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Dynamic sheath expansion and ion current in transient ion sheath experiments

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Abstract. Experimental results on the measurement of current collected by an electrode immersed in a plasma for a pulsed negative bias are presented. The measured current is compared with a model based on the concept of an expanding capacitor. The scaling laws predicted by the model are verified for the measured current which agree each other. The paper emphasizes the role of displacement current in an expanding ion sheath.

Keywords. Transient sheath evolution; displacement current; dynamic capacitor.

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1. Introduction

Plasma sheaths are rather ubiquitous in laboratory plasmas; occurring under a variety of conditions; e.g. probes [1], dusty plasmas [2], radio frequency (RF) antenna plasma interactions [3], etc. The recent upsurge of interest in the study of transient and equilibrium ion sheaths is motivated by the necessity to understand their role in technological applications. In plasma source ion implantation [4] (PSII) which is relatively new and a non-line of sight technique for surface modification of materials, the sheath physics plays a crucial role. In this technique a large negative potential is applied to the target immersed in a plasma. On a time scale short compared to the inverse of ion plasma frequency (f_{pi}^{-1}) the electrons are repelled away from the electrode exposing stationary ions thus forming on ion matrix sheath. On a larger time scale compared to (f_{pi}^{-1}), the ions are accelerated towards the target while the sheath expands away from the target.

The transient sheath evolution resulting from an instantaneous pulse of negative voltage applied to the electrode has been widely studied in PSII literature. Widner *et al* [5] have found the sheath propagation by experiments and numerical calculations using fluid equations. In the case of mercury arc rectifiers, Chester [6] applied the Child–Langmuir law quasi-statically to describe the sheath expansion. Liberman [7] and Scheurer *et al* [8] have developed analytical models for instantaneous pulse application. Stewart *et al* [9] have extended the model analytically to cases with finite rise time of the pulse. The basic assumption in all these models is that the conduction current is continuous across the sheath, i.e. the charge uncovered by the expanding sheath is equal to the implantation current. Recently Wood [10] has reexamined this assumption on the basis of numerical simulation. He has shown that in an expanding sheath the displacement current due to changing electric field is significant. The presence of finite displacement current in PSII experiments has also been noticed. The general conclusion seems that in the rising part of the pulse the contribution from the displacement current is substantial.

In this paper we report an experimental study to ascertain the role of displacement current in the sheath expansion. In the experiment a large negative bias is applied to a disc electrode immersed in a uniform plasma and the attention is focussed on the flat top phase of the pulse where the applied voltage is constant. The total current collected by the electrode is recorded as a function of time. The results are explained in terms of a model where the expanding sheath is regarded as a variable capacitor. Our results show that even during the flat top phase of the pulse the contribution of the displacement current is substantial and it reduces the total current to the electrode. Apart from this we have also explored the parameter space in order to verify the various scaling laws relating the electrode current to the plasma density and peak voltage.

2. Experimental set up and measurement techniques

The experimental set up [figure 1] consists of a stainless steel cylindrical chamber of length 120 cm and diameter 50 cm and is pumped down to $1{\cdot}0 \times 10^{-5}$ torr by using a combination of rotary and diffusion pumps. Argon gas is filled is the chamber to a pressure of $1{\cdot}0 \times 10^{-4}$ torr. Filaments located at the ends of the chamber are heated to give thermionic electrons. The electrons are accelerated into the chamber where they impact ionize the argon atoms to produce a plasma. Langmuir probe measurements show, that the plasma produced has densities in the range of 10^{8-9} per cc and T_e of 2 eV. In the centre of the chamber a disc of diameter 20 cm and thickness 1 mm is kept on which a pulsed negative bias is applied. The bias applied on the disc has a rise time of 50 to 70 ns depending on the amplitude of the bias applied. Thereafter the bias is maintained at its peak value for 3 μs and is made to go to zero in 2 μs.

