of the neutrino. It was pointed out by a number of authors [20] that the Zee model at one-loop level cannot induce a large enough magnetic moment for a naturally light electronic neutrino in spite of an erroneous claim made by Pal [21]. Moreover, the spin precession mechanism in Voloshin *et al* (VVO) [22] or Lim *et al* (LMA) [23] scenarios is not efficient enough in the Zee model because there is nothing to guarantee the requisite high degree of mass degeneracy between the two different Majorana neutrino states.

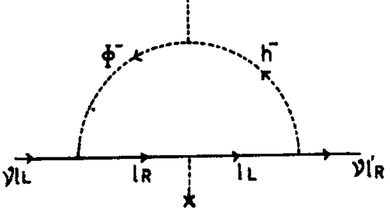


**Figure 2.** A generic one loop diagram for induced mass in the Zee model.

## 4. A model with an approximate horizontal symmetry

One possible way to achieve the required suppression of neutrino mass in comparison with the neutrino magnetic moment is to introduce an additional symmetry which forbids neutrino masses but allows electromagnetic dipole moments. Voloshin [24] noted the different transformation properties of mass and magnetic moment operators under a $SU(2)_\nu$ symmetry under which $(\nu\nu^c)$ form a doublet. The antisymmetric combination being a singlet under $SU(2)_\nu$ would be allowed but the symmetric mass terms belonging to a triplet of $SU(2)_\nu$ would be forbidden. As a consequence, Voloshin's symmetry would decouple neutrino mass from its magnetic moment. However, there is no evidence for $\nu$ transforming into $\nu^c$ in nature. Rather, $\nu$ transforms as a doublet of weak isospin $SU(2)_L$ whereas $\nu_C$ transforms as a singlet so that Voloshin's $SU(2)_\nu$ does not even commute with the standard model gauge group. Difficulties encountered in the implementation of Voloshin's symmetry in realistic models have been discussed by Barbieri and Mohapatra [25].

In this context, it is important to note that within the framework of the standard model certain parameters are magically small numbers. Electron and muon masses are a case in point. There is no satisfactory theory to explain their small values even though some models are successful in generating small values for them. In the standard model, when the Yukawa couplings associated with electron and muon masses are set to zero, the theory possesses a leptonic chiral $SU(2) \times SU(2)$ symmetry. In this limit, a custodial horizontal symmetry automatically emerges under which neutrino masses are zero but magnetic moments are non-zero. In this class of models, the smallness of neutrino masses relative to the magnetic moments is linked to the smallness of charged lepton masses relative to the gauge boson masses. Thus neutrino mass can be decoupled from its magnetic moment by embedding the standard model gauge group in a larger group. The bigger group is endowed with additional Higgs particles. To each diagram generating neutrino mass, there exists an analogous diagram involving just an additional external photon which contributes to the neutrino magnetic moment. However, each element of the mass and magnetic moment matrices is generated by the two one-loop diagrams. The extended symmetry guarantees that the two diagrams add in the case of the magnetic moment leading to a larger $\mu_\nu/m_\nu$. One of the first models of this type proposed by Barbieri and Mahapatra [25] is based on the extended symmetry $SU(3)_L \times U(1)$. The scalars belong to a $SU(3)_L$ triplet $(\eta_1^+, \eta_2^+, \eta^\sigma)$. The resulting Dirac neutrinos can have $m_{\nu_e} = 0$ and $\mu_\nu = 10^{-11}\mu_B$ provided $m_{\eta_1} = m_{\eta_2}$ which is possible for a reasonable choice of parameters. However, this can only be ensured by fine tuning after symmetry breaking [26]. Moreover, there is nothing in the model to guarantee the high degree of mass degeneracy required for spin precession mechanism to be effective. Another, model of this class proposed by Babu and Mahapatra [8] is based on the extended gauge group $SU(2)_H \times SU(2)_L \times U(1)_Y$ and contains among others a $SU(2)_H$ scalar doublet $(\eta_1^+, \eta_2^+)$ which contributes to both the neutrino mass and magnetic moment yielding a large enough $\mu_\nu/m_\nu$. This model exhibits advantages and disadvantages similar to the one proposed by Barbieri and Mahapatra [25]. The $SU(2)_H$ in this model is gauged. The model utilizes two additional real triplets for spontaneous symmetry breaking at very high energies thus reintroducing the fine tuning. In addition $SU(2)_H$ in this model has global anomalies since there is an odd number of Weyl doublets.

