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Charge screening in a two potential theory of electromagnetism

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Abstract. By surrounding an Abelian Dyon with an axion-like domain wall we calculate the screening effects generated by the domain wall on the dyon degrees of freedom.

Keywords. Dyon; charge screening; two potential theory.

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1. Introduction

In theories of elementary particle phenomena screening effects play an important role firstly because they determine the attractive, repulsive and confining properties of a gauge field [1] and secondly because screening effects alter secondary phenomena such as proton decay [2] and phase transitions such as the deconfinement transition and the chiral phase transition in QCD [3]. It is known in QED that a bare electric charge is screened by a vacuum polarized surrounding cloud of opposite charge to give rise to a renormalized finite charge [4] and in QCD a given coloured quark is screened by anti-coloured quarks but anti-screened by like coloured gluons to generate the confining QCD potential [5]. In a previous note [6] we have discussed a somewhat related but different phenomena, namely the screening of the electric charge degree of freedom of an Abelian dyon by an axion cloud. The analysis of that investigation showed that the electric charge could be screened, reversed in sign or anti-screened by a surrounding axion cloud depending on the boundary conditions of the axion field at the interior and exterior of the configuration.

In an interesting study Fischler and Preskill [7] have shown how monopoles might deplete the axion density to acceptable levels in the early universe so as not to overclose the universe by forcing (θ_{QCD}) the vacuum angle of QCD to approach 0. (Here θ_{QCD} is the coefficient of the topological CP violating term induced by the non-trivial vacuum structure of QCD). In that investigation the authors demonstrated that the axion field would screen the electric charge of a dyon but not its magnetic charge. Thus in addition to altering the properties of the dyon degree of freedom the interplay between the axion-like field and the dyon parameters may resolve problems in early universe cosmology that have no other solution.

In a subsequent note we have studied screening of a $U(1)\times U(1)$ gauge theory where each $U(1)$ group was represented by an electric-like and magnetic-like potential, the analysis demonstrated that screening and anti-screening were dependent on the strength of the coupling [8]. Following this note we discussed the screening of an Abelian dyon by a spherically symmetric domain wall where the positions of the degenerate minimum determine the screening effects of the domain wall [9]. As mentioned in [9], topological defects such as strings, monopoles and domain walls occur in a spontaneously broken gauge theory and the trapping of a monopole or dyon by a domain wall might be a likely occurrence in the early universe. It was also stressed that the screening of a dyon by a domain wall should have far ranging consequences in the space surrounding the exterior of the domain wall altering such phenomena as the catalysis of proton decay and plasma generated density perturbations that may serve as the origin of large scale structure. In the following note we discuss the screening of both electric and magnetic charge by a domain wall in a theory where the electromagnetic field is represented by a two potential theory (electric and magnetic potential) [10].

To-date there is no conclusive evidence that would either confirm or refute the two potential theory since magnetic monopole (magnetic charge) searches have to-date not produced any conclusive evidence for their existence [11]. Since monopoles are a necessary consequence of spontaneously broken gauge theories [12] their detection may be difficult due to their extraordinarily large masses. In a recent paper S Davidson and M Peskin [13] have discussed the existence of a paraphoton interacting with milli-charged particles and given astrophysical bounds on such particles. Holdom [14] has shown how a second photon can be a natural consequence of GUT models and can be produced in the laboratory via Compton scattering of normal photons. Possibly these developments might suggest that the existence of a second photon could be accommodated by a two potential theory. Actually, Callegari *et al* [15] have shown how a two potential theory of massive photons could be tested for by looking for corrections to the earth's magnetic field. One of the shortcomings of the two potential theory is that when electromagnetism interacts with fermions it leads to an effective non-linear lagrangian which implies a coupling between the electric like and magnetic like photons, this would suggest that high fields would necessarily generate magnetic like photons that would effect phenomena such as photon splitting in a magnetic field [16] which has been calculated to many orders of perturbation theory in normal QED and awaits experimental confirmation perhaps in the radiation coming from pulsars where the magnetic field is high. Using the two potential theory we calculate the screened values of the electric and magnetic charge as seen exterior to the domain wall as well as an approximate expression for the electric field, magnetic field and pseudo-scalar axion field within the wall. Finally, we develop an approximate expression for the mass of the domain wall in terms of the parameters in the lagrangian and the central dyon charges.