The pulse described above is made by discharging a charged coaxial cable into its characteristic impedance [11]. Between the disc electrode and the characteristic impedance (50Ω) a 0·1 μF capacitor is kept so that the disc is initially floating. A Pearson current transformer is kept in between the capacitor and disc to measure the current. The capacitor also ensures that there is no dc current through the current transformer [figure 2]. The peak voltages are varied from 350 V to 5 kV.

In figure 3 we show the general nature of the applied bias and the current as measured by the current transformer. In the subsequent figures the time $t = 0$ ns corresponds to the time when the voltage has reached its peak. Figures 4 and 5 are the typical nature of the experimental current measured as function of time in the flat top phase of the pulse, and it decreases monotonically as a function of time.

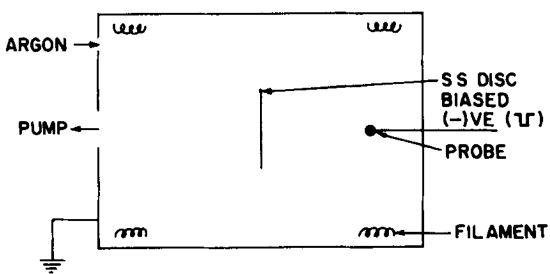

Figure 1. Experimental set up.

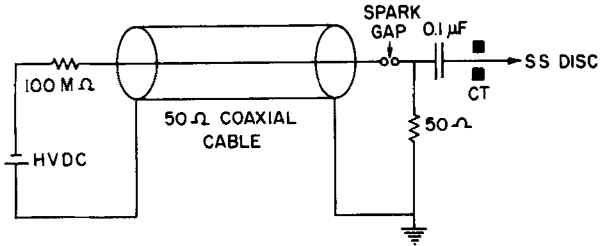

Figure 2. Pulse forming network, CT is the Pearson current transformer.

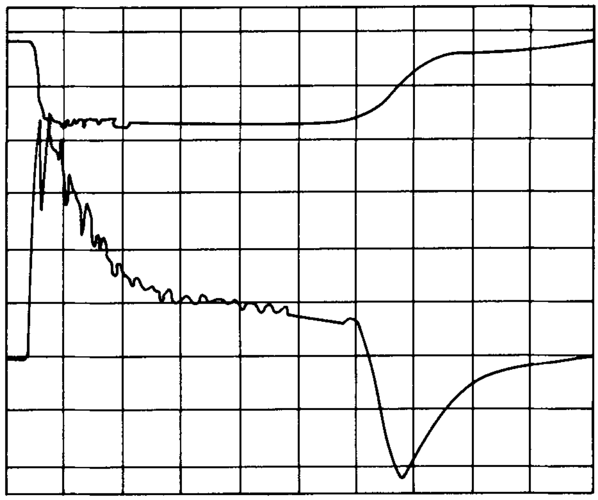

Figure 3. A typical oscilloscope trace for the applied bias (top) and the corresponding current through current transformer (bias). Top trace has y-axis 2000 V/div and x-axis 500 ns/div. Bottom trace has y-axis 100 mA/div and x-axis 500 ns/div. Plasma density $6{\cdot}26 \times 10^8$ per cc. Note the current behaviour in the flat top of the applied bias. The model predicts the current in this portion.

3. Model and comparison with experimental data

The results can be understood in terms of a simple model which represents the sheath as dynamic capacitor. We consider a planar, one dimensional model where at a time $t = 0^+$ a voltage pulse of $V = -V_0$ is applied on the planar electrode. The electrons respond instantaneously and are driven away. The exposed ions form an ion matrix sheath. On a longer time scale the ions move towards the electrode while the sheath expands in the other direction to supply the extra ions. We assume,

a) the ions while going through the sheath does not undergo any collisions.
b) the transit time of ions through the sheath is much faster than the rate of sheath expansion.
c) at each instant of time the current flow in the sheath satisfies Child's law
d) the applied bias is much larger than T_e.