A $SU(2)_H$ symmetric model with Zee model type scalar spectrum was proposed by Dhar and Dev [12] and 1-loop induced transition magnetic moments and masses for

Majorana neutrinos were studied in this model. The model utilizes the lepton flavour symmetry $SU(2)_H$ acting between the first two leptonic generations which is explicitly broken only by the weak gauge interaction leaving a subgroup $U(1)_N$ of $SU(2)_H$ unbroken to ensure $\Delta m^2_{\nu_e\nu_\mu}$ to be sufficiently small for the spin precession mechanism to be effective in the convective zone of the sun. In this model $\nu_e$ and $\nu^C_\mu$ pair up to form a ZKM [27] type pseudo Dirac neutrino.

In the exact $SU(2)_H$ limit, the leptons and the scalar particles have the following assignments:

$$\psi^{a\in SU(2)_H}_{Li\in SU(2)_L} = \begin{pmatrix} \nu_e & \nu_\mu \\ e & \mu \end{pmatrix}; \quad T = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix},$$

$$\psi^a_R = (e, \mu)_R, \quad \tau_R.$$

The scalar sector is enlarged which now comprises of two additional $SU(2)_H$ doublets $D$ and $S$ apart from the standard Higgs which is a singlet under $SU(2)_H$:

$$D^a_i = \begin{pmatrix} d^0_e & d^0_\mu \\ D^-_e & D^-_\mu \end{pmatrix}; \quad S^a_i = (S^-_e\, S^-_\mu); \quad H = \begin{bmatrix} H^+ \\ H^0 \end{bmatrix}.$$

The most general $SU(2)_H$ invariant Higgs potential for the model can be written as

$$\begin{aligned}
\mathscr{L}_{\text{Higgs}} = {} & \lambda_1(H^\dagger H - \vartheta^2)^2 + \lambda_2(\mathrm{Tr}(D^\dagger D))^2 + M_D^2\mathrm{Tr}(DD^\dagger) \\
& + \lambda_3(SS^\dagger)^2 + M_S^2 SS^\dagger + \lambda_4(H^\dagger H - \vartheta^2)\mathrm{Tr}(DD^\dagger) \\
& + \lambda_5(H^\dagger H - \vartheta^2)SS^\dagger + \lambda_6\mathrm{Tr}(DD^\dagger)SS^\dagger + \lambda_7\mathrm{Tr}(DD^\dagger DD^\dagger) \\
& + \lambda_8 H^\dagger DD^\dagger H + \lambda_9 SD^\dagger DS^\dagger + \lambda_{10}(\vartheta S + H^\dagger i\tau_2 D)(\vartheta S^\dagger - D^\dagger i\tau_2 H^*) \\
& + \mu_D^2\mathrm{Tr}(D\sigma_3 D^\dagger) + \mu_S^2 S\sigma_3 S^\dagger + \lambda_{11}[\mathrm{Tr}(D\sigma_3 D^\dagger)]^2 \\
& + \lambda_{12}(S\sigma_3 S^\dagger)^2 + \lambda_{13}(H^\dagger H - \vartheta^2)\mathrm{Tr}(D\sigma_3 D^\dagger) \\
& + \lambda_{14}(H^\dagger H - \vartheta^2)S\sigma_3 S^\dagger + \lambda_{15}\mathrm{Tr}(DD^\dagger)\mathrm{Tr}(D\sigma_3 D^\dagger) \\
& + \lambda_{16}\mathrm{Tr}(DD^\dagger)S\sigma_3 S^\dagger + \lambda_{17}SS^\dagger\mathrm{Tr}(D\sigma_3 D^\dagger) \\
& + \lambda_{18}SS^\dagger S\sigma_3 S^\dagger + \lambda_{19}\mathrm{Tr}(D\sigma_3 D^\dagger)S\sigma_3 D^\dagger \\
& + \lambda_{20}H^\dagger D\sigma_3 D^\dagger H + \lambda_{21}S(1 - i\sigma_3)D^\dagger D(1 + i\sigma_3)S^\dagger \\
& + \lambda_{22}(\vartheta S + H^t i\tau_2 D)\sigma_3(\vartheta S^t - D^\dagger i\tau_2 H^*) \\
& + \delta(H^\dagger D + e^{i\alpha}H^t D^* i\sigma_2)\sigma_3(D^\dagger H - e^{-i\alpha}i\sigma_2 D^t H^*).
\end{aligned}$$