2. Electric and magnetic charge screening in a two potential theory

We begin our analysis by writing the following axion-like electromagnetic lagrangian of a pseudo-scalar field coupled to electromagnetism and gravity as

$$
\begin{aligned}
\mathscr{L} &= \frac{c^4}{16\pi G} R\sqrt{-g} + \left[\frac{\partial^\mu\phi\partial_\mu\phi}{2} - \frac{A_2}{4}\left(\phi^2 - \frac{A_1}{A_2}\right)^2\right]\sqrt{-g} \\
&\quad + \alpha\phi\left(\frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{\sqrt{-g}}\right)\sqrt{-g} - \frac{1}{16\pi} + F_{\mu\nu}F^{\mu\nu}\sqrt{-g} \qquad (2.1) \\
&= \mathscr{L}_G + \mathscr{L}_M
\end{aligned}
$$

where α is the coupling constant.

The kinetic term in (2.1) is the usual kinetic term in any scalar or pseudo-scalar theory and the potential has degenerate minimum at

$$\phi = \pm (A_1/A_2)^{1/2}$$

The phenomenological coupling between the pseudo-scalar field and the electromagnetic field is analogous to the axion electromagnetic interaction discussed by Peccei and Quinn [17], Weinberg [18], Wilczek [19] and Kim [20] which represents a low energy effective term after the heavy fields in the theory are integrated out. Here since α depends on the inverse mass of the heavy fields we expect it to be small and thus screening effects depending on α to be small. The last term is the usual kinetic term of electromagnetism.

For the metric in this problem we choose

$$(\mathrm{d}S)^2 = e^{\nu}(\mathrm{d}x^4)^2 - e^{\lambda}(\mathrm{d}r)^2 - r^2(\mathrm{d}\theta)^2 - r^2\sin^2\theta(\mathrm{d}\bar{\phi})^2$$

where e^{ν} e^{λ} depend only on r. The electromagnetic field is described by [10]

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial X^\nu} - \frac{\varepsilon_{\mu\nu}{}^{\alpha\beta}}{2\sqrt{-g}}\left(\frac{\partial B_\alpha}{\partial x^\beta} - \frac{\partial B_\beta}{\partial x^\alpha}\right)$$

where A_μ = electric four potential, B_μ = magnetic four potential, ϕ = pseudo-scalar field with the potential term containing degenerate minimum at

$$\phi = \pm (A_1/A_2)^{1/2}.$$

In the above expression for $F_{\mu\nu}$ we note we have two potentials A_μ, B_μ; this does not necessarily imply the existence of two photons since the theory is equivalent to the usual theory in the absence of magnetic charge and interactions. In the presence of magnetic charge and interactions quanta of the B field will be associated with the magnetic charge existing as virtual or free photons and possibly mixing with the normal photon.

Varying (2.1) with respect to ϕ gives

$$-\Box\phi - A_2\phi\left(\phi^2 - \frac{A_1}{A_2}\right) + \frac{\alpha\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{\sqrt{-g}} = 0 \tag{2.2}$$

Varying (2.1) with respect to A_μ gives

$$\frac{\partial}{\partial x^\nu}\left(\frac{1}{4\pi}\sqrt{-g}F^{\mu\nu}\right) - 4\alpha\frac{\partial}{\partial x^\nu}(\phi\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}) = 0 \tag{2.3}$$

Varying (2.1) with respect to B_μ gives

$$\frac{\partial}{\partial x^\nu}\left(\frac{1}{4\pi}\tilde{F}^{\mu\nu}\right) - 4\alpha\frac{\partial}{\partial x^\nu}\left(\frac{\phi\varepsilon^{\mu\nu\alpha\beta}\tilde{F}_{\alpha\beta}}{\sqrt{-g}}\right) = 0$$

$$\tilde{F}^{\mu\nu} = \frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}}{2} \tag{2.4}$$

The energy momentum tensor for the pseudo-scalar field and EM field is from (2.1)

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial L_M}{\partial g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - \frac{g_{\mu\nu}}{2}(\partial^\alpha\phi\partial_\alpha\phi) + g_{\mu\nu}\frac{A_2}{4}\left(\phi^2 - \frac{A_1}{A_2}\right)^2$$
$$+ \frac{1}{16\pi}g_{\mu\nu}(F_{\alpha\beta}F^{\alpha\beta}) - \frac{1}{4\pi}F_{\mu\alpha}F^\alpha_\nu. \quad (2.5)$$