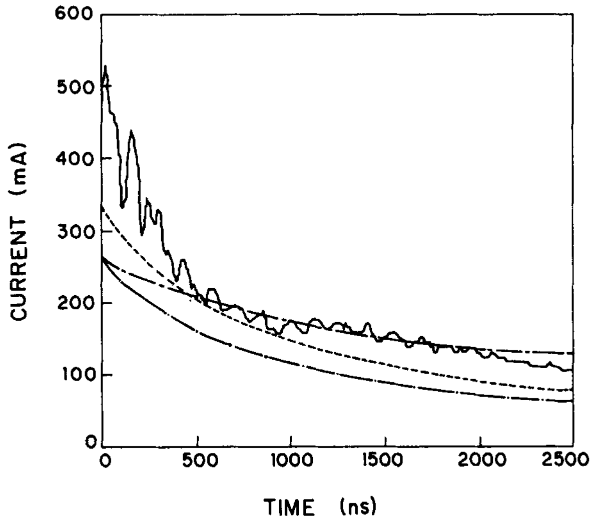

Figure 4. Comparison between the current obtained in (i) experiment (——) (ii) the model [Eq. (7)] (—·—) (iii) conduction current [Eq. (2)] (——·——) (iv) model [Eq. (7)] and contribution from the emitted secondary electrons (----). Applied bias is 3010 V and plasma density is $9{\cdot}4 \times 10^8$/cc.

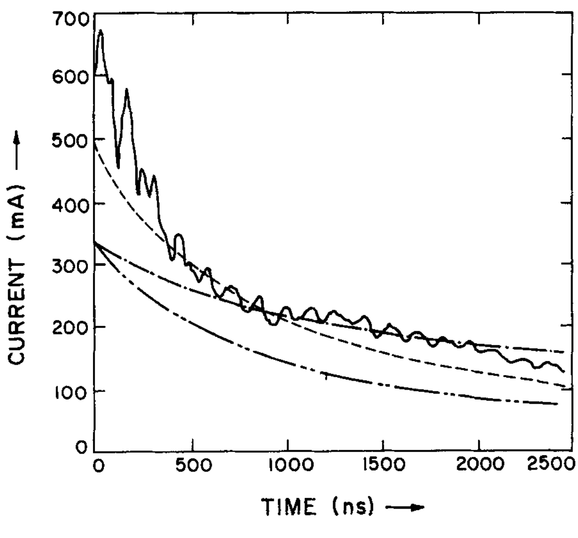

Figure 5. Comparison between the current obtained in (i) experiment (——) (ii) the model [Eq. (7)] (—··—) (iii) conduction current [Eq. (2)] (——·——) (iv) model [Eq. (7)] and contribution from the emitted secondary electrons (----). Applied bias is 4900 V and plasma density is $9{\cdot}4 \times 10^8$/cc.

The first assumption allows the ions to impinge on the electrode with the energy corresponding to the entire potential difference existing between the electrode and the dynamic sheath plasma interface. The third assumption is a consequence of the second assumption being valid. The fourth assumption neglects the effect of the electron dynamics on the transient sheath properties.

Dynamic sheath expansion

We first neglect the displacement current and calculate the electrode current with the assumption that the conduction current is continuous across the sheath. This requires that the Child's law current density is equal to the charge crossing the sheath boundary per unit time, i.e.,

$$[(4/9)\varepsilon_0(2e/M)^{1/2}V^{3/2}]/x^2 = en(\mathrm{d}x/\mathrm{d}t + u_0) \tag{1}$$

where x is the sheath position, M is the ion mass, n is the number density while u_0 is the speed with which the ions enter. Following Wood and Liberman we assume that since the sheath thickness is much small compared to the final equilibrium sheath, the term u_0 is small compared to $\mathrm{d}x/\mathrm{d}t$ and hence can be neglected. The resulting equation can be solved for $x(t)$, with the following boundary condition,

$$x(0) = (2\varepsilon_0 V_0/ne)^{1/2},$$

where $x(0)$ is the ion matrix sheath thickness. The value of $x(0)$ when substituted in (1) gives the conduction current as