In these terms, the $\sigma_i$'s are Pauli matrices acting in the $SU(2)_H$ space, while $i\tau_2$ acts in the weak isospin space. $\delta$ is a measure of weak isospin symmetry breaking and can be fine tuned to be as small as one desires. The scalar potential contains the following invariant term $SD^\dagger i\tau_2 H^* + H.C$ apart from others. In the exact $SU(2)_H$ limit and broken $SU(2)_{WS} \times U(1)_y$ via $\langle H^0 \rangle = \vartheta$, the fields $D$ and $S$ will mix but $(S^-_e, S^-_\mu)$ as well as $(D^-_e, D^-_\mu)$ will remain degenerate to give the following physical scalars:

$$\begin{bmatrix} K \\ K' \end{bmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{bmatrix} D^a_i \\ S^a_i \end{bmatrix}$$

where $\beta$ is the mixing angle and the mass eigenstates are $K$ and $K'$. The charged physical scalars $K$ and $K'$ are mixtures of doublet and singlet under weak isospin.

The most general leptonic Yukawa couplings of the model are

$$= h_1 \operatorname{Tr}(\bar{\psi}_L H \psi_R) + h_2 \bar{T} \mathscr{H} \tau_R + h_3 \bar{T} D i \tau_2 \psi_R^t$$
$$+ f S \tau_2 \psi_L^T \tau_2 C^{-1} T + f' \operatorname{Tr}(\bar{\psi}_L D) \tau_R + \text{h.c.}$$

The last two terms are crucial for generating neutrino magnetic moment. Neutrino masses and magnetic moments are generated via the pairs of graphs given in figure 3. Contributions to induced mass and magnetic moment come from pairs of graphs through $(S_e, D_e)$ and $(S_\mu, D_\mu)$ exchange. In the exact $SU(2)_H$ limit, the masses of the scalar particles circulating inside the loop are the same, so the two graphs contribute equally. But because of the relative minus sign at one of the vertices, their contribution to induced Majorana mass cancels out but attaching a photon line on the internal loop brings in another relative minus sign (because of the difference in the direction of charged flow in the two graphs) so that the contributions of the two graphs add for the magnetic moment. However, the $SU(2)_H$ is only an approximate symmetry of the standard model and is broken explicitly by the difference in the Yukawa couplings of the first two leptonic generations characterized by the parameter

$$\frac{f_\mu - f_e}{2} = \frac{m_\mu - m_e}{2\langle H_0 \rangle} \approx 3 \times 10^{-4}$$

where $f_e$ and $f_\mu$ are the leptonic Yukawa couplings of the first two generations. The $SU(2)_H$ breaking manifests itself in a finite mass difference $m_\mu \neq m_e$. Now we choose to break $SU(2)_H$ only in the Yukawa coupling of $e$ and $\mu$ with the Higgs doublet (so as to yield a finite mass difference for the electron and the muon while leaving the subgroup $U(1)_N$ intact to ensure conservation of $N\mu - N_e$) and require all other interactions involving new scalars to be $SU(2)_H$ symmetric. This is significantly different from the pattern of horizontal symmetry breaking chosen earlier [12]. One loop contribution to neutrino mass involves only the $\tau$-lepton and the new scalars but not $e$ and $\mu$ and hence vanishes. The neutrino mass only arises at the two-loop level from one-loop corrections involving $e$ and $\mu$ to new scalar propagators. A typical 2-loop diagram contributing to neutrino mass is given in figure 4. The contribution of the fermionic subloop on the scalar propagator to neutrino mass can be estimated to be of the following order

$$m_\nu^{2\text{-loop}} \sim \frac{f^2}{16} m_\mu^2.$$

The relevant diagrams for magnetic moment are obtained by attaching an external photon line to any of the charged scalars or the internal lepton in figure 4. But the