We now position an Abelian dyon of mass M and electric and magnetic charge e, q so that its centre is at $r = 0$, we call the effective radius of the dyon R_D, for $R_D \leqslant r \leqslant R_1$ we have the minimum

$$\phi = -(A_1/A_2)^{1/2}.$$

Actually by (2.2) we have a variation of ϕ over the region $R_D \leqslant r \leqslant R_1$ but if R_1 and R_D are close to one another we may neglect the variation of ϕ over $R_D \leqslant r \leqslant R_1$ and set $\phi = -(A_1/A_2)^{1/2}$to a good approximation. At $r = R_2$ (outer boundary of domain wall) we set

$$\phi = (A_1/A_2)^{1/2}$$

and neglect the variation of ϕ for $r > R_2$ since ϕ will change very slowly by (2.2) if R_2 is far enough from the dyon centre where E, B will vary as $1/r^2$. Here R_1 and R_2 are arbitrary except for the fact that R_1 is close to R_D so as to ensure that the variation of the axion-like field is small for $R_D \leqslant r \leqslant R_1$ and R_2 is large enough so that the variation of ϕ for $r > R_2$ can be neglected. Thus

$$\phi = -(A_1/A_2)^{1/2} \text{ for } R_D \leqslant r \leqslant R_1,$$
$$\phi = (A_1/A_2)^{1/2} \text{ for } r > R_2$$

(R_1, R_2 = inner and outer radii of domain wall respectively). The two degenerate minima are separated by a domain wall where the potential attains a maximum somewhere between R_1 and R_2. Also $F_{14} = E(r)$, $F_{23} = r^2 \sin\Theta B(r)$. ($E, B$ = radial electric and magnetic field). For $R_D < r < R_1$ we may integrate (2.3) to give

$$\frac{r^2 E}{4\pi} - 8\alpha\phi r^2 B = \frac{e}{4\pi} \quad (2.6)$$

and (2.4) integrates to give

$$\frac{r^2 B}{4\pi} + 8\alpha\phi r^2 E = \frac{q}{4\pi} \quad (2.7)$$

where

$$\phi = -(A_1/A_2)^{1/2} \text{ for } R_D < r \leqslant R_1.$$

(e and q are electric and magnetic charges of the central dyon.)
In (2.6) and (2.7) we have used $\lambda + \nu = 0$ for $R_D < r < R_1$ which follows from the form of the energy momentum tensor ($T^1_1 = T^4_4$) in (2.5) for $R_D < r < R_1$ and the Einstein equations.

Solving (2.6) and (2.7) gives

$$E = \frac{1}{r^2}\left(\frac{e - 32\pi\alpha(A_1/A_2)^{1/2}q}{1 + (32\pi\alpha(A_1/A_2)^{1/2})^2}\right) = \frac{\bar{e}}{r^2} \tag{2.8}$$

$$B = \frac{1}{r^2}\left(\frac{q - 32\pi\alpha(A_1/A_2)^{1/2}e}{1 + (32\pi\alpha(A_1/A_2)^{1/2})^2}\right) = \frac{\bar{q}}{r^2} \tag{2.9}$$

($\bar{e}, \bar{q}$ are the effective electric and magnetic charge for $R_D < r \leqslant R_1$).
For $r > R_2$ we have upon integrating (2.3) and (2.4) and setting

$$\phi = (A_1/A_2)^{1/2}$$

$$E = \frac{1}{r^2}\left[\frac{e_E + 32\pi\alpha(A_1/A_2)^{1/2}q_E}{1 + (32\pi\alpha(A_1/A_2)^{1/2})^2}\right] = \frac{\bar{\bar{e}}}{r^2} \tag{2.10}$$

$$B = \frac{1}{r^2}\left[\frac{q_E + 32\pi\alpha(A_1/A_2)^{1/2}e_E}{1 + (32\pi\alpha(A_1/A_2)^{1/2})^2}\right] = \frac{\bar{\bar{q}}}{r^2} \tag{2.11}$$

($\bar{\bar{e}}, \bar{\bar{q}}$ are the effective electric and magnetic charge seen for $r \geqslant R_2$).