$$I = (2/9)A(2e/M)^{1/2}enV^{1/2}/(1 + (2/3)w_{pi}t)^{2/3} \tag{2}$$

where A is the area of the electrode. In figures 4 and 5 we plot $I(t)$ obtained from (2). Initially it is smaller than the experimental $I(t)$ while at later times it is in excess. This clearly shows the inadequacy of the assumption of the continuity of the conduction current across the sheath. This also indicates, that if the pulse duration is increased more, keeping the applied bias as constant, the difference between the experimental current and the current predicted by (2) would also increase.

In an expanding sheath the electric field changes with time (even when the applied voltage is constant, because the sheath plasma interface penetrates in the plasma starting from the ion matrix sheath thickness) and the effect of the resultant displacement current must be taken into account. This implies that we must assume the continuity of the total current (conduction current + displacement current) rather than that of the conduction current alone. As shown by Wood this can be elegantly done by considering the sheath as a variable capacitor with varying thickness. Wood has considered the effect when the voltage at the electrode is a function of time ($\mathrm{d}V/\mathrm{d}t \neq 0$), and found the expression for the current. But his general prescription (Eq. (9) of Ref. 10) is valid as long as the net charge (Q) in the dynamic sheath is a function of time. The total charge collected by the electrode is $Q = CV$ so that the total electrode-current at constant voltage is,

$$I = \mathrm{d}Q/\mathrm{d}t = C\mathrm{d}V/\mathrm{d}t + V\mathrm{d}C/\mathrm{d}t = V\mathrm{d}C/\mathrm{d}t. \tag{3}$$

The capacitance of the sheath is greater than that of the vaccum capacitor with same parameters, because of space charge effect. This problem has been considered in detail by Bull [12] who has shown that space charge limited capacitance is roughly twice the value of the vacuum capacitance. We assume that the result holds for an expanding sheath as well, so that,

$$C = \frac{2\varepsilon_0 A}{x}. \tag{4}$$

Then

$$I = -\frac{2\varepsilon_0 A V}{x^2}(dx/dt). \tag{5}$$

The sheath expansion velocity is given by,

$$\frac{dx}{dt} = \frac{4}{9}\varepsilon_0\left(\frac{2e}{M}\right)^{1/2}\frac{V_0^{3/2}}{enx^2}. \tag{6}$$

Substituting (6) in (5) we have,

$$I = [(2/9)A(2e/M)^{1/2}en V^{1/2}]/(1 + (2/3)w_{pi}t)^{4/3}. \tag{7}$$

In figures 4 and 5, $I(t)$ obtained from (7) is also plotted as a function of time. It can be seen that the current obtained from (7) is smaller than that obtained from (2). This is clear because of the contribution of the displacement current. From figures 4 and 5 we see that in this case the general agreement with the experimental current, as far as the shape is considered, is better. There is a consistent residual difference. This residual difference is because of a number of factors not included in our model. These factors will be discussed at the end.

To find the validity of the scaling laws that is predicted from the model, we assume that the measured current has $I \propto V^b$ for constant t and n. The value of b is obtained by curve fitting techniques and is tabulated in table 1. The experimental data used to find b is at a time $t = 2000$ ns). From the table it is seen that the value of b is around 0·5 which agrees with the model. To ;e the scaling with density for constant bias and time we again assume that $I \propto n^c$. A dependence like this is valid when the denominator in our model is close to unity. The experimental data used to find c is at a time $t = 50$ ns. The value of c is obtained by curve fitting techniques and is tabulated in table 2. The value of c is closer to 1 indicating the validity of the scaling laws predicted by our model.

4. Discussion

To summarize we have experimentally measured the electrode current as a function of time during the flat top of the voltage pulse. It is also shown that even during this phase the contributions from the displacement current in the sheath are significant.

