PRAMANA
— journal of
physics

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Vol. 44, No. 3,
March 1995
pp. 257–262

The evolution of the resonant mode in plasma-maser interaction

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MS received 18 February 1994; revised 16 January 1995

Abstract. We study the evolution of the resonant waves considering its interaction with the nonresonant waves and the plasma particles due to plasma-maser effect. The nonlinear dielectric function of the resonant wave is calculated and is found to consist of two parts: the direct and the polarization coupling terms. On the other hand, the nonlinear dielectric function of the nonresonant wave consists only of the direct coupling term. The significance of our results is discussed.

Keywords. Mode-mode coupling; nonlinear interaction; resonant and nonresonant waves; dielectric function.

PACS No. 52·35

1. Introduction

Since the prediction of the plasma-maser effect [1, 2], also known as the induced Bremsstrahlung radiation, the theory has been applied to expalin numerous radiation phenomena occurring in space [3]. The plasma-maser effect occurs when nonresonant as well as resonant waves are present in a plasma. The resonant waves are those for which the Cherenkov resonance condition ($\omega - \mathbf{k}.v = 0$) is satisfied (in a magnetized plasma we have the cyclotron resonance), while the nonresonant waves are those for which the Cherenkov resonance and the scattering resonance conditions are not satisfied [($\Omega - \mathbf{K}.v \neq 0$) and ($\Omega - \omega - (\mathbf{K} - \mathbf{k}).v \neq 0$)]. Here $\omega, \mathbf{k}$ and $\Omega, \mathbf{K}$ are frequencies and wave numbers of the resonant and the nonresonant waves respectively. The most important characteristics of the plasma-maser interaction is its efficiency for up-conversion of wave energy from a turbulent low frequency resonant mode to a high frequency nonresonant mode through resonant interaction with the particles.

In most of the earlier works on the plasma-maser, only the evolution of the nonresonant waves were studied, and the nonlinear dielectric function of the nonresonant modes were derived to obtain growth rate [4, 5]. There are also works which studied the simultaneous effect of the resonant and the nonresonant waves on the particle distribution function [6]. It is demonstrated that in a plasma where energy-momentum sources are available (open system), a test nonresonant mode is highly unstable due to plasma-maser effect; while in a closed system, where there is no energy-momentum sources, the growth due to plasma-maser is exactly equal and opposite to the reverse absorption effect due to nonstationarity of the system [7, 8]. Thus in a closed system the number of nonresonant quanta is conserved. Recently, the energy-momentum conservation and the Manley-Rowe relations were investigated for unmagnetized [9] and magnetized systems [10].

In the present paper, we investigate the evolution of the resonant mode in plasma-maser interaction. We obtain an expression for the nonlinear dielectric function of the resonant mode, and its basic differences with the nonlinear dielectric function of the nonresonant mode is pointed out. We obtain nonlinear dielectric function of the resonant (ion sound) wave in presence of the nonresonant (Langmuir) wave. For simplicity of calculations we assume one dimensional unmagnetized plasma with $k_e > k > K$, where k_e is the electron Debye wave number, and k and K are the respective wave numbers of the resonant and nonresonant modes.

Here we must mention that there are two competing processes which potentially give the same order contribution as that of plasma-maser. The first v one is the nonlinear Landau resonance [11]. The condition is $\Omega \pm \omega = (\mathbf{K} \pm \mathbf{k})\cdot v$. This gives the resonance velocity $v = \omega_{pe}/(K \pm k)$ for $\Omega \sim \omega_{pe} \gg \omega$. Accordingly, for $k_e \gg K$ and $k_e \gg k$, the resonance velocity is much larger than the electron thermal velocity ($v \gg v_e$). The second process is the resonant decay interaction [12]. The relevant conditions are $\Omega(\mathbf{K}) \pm \omega(\mathbf{k}) = \Omega'(\mathbf{K}')$ and $\mathbf{K} \pm \mathbf{k} = \mathbf{K}'$. In addition, to identify the plasma wave as the high frequency mode, we need $\Omega > \omega_{pi}$, where ω_{pi} is the ion plasma frequency. Thus the resonant decay interaction is kinematically possible only when $\omega_{pe} \sim kc_s$ (where c_s is the ion sound velocity). This condition obviously does not coexist with our earlier assumption that $k_e > k > K$. Thus, at least for $k_e > k > K$ considered in this paper, we can rule out the two competing processes: nonlinear Landau resonance and the resonant three wave decay process. Furthermore, both these processes are energy down conversion processes, whereas the plasma-maser is basically an energy up-conversion process.

The paper is organized in the following fashion. In § 2, the nonlinear dielectric function of the resonant mode is derived. The growth (damping) rate of the resonant mode is considered and compared with that of nonresonant mode in § 3. Section 4 contains a brief discussion of our work and its importance in nonlinear wave-particle interaction.