In (2.10) and (2.11) we have again used $\lambda + \nu = 0$ for $r > R_2$ which follows from (2.5) and the Einstein equation for $r > R_2$. Here e_E, q_E = exterior electric and magnetic charge seen for $r > R_2$ and correspond to the rhs in (2.6) and (2.7) for $r > R_2$.
Now integrating (2.3) and (2.4) between R_1 and R_2 with

$$\phi_{R_1} = (A_1/A_2)^{1/2}, \ \phi_{R_2} = (A_1/A_2)^{1/2}$$

$$\bar{\bar{e}} - \bar{e} - 32\pi\alpha(A_1/A_2)^{1/2}\bar{\bar{q}} - 32\pi\alpha(A_1/A_2)^{1/2}\bar{q} = 0 \tag{2.12}$$

$$\bar{\bar{q}} - \bar{q} - 32\pi\alpha(A_1/A_2)^{1/2}\bar{\bar{e}} + 32\pi\alpha(A_1/A_2)^{1/2}\bar{e} = 0 \tag{2.13}$$

or

$$\bar{\bar{e}} - 32\pi\alpha(A_1/A_2)^{1/2}\bar{\bar{q}} = 32\pi\alpha(A_1/A_2)^{1/2}\bar{g} + \bar{e} \tag{2.14}$$

$$\bar{\bar{e}}(32\pi\alpha(A_1/A_2)^{1/2}) + \bar{\bar{q}} = \bar{q} - 32\pi\alpha(A_1/A_2)^{1/2}\bar{e}. \tag{2.15}$$

Solving (2.14) and (2.15) for $\bar{\bar{e}}$, $\bar{\bar{q}}$ gives

$$\bar{\bar{e}} = \frac{\bar{e}(1 - (32\pi\alpha(A_1/A_2)^{1/2})^2) + 64\pi\alpha(A_1/A_2)^{1/2}\bar{q}}{(1 + (32\pi\alpha(A_1/A_2)^{1/2})^2)} \tag{2.16}$$

$$\bar{\bar{q}} = \frac{\bar{q}(1 - (32\pi\alpha(A_1/A_2)^{1/2})^2) - 64\pi\alpha(A_1/A_2)^{1/2}\bar{e}}{(1 + (32\pi\alpha(A_1/A_2)^{1/2})^2)}. \tag{2.17}$$

Using (2.16), (2.17) along with (2.8), (2.9), (2.10) and (2.11) for the definitions of $\bar{e}, \bar{q}, \bar{\bar{e}}, \bar{\bar{q}}$ we have upon solving for e_E, q_E in terms of e, q

$$e_E = e$$

$$q_E = q.$$

From (2.16) we see that the domain wall screens $\bar{e}$ through the first term in (2.16) and anti-screens $\bar{e}$ through the second term which contains the effective magnetic charge $\bar{q}$. If $32\pi\alpha(A_1/A_2)^{1/2} < 1$ the dominant term is the anti-screening term in (2.16) which contains the effective magnetic charge $\bar{q}$.

From (2.17) we see that the effective magnetic charge $\bar{q}$ is screened by the effective electric charge $\bar{e}$ and the first term in (2.17) also screens the effective magnetic charge $\bar{q}$. If $32\pi\alpha(A_1/A_2)^{1/2} > 1$ then both the effective electric charge $\bar{e}$ and effective magnetic charge $\bar{q}$ can be screened through the first term in (2.16) and both terms in (2.17). If instead of the degenerate minima

$$\phi_{R_1} = -(A_1/A_2)^{1/2}$$

and

$$\phi_{R_2} = (A_1/A_2)^{1/2}$$

we have

$$\phi_{R_1} = (A_1/A_2)^{1/2}, \phi_2 = -(A_1/A_2)^{1/2}$$

then (2.16) and (2.17) would be altered by letting

$$(A_1/A_2)^{1/2} \rightarrow -(A_1/A_2)^{1/2}.$$

This would lead to a screening of the electric charge by the magnetic charge $\bar{q}$ and anti-screening of the magnetic charge by the electric charge for