Table 1. Tabulated values of b for various plasma densities, where $I_{\text{experiment}} \propto (\text{bias})^b$; $t = 2000$ ns; the model predicts $b = 0{\cdot}5$.

Density (10^8/cc)	Value of b
1·56	0·63
3·13	0·58
4·69	0·55
6·26	0·53
7·83	0·55
9·38	0·52

Table 2. Tabulated values of c for various applied bias, where $I_{\text{experiment}} \propto (\text{density})^c$; $t = 50$ ns, model predicts $c = 1$ when denominator in (7) is unity.

Bias (V)	Value of c
380	1·078
1130	1·088
2130	1·095
3010	1·236
4900	1·275

Following Wood the results are interpreted in terms of a variable capacitor model which elegantly takes into account the effect of displacement current.

The experimental conditions satisfy the requirements of the model. The applied pulse has a rise time of the 50 to 70 ns which is much faster than the inverse of ion plasma frequency. Thus the concept of initial formation of the ion matrix sheath is valid. The current generated by the expansion of the ion matrix sheath during this rise time is not computed in the present work, as it has been dealt already in [10]. Secondly, the model assumes that the sheath starts developing from the ion matrix sheath thickness, and this assumption is satisfied when the pulse reaches its flat top. The model cannot be extrapolated to the limits of $t = 0$ and $t = \infty$. The second limit gives the current for the equilibrium sheath. Thus our model is valid in the limit when the sheath is expanding. Wood has also shown that in about 3 to 4 ion plasma periods, equilibrium flow of ions is established. The experiment is stopped before that so that the assumptions made in the model does not break down.

We now comment on the factors which are responsible for the residual difference between experimental and theoretical current. One of the reasons is the secondary electron emission from the disc, giving an additional current which is to be added to the current predicted by the model (Eq. (7)). It has been proved experimentally [13], that the current generated by the secondary electrons can be substantial and it is necessary to add this current to the current generated by the sheath motion. Thus the net current to the electrode increases by a factor of 1·1 to 1·4 [14] to that predicted by the model depending on the bias voltage. In figures 4 and 5 we also plot the total current as a function of time, where total current is a sum of current generated by (7) and contribution from secondary electron emitted from the electrode. The total current thus obtained, is in better agreement with the experimental current. If the contribution of the secondary electrons is added to the conduction current, then it is obvious that the resultant current will be much larger than the measured current.

The other source of error in the model can be the assumption made regarding the expansion of the sheath. It is assumed, that the sheath expands with sheath plasma interface parallel to the disc. Expansion in any other direction is neglected. This assumption may not be true at later times of the evolution. If the sheath expands in other directions more ions will be exposed to the sheath electric field and the net current will increase. It has been shown by Malik *et al* [15] that the sheath formed by a disc takes a spherical shape. Hence the current created by the two dimensional expansion of the sheath, can add to the total current and make even better agreement with the experiment.

Apart from this there are a few prominent peaks seen in the initial part of the measured current. These peaks are initially large and gradually decrease in amplitude with time. The model which is presented does not explain this observation. A recent simulation by Calder *et al* [16], where a negative bias is applied on an electrode immersed in a plasma, the current collected also shows oscillatory behaviour. The simulation indicates, that the electrons which are displaced by the negative bias overshoot their dynamic equilibrium position and then oscillate about this position. As the sheath expands the centre of oscillation also shifts with it. As the sheath thickness approaches its equilibrium value, the oscillations damp in amplitude. This picture can qualitatively explain the peaks observed in the experimental current. When the sheath expands, the sheath edge is not uniquely defined because of the presence of these oscillations at the sheath edge. Hence the sheath thickness also oscillates, though its mean value is always increasing. As the current to the electrode is defined by (3), the current also oscillates because the oscillating sheath edge changes the sheath capacitance. The shifting of the mean position defines the sheath expansion which gives rise to the current which the model tries to explain. On top of this current is a small oscillatory current created by the sheath edge oscillation. A detailed study of the problem is in progress.

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