2. Nonlinear dielectric function

We use the Vlasov-Poisson system of equations to obtain the nonlinear dielectric function of the ion sound mode as,

$$\varepsilon_{\omega,k} = \varepsilon^0_{\omega,k} + \varepsilon^d_{\omega,k} + \varepsilon^p_{\omega,k} \tag{1}$$

where $\varepsilon^0_{\omega,k}$ is the linear part given by,

$$\varepsilon^0_{\omega,k} = 1 + \frac{\omega^2_{pe}}{k}\int \frac{1}{\omega - kv + i0}\frac{\partial}{\partial v} f_{0e}\mathrm{d}v + \frac{\omega^2_{pe}}{k}\int \frac{1}{\omega - kv}\frac{\partial}{\partial v} f_{0i}\mathrm{d}v. \tag{2}$$

$\varepsilon^d_{\omega,k}$ is the direct nonlinear coupling term given by,

$$\varepsilon^d_{\omega,k} = \frac{\omega^2_{pe}}{k}\left(\frac{e}{m}\right)^2 \sum_{\Omega,K} \left|E_{\Omega,K}\right|^2 \int \frac{1}{\omega - kv + i0}\frac{\partial}{\partial v}\frac{1}{\omega - \Omega - (k-K)v} \times \frac{\partial}{\partial v}\left(\frac{1}{\Omega - Kv} + \frac{1}{kv - \omega - i0}\right)\frac{\partial}{\partial v} f_{0e}\mathrm{d}v. \tag{3}$$

$\varepsilon^p_{\omega,k}$ is the polarization coupling term,

$$\varepsilon^p_{\omega,k} = -\frac{\omega_{pe}^2}{k}\left(\frac{e}{m}\right)^2 \sum_{\Omega,K} \left|E_{\Omega,K}\right|^2 \frac{1}{(k-K)\varepsilon^0_{\omega-\Omega,k-K}} A \times B \tag{4}$$

where

$$A = \int \frac{1}{\omega - kv + i0} \frac{\partial}{\partial v}\left(\frac{1}{\omega - \Omega - (k-K)v} + \frac{1}{\Omega - Kv}\right)\frac{\partial}{\partial v} f_{0e} \mathrm{d}v \tag{5}$$

and

$$B = A(\omega \leftrightarrow \omega - \Omega, k \leftrightarrow k - K, \Omega \leftrightarrow -\Omega, K \leftrightarrow -K). \tag{6}$$

Thus we see that the nonlinear growth (or damping) rate of the resonant waves due to plasma-maser interaction originates from two processes: the direct coupling term and the polarization coupling term, which can be written as,

$$\gamma_{\omega,k} = -\frac{\mathrm{Im}\varepsilon^d_{\omega,k} + \mathrm{Im}\varepsilon^p_{\omega,k}}{\dfrac{\partial}{\partial\omega}\mathrm{Re}\varepsilon^0_{\omega,k}} \tag{7}$$

Here Re and Im show the real and imaginary parts of the respective dielectric function.

It is to be noted here that in a closed system, an additional imaginary part of the dielectric function proportional to $|E_{\omega,k}|^2$ due to nonstationarity of the system is to be taken into account in evaluating the growth (damping) rate of the nonresonant wave. This corresponds to the quasilinear interaction of the particles with the resonant waves. But the nonresonant waves give no contribution proportional to $|E_{\Omega,\mathbf{K}}|^2$ to the quasilinear interaction owing to the assumption $\Omega - \mathbf{K}\cdot v \neq 0$. In case of the nonresonant waves it can be shown that this term is equal and opposite to the imaginary part of the direct nonlinear coupling term of the dielectric function in closed system. However, in this paper we are primarily concerned with the contribution of the polarization coupling term in respect of resonant and nonresonant waves. Further, we assume the system to be open where constant particle distributions are maintained.

3. The growth (damping) rate of the resonant mode

Assuming Maxwellian distribution for unperturbed distribution function, we evaluate the imaginary part of the direct nonlinear term (3) to obtain the damping rate as,

$$\frac{\gamma^d_{\omega,k}}{\omega} = -2\sqrt{\frac{\pi m}{M}} \sum_{\Omega,K} W_L \frac{k}{|k|} \tag{8}$$

where $W_L = |E_{\Omega,K}|^2/4\pi N T$ is the normalized energy of the nonresonant mode. Here we take the real part of the linear dispersion function of the resonant (ion sound) mode as,

$$\mathrm{Re}\varepsilon^0_{\omega,k} = 1 + \left(\frac{k_e}{k}\right)^2 - \left(\frac{\omega_{pi}}{\omega}\right)^2 \tag{9}$$

Next, we evaluate the damping rate due to the polarization coupling term (4). From (5) and (6), we obtain,

$$A = -\frac{k}{\Omega^2}\int \frac{1}{\omega - kv + i0}\frac{\partial}{\partial v} f_{0e}\mathrm{d}v \tag{10}$$

and

$$B = -\frac{(k-K)}{\Omega^2}\int \frac{1}{\omega - kv - i0}\frac{\partial}{\partial v} f_{0e}\mathrm{d}v \tag{11}$$

which gives,

$$\mathrm{Im}(A \times B) = 2\sqrt{\frac{\pi m}{M}}\left(\frac{m}{T}\right)^2 \frac{(k-K)}{\Omega^4 |k|}. \tag{12}$$