$$32\pi\alpha(A_1/A_2)^{1/2} < 1.$$

This would be a reverse effect as that contained in (2.16) and (2.17). For

$$32\pi\alpha(A_1/A_2)^{1/2} > 1$$

again both electric charge and magnetic charge would be screened. This curious property of a transition from screening to anti-screening or anti-screening to screening brought about by interchanging the degenerate minimum of ϕ suggests that the symmetry $\phi \rightarrow -\phi$ enjoyed for a pure scalar field theory is broken locally in the lagrangian of (2.1) and globally by the effective dyon charges seen by a distant observer. In Ref. [9] we demonstrated that for a one potential theory the magnetic charge remains unscreened, but the electric charge is either screened or anti-screened depending on which degenerate minima of ϕ is on the interior of the domain wall and which is on the outside. In another investigation [21] we have discussed the screening of a $U(1) \times U(1)$ dyon in a theory with two scalar fields each having degenerate minimum of

$$\phi_1 = \pm(A_1/A_2)^{1/2}, \quad \phi_2 = \pm(\bar{A}_1/\bar{A}_2)^{1/2}$$

on either side of the domain wall, here we have four different possible configurations and we found if the negative minimum are on the interior of the wall, the electric-like charges of both $U(1)$ factors are anti-screened, if the positive minimum of ϕ_1, ϕ_2 are on the interior of the wall both electric-like charges are screened. Here for the actual magnitude of the screen charges we have four different possibilities since the interior of the wall can be at either

$$\phi_1 = \pm(A_1/A_2)^{1/2}, \quad \phi_2 = \pm(\bar{A}_1/\bar{A}_2)^{1/2}.$$

The strength of the coupling constant also effects the magnitude of the screened charges. Again, as in [ref. 9] the magnetic-like charges remained unscreened.

To test the above theories we would first have to identify a dyon in an astrophysical setting, if the magnetic charges of a series of dyons remain the same we would be driven to choose a one-potential theory. If however the magnetic charge varied for a series of similar dyons it would suggest a two potential theory. Phenomena sensitive to the screened charges include, Zeeman like splitting of atomic hydrogen in the field of a dyon, dyon induced plasma oscillations and photon splitting for high magnetic fields. The multiplicity of plasma frequencies, Zeeman like splittings and photon orientations from splitting phenomena could be used to distinguish amongst the above theories.

Also all elementary particle phenomena in the field of a dyon would be sensitive to high magnetic fields and this would lead to altered life-times, altered left right asymmetries and altered back-forth asymmetries which could be used to probe the dyon properties if cosmic rays from the dyon region could be analyzed. The reversal or transition from anti-screening to screening by adjustment of the coupling constant (α) and the vacuum expectation values $\pm(A_1/A_2)^{1/2}$ suggests that any cosmological conditions that alter the strength of α and $(A_1/A_2)^{1/2}$ can alter the electromagnetic effect of dyons and thus alter such phenomena as the catalysis of proton decay as well as dyon induced density fluctuations giving rise to the seeds of large scale structure.

To calculate the evolution of the pseudo-scalar field within the domain wall we first integrate (2.3) and (2.4) from R_1 to r within the domain wall to obtain (using the approximation $e^{\lambda} \approx e^{\nu} \approx 1$ for $R_1 \leqslant r \leqslant R_2$)

$$\frac{r^2 E(r)}{4\pi} - \frac{1}{4\pi}\bar{e} = 8\alpha\phi r^2 B(r) + 8\alpha(A_1/A_2)^{1/2} r^2 \frac{\bar{q}}{r^2} \tag{2.18}$$

$$\frac{r^2 B(r)}{4\pi} - \frac{1}{4\pi}\bar{q} = -8\alpha\phi E(r) r^2 - 8\alpha(A_1/A_2)^{1/2}\left(\frac{\bar{e}}{r^2}\right) r^2. \tag{2.19}$$

Solving (2.18) and (2.19) for E, B gives for $R_1 < r < R_2$

$$E(r) = \frac{\bar{e}\left(r^2 - 102\pi^2\alpha^2\phi(A_1/A_2)^{1/2} r^2\right) + \bar{q}\left(32\pi\alpha(A_1/A_2)^{1/2} r^2 + 32\pi\alpha\phi r^2\right)}{r^4 + 1023\pi^2\phi^2 r^4} \tag{2.20}$$

$$B(r) = \frac{\bar{q}\left(r^2 - 1024\pi^2\alpha^2 r^2 (A_1/A_2)^{1/2}\phi\right) - \bar{e}\left(32\pi\alpha\phi r^2 + 32\pi\alpha(A_1/A_2)^{1/2} r^2\right)}{r^4 + 1024\pi^2\alpha^2\phi^2 r^4} \tag{2.21}$$