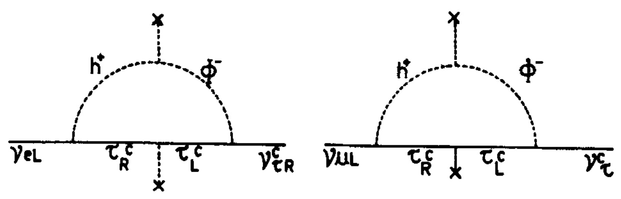

**Figure 3.** Dominant contribution to one loop induced Majorana neutrino mass in the Zee model.

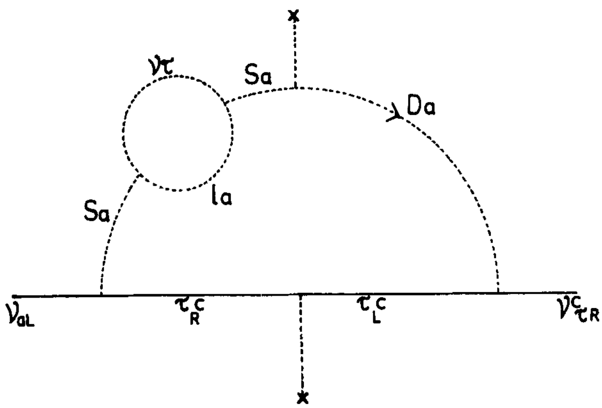


**Figure 4.** A typical two loop diagram contributing to Majorana neutrino mass.

one-loop contribution to $\mu_\nu$ estimated in [12] will be the dominant contribution:

$$\mu_\nu \sim \frac{e}{16\pi^2}\frac{f_{e\tau}f_{\mu\tau}}{M^2}m_\tau \ln(M^2/m_\tau^2)$$

One, thus, expects

$$m_\nu \sim \frac{\mu_\nu}{e}\frac{f^2}{16\pi^2}m_\mu^2 \sim 10^{-3}f^2\,\mathrm{eV}$$

yielding a neutrino mass which is experimentally unobservable. New scalar masses could now be as large as 1 TeV. Thus the model parameters are now more flexible than in [12]. The prediction of new scalars with mass of the order of 50 GeV in [12] cannot thus be taken as firm as the lightest charged scalars in this scenario could have masses of the order of 1 TeV while the resulting ZKM pseudo Dirac neutrino could be as light as $10^{-3}$ eV. Moreover, DDM constraint is naturally satisfied since the mass and magnetic moment are not induced at the same loop level now.

## 5. Conclusions

Decaying neutrino dark matter hypothesis would solve several ionization problems in cosmology and astrophysics provided the heavy neutrino decays into a light neutrino and hydrogen ionizing photons via magnetic moment interactions. The requisite lifetime of the order of $10^{23}$ s is obtained with a transition magnetic moment $\mu_{\nu_h\nu_e} \sim 10^{-14}\mu_B$. However, the same magnetic moment interactions would generate dangerously large neutrino oscillations whenever the transition mass and transition magnetic moment are induced at the same loop level and the necessary chirality flip is realized on an internal fermion line. It is noted that this is precisely the situation with the most viable models for large neutrino magnetic moment proposed so far. This would render the DDM scenario rather unattractive from the viewpoint of particle physics in spite of its tremendous success in astrophysics and cosmology. The need to construct gauge models for large neutrino magnetic moment not in conflict with DDM hypothesis has been emphasized. A model for large neutrino magnetic moment consistent with DDM hypothesis has been presented. Two loop contributions to induced Majorana mass and transition magnetic moment have been estimated. The induced neutrino mass in this model only arises at the two loop level from one loop corrections involving the electron and the muon to the new scalar propagators and is too small to be experimentally observable. The one loop induced neutrino magnetic moment, on the other hand, is the dominant contribution in this model and is large enough for the solution of the solar neutrino puzzle. The masses of the new charged scalars are pushed to the TeV range as an additional advantage.

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