Thus the damping rate due to the polarization term reduces to,

$$\frac{\gamma^p_{\omega,k}}{\omega} = -\sqrt{\frac{\pi m}{M}} \sum_{\Omega,K} W_L \left(\frac{k_e}{k}\right)^2 \frac{k}{|k|} \tag{13}$$

In evaluating (13), we use

$$\frac{1}{\varepsilon^0_{\omega-\Omega,k-K}} \approx -\left(\frac{k_e}{k}\right)^2.$$

This result is markedly different from the results obtained for nonresonant waves. For nonresonant waves, the imaginary part of the polarization term of the dielectric function vanishes irrespective of the modes of the coupling waves in unmagnetized system [4, 5]. Following ref. [4] we obtain the polarization term of the dielectric function for the nonresonant waves as,

$$\varepsilon^p_{\Omega,K} = \frac{\omega^2_{pe}}{K}\left(\frac{e}{m}\right)^2 \sum_{\omega,K} |E_{\omega,k}|^2 \frac{\omega^2_{pe}}{(K-k)\varepsilon^0_{\Omega-\omega,K-k}} C \times D \tag{14}$$

where

$$C = \int \frac{1}{\Omega - Kv}\frac{\partial}{\partial v}\left(\frac{1}{\Omega - \omega - (K-k)v} + \frac{1}{\omega - kv + i0}\right)\frac{\partial}{\partial v} f_{0e}\mathrm{d}v \tag{15}$$

and

$$D = C(\Omega \leftrightarrow \Omega - \omega, K \leftrightarrow K - k, \omega \leftrightarrow -\omega, k \leftrightarrow -k) \tag{16}$$

It is straightforward to show that,

$$C \propto \int \frac{1}{(\Omega - Kv)(\Omega - \omega - (K-k)v)(\omega - kv + i0)}\frac{\partial}{\partial v} f_{0e}\mathrm{d}v$$

$$D \propto \int \frac{1}{(\Omega - \omega - (K-k)v)(\Omega - Kv)(\omega - kv - i0)}\frac{\partial}{\partial v} f_{0e}\mathrm{d}v = -C^* \tag{17}$$

Here * represents the complex conjugate. Thus $\mathrm{Im}[C \times D] = 0$, and accordingly, the imaginary part of the polarization coupling term of the dielectric function of the nonresonant wave for an unmagnetized plasma becomes zero. Thus we see that the polarization term gives non-zero contribution in case of resonant waves for the breaking symmetry of the term (i.e., $B \neq -A^*$).

The growth rate of the nonresonant wave comes only from the direct coupling term, which can be written as [13],

$$\frac{\gamma^d_{\Omega,K}}{\omega_{pe}} = \frac{1}{2}\sqrt{\frac{\pi m}{M}} \sum_{\omega,k} W_S \frac{kK}{k_e^2}. \tag{18}$$

As an illustration, we give a numerical comparison of the growth (damping) rates of the nonresonant and resonant waves in plasma-maser interaction. If we assume $k \sim K$, $k_e/k \sim 10$ and $\omega/\omega_{pe} \sim \sqrt{m/M}$, we obtain from (8) and (13) $|\gamma^d_{\omega,k}| \sim 10^{-3} W_L$ and $|\gamma^p_{\omega,k}| \sim 10^{-2} W_L$, respectively. Taking the nonresonant wave as test wave, we obtain from (18) $\gamma^d_{\Omega,K} \sim 10^{-4} W_S$ and $\gamma^p_{\Omega,K} = 0$. Thus it is clearly seen that the total wave energy $(W_S + W_L)$ decreases due to the absorption of energy by the resonant particles in the plasma-maser instability.

4. Discussion

We have obtained the nonlinear dielectric function and the evolution of the resonant waves considering its interaction with nonresonant waves and particles due to plasma-maser effect. It is found that in contrast to the growth rate of the nonresonant waves, which comes only from the direct nonlinear coupling of the waves, the damping rate of the resonant waves comes from both the direct and the polarization coupling terms. This result is significant, because the question of energy exchange in the plasma-maser interaction is important. Furthermore, the conservation of the energy in plasma-maser interaction [9, 10] indicates an effective energy variation of the resonant mode. It can also be expected that under certain conditions, the plasma-maser can also lead to the increase of energy of the resonant mode. This may occur, for instance, in presence of an external current. In presence of an external current, the unperturbed distribution function can be written as,

$$f_{0e} = \left(\frac{m}{2\pi T}\right)^{1/2} \exp\left[-\frac{m(v-v_0)^2}{2T}\right],$$

where v_0 is the drift velocity of the electrons. Then, under the condition $v_0 > \sqrt{m/M} v_e$, the energy of the resonant mode may increase for positive phase velocity of the nonresonant mode ($\Omega/K > 0$). The physical reason is that the drifting electrons carry some extra free energy from the beginning.

Acknowledgement

The work of BS is supported by the Council of Scientific and Industrial Research, India.

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