To solve for ϕ within the wall we write out (2.2) as

$$\frac{\mathrm{d}}{\mathrm{d}r}(r^2\phi_{,r}) - r^2 A_2 \phi(\phi^2 - A_1/A_2) - 8\alpha r^2 E(r) B(r) = 0. \tag{2.22}$$

For ϕ we have

$$\phi(r = R_1) = -(A_1/A_2)^{1/2}, \quad \phi(r = R_2) = (A_1/A_2)^{1/2} \tag{2.23}$$

we set

$$\frac{\mathrm{d}\phi}{\mathrm{d}r}(r = R_1) = K_0. \tag{2.24}$$

We may develop a power series solution for ϕ about $r = R_1$ as

$$\phi(r) = \phi(R_1) + \left(\frac{\mathrm{d}\phi}{\mathrm{d}r}\right)_{R_1}(r - R_1) + \frac{1}{2}\left(\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2}\right)_{R_1}(r - R_1)^2 \ldots \tag{2.25}$$

by using $\phi(r = R_1)$ from (2.23) and K_0 for $\left(\frac{\mathrm{d}\phi}{\mathrm{d}r}\right)_{r=R_1}$ and calculating higher derivatives of ϕ at $r = R_1$ from Eq. (2.22) after substituting E, B from (2.20) and (2.21), in this way we may calculate ϕ to any order about $r = R_1$ from (2.25). To evaluate K_0 in an approximate manner we take a finite number of terms in (2.25) and set $\phi(r = R_2) = (A_1/A_2)^{1/2}$ and solve for K_0.

To obtain an approximate expression for the mass of the domain wall we have the energy momentum tensor from (2.5)

$$T^\mu_\nu = \partial^\mu\phi\partial_\nu\phi - \frac{1}{2}\delta^\mu_\nu(\partial^\alpha\phi\partial_\alpha\phi) + \frac{\delta^\mu_\nu}{4}A_2\left(\phi^2 - \frac{A_1}{A_2}\right)^2 + \frac{1}{16\pi}\delta^\mu_\nu(F_{\alpha\beta}F^{\alpha\beta}) - \frac{1}{4\pi}F^\mu_\alpha F^\alpha_\nu. \tag{2.26}$$

For $R_D < r < R_1$ we have using $\lambda + \nu = 0$ for $R_1 > r > R_D$

$$T^1_1 = T^4_4 = \frac{1}{8\pi}\left(\frac{\bar{e}^2}{r^4}\right) + \frac{1}{8\pi}\left(\frac{\bar{q}^2}{r^4}\right).$$

For $\infty > r > R_2$ we have using $\lambda + \nu$ for $\infty > r > R_2$.

$$T^1_1 = T^4_4 + \frac{1}{8\pi}\left(\frac{\bar{e}^2}{r^4}\right) + \left(\frac{\bar{q}^2}{R^4}\right).$$

For $R_2 > r > R_1$ we have using (2.26)

$$T^4_4 = \frac{1}{2}(\phi_{,r})^2 + \frac{A_2}{4}\left(\phi^2 - \frac{A_1}{A_2}\right)^2 + \frac{E^2}{8\pi} + \frac{B^2}{8\pi}. \tag{2.27}$$

Here we have approximated $e^\nu \approx e^\lambda \approx 1$ for $R_1 < r < R_2$.In (2.27) we substitute ϕ from (2.25) and E, B from (2.20) and (2.21). From the $\binom{4}{4}$ Einstein equation we have

$$\frac{\mathrm{d}}{\mathrm{d}r}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4}T^4_4 r^2$$

integrating between R_1 ad R_2 we have

$$(re^{-\lambda})_{R_2} - (re^{-\lambda})_{R_1} = R_2 - R_1 - \frac{8\pi G}{c^4}\int_{R_1}^{R_2} r^2 T^4_4 \mathrm{d}r \tag{2.28}$$

here

$$(e^{-\lambda})_{R_1} = 1 - \frac{2GM}{R_1c^2} + \frac{G\bar{e}^2}{R_1^2c^4} + \frac{G\bar{q}^2}{R_1^2c^4} \tag{2.29}$$

$$(e^{-\lambda})_{R_2} = 1 - \frac{2G(M + M_D)}{R_1c^2} + \frac{G\bar{e}^2}{R_2^2c^4} + \frac{G\bar{q}^2}{R_2^2c^4}. \tag{2.30}$$

Equations (2.29) and (2.30) follow from the $\binom{4}{4}$ Einstein equations in the regions $R_D < r < R_1$ and $r > R_2$ respectively, here we have used the continuity of the metric at $r = R_1$ and R_2 and the expressions for T_4^4 in each region when we integrate the $\binom{4}{4}$ Einstein equation.

Also M_D = mass of domain wall and M = mass of central dyon. When (2.29), (2.30) and T_4^4 from (2.27) are substituted into (2.28) we may solve for M_D (mass of domain wall).

3. Conclusion

The two potential theory of electromagnetism coupled to the axion-like pseudo-scalar has led to electric charge anti-screening and magnetic charge screening for $32\pi\alpha(A_1/A_2)^{1/2} < 1$ and electric charge screening and magnetic charge screening for $32\pi\alpha(A_1/A_2)^{1/2} > 1$ from (2.16) and (2.17). The transition from screening to anti-screening may be determined by the cosmological conditions affecting α (the coupling constant in (2.1), thus the electric charge degree of freedom of a dyon may be more effective than the magnetic charge as the universe cools which in turn produces a delayed effect on particle phenomena such as the catalysis of proton decay (since the magnetic field is diminished) as well as plasma induced density perturbations stimulated by the dyon field. Our results here differ from previous results [Ref. 9] for a one potential electromagnetic theory which always produced an anti-screening effect on the electric charge for a domain wall with interior minimum at $-(A_1/A_2)^{1/2}$ and exterior minimum at $(A_1/A_2)^{1/2}$.

The fascinating feature of this investigation is that it offers us a test for a two potential theory because the magnetic charge is also effected by the aomain wall. It might very well be that the domain wall transforms a quantized magnetic charge into a continuum of values (dependent on parameters of a wall) which could show up in exterior secondary phenomena sensitive to magnetic fields such as the rate of elementary particle interactions, magnetic charge plasma oscillations, and Zeeman like spliting of surrounding atomic hydrogen atoms in a dyon induced magnetic field. In closing, if dyons were discovered in an astrophysical setting, their properties probed external to the topological defect could not only distinguish between the one potential theory and two potential theory, but also serve to probe the precise nature of the gauge-field pseudo-scalar coupling.

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References

[1] M Creutz, *Phys. Rev.* **D21**, 2308 (1980)
[2] C G Callan Jr. *Phys. Rev.* **D26**, 2058 (1982)
[3] K G Wilson, *Phys Rev.* **D10**, 2445 (1974)
[4] A Euler and W Heisenberg, *Z. Phys.* **98**, 714 (1936)
[5] C Rebbi, *Lattice gauge theories and Monte Carlo simulations* (World Scientific Pub. Co., Singapore, 1983)
[6] C Wolf, *Acta Phys. Slovaca* **41, (No. 2)** 109 (1991)

[7] W Fischler and J Preskill, *Phys., Lett.* **B125**, 165 (1983)
[8] C Wolf, *An. Found. Louis de Broglie* **18**, 15 (1993)
[9] C Wolf, *Can. J. Phys*, **70**, 391 (1992)
[10] M Cabibbo and E Ferrari, *Il. Nuovo Cimento* **23**, 1147 (1962)
[11] W P Trower, *Phys. Rev.* **D50**, 1793 (1994)
[12] E W Kolb and M S Turner, *The early universe* (Addison-Wesley Pub. Comp., Reading Mass, 1990), pp. 233
[13] S Davidson and M Peskin, *Phys. Rev.* **D49**, 2114 (1994)
[14] B Holdom, *Phys. Lett.* **B166**, 196 (1986)
[15] P Callegari, P Fortini and D Tartari, *Acta Phys. Hung.* **67**, 359 (1990)
[16] S Adler, *Ann. Phys. (N.Y.)* **67**, 599 (1971)
[17] R Peccei and H Quinn, *Phys. Rev. Lett.* **38**, 1440 (1978)
[18] S Weinberg, *Phys. Rev. Lett.* **40**, 223 (1978)
[19] F Wilczek, *Phys. Rev. Lett.* **40**, 279 (1978)
[20] J E Kim, *Phys. Rev. Lett.* **43**, 103 (1979)
[21] C Wolf, *Pramana – J. Phys.* **39**, 37 (1992)