PRAMANA
— journal of
physics

Vol. 44, No. 2,
February 1995
pp. 167–176

Momentum- and energy-dependence of J/ψ suppression in relativistic heavy ion collisions

R S KAUSHAL and S K KARN
Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India

MS received 29 August 1994; revised 24 November 1994

Abstract. With a view to understand the available data on the momentum- and energy-dependences of the J/ψ suppression, we compute the suppression rate within the framework of a hydrodynamical evolution model for the quark-gluon plasma (QGP) formation. We make use of an ansatz for the temperature profile function which accounts not only for the space and time evolutions of QGP but also for the possiblity of a mixed phase in the boundary region of the deconfined phase. While the predictions are made for the longitudinal momentum- and energy-dependences of the suppression rate, a satisfactory agreement with the available data is found for its dependences on the transverse momentum and the transverse energy.

Keywords. Quark-gluon plasma; suppression rate; energy- and momentum-dependence.

PACS Nos 12·38; 25·70

1. Introduction

It is believed that heavy ion collision experiments at ultra-relativistic energies ($\geqslant 200$ GeV/nucleon) can offer [1, 2] the conditions well-suited for the formation of a deconfined state of quarks and gluons (called quark-gluon plasma (QGP)). Theoretically, the J/ψ suppression in these experiments has been argued [3] as one of the important signals for the QGP formation in the laboratory. In fact, J/ψ resonances produced in these collisions are not able to come out in the presence of QGP mainly due to the screening effect.

No doubt the mechanism responsible for the formation of QGP in the laboratory can throw light on the theories [2] of early Universe, but the situation regarding the understanding of J/ψ suppression still seems to be fluid in both experiment and theory. Not only the data [4, 5] on p_T-dependence clearly show large error bars throughout but also the data on E_T-dependence show the errors of the same magnitude at least for small values of E_T. In spite of this fact several [6–11] theoretical attempts have been made to understand this anomalous phenomenon. Unfortunately, none of the calculations has been able to reproduce both the pieces of available data, namely p_T (transverse momentum)- and E_T (transverse energy)-dependences of the suppression rate at the same time.

In the present work, we attempt to understand both pieces of data simultaneously within the framework of the hydrodynamical evolution model for the QGP formation. For this purpose an exponential-type temperature profile function is used which not only accounts for the time and space evolutions of QGP in the transverse and longitudinal directions, but also for the possibility of mixed phase in the boundary-region of the deconfined phase. In the next section, we briefly outline various theoretical works carried out in the recent past on the understanding of J/ψ suppression. In § 3, we compute the suppression rate within the framework of the present model. The calculated survival probability is then compared with experiments and with other theoretical results in § 4. Finally, the results are discussed and summarized in § 5.

2. Various theoretical approaches

The formation and subsequent evolution of baryonless QGP in the relativistic heavy ion collision experiment is fairly well described [12] within the framework of the Björken hydrodynamical model and in terms of thermodynamical variables like temperature (T), energy density (ε), entropy density (s), etc. In fact one calculates the survival probability (S) which is related [8] to the probability of disintegration (P) by [13]

$$P = 1 - S = 1 - \exp(-T). \tag{1}$$

Ftacnik *et al* [8] have considered the J/ψ suppression mainly due to J/ψ- and χ-disintegration in collisions with other hadrons in a dense hadronic gas formed in the collision. Karsch and Petronzio [7] (KP), on the other hand, study the E_T- and nuclear size-dependences of J/ψ suppression within the framework of the hydrodynamical model. They assume that QGP is formed in a finite volume and for a short time. While reasonably good results are obtained by both KP and Ftacnik *et al* for the E_T-dependence of S, the results for the p_T-dependence somehow do not turn out to be that good.

Recently, Gazdzicki and Mrowczynski [9] have tried to understand both strangeness enhancement and J/ψ suppression in terms of a common model for the nuclear interactions at high energies. However, the computed E_T-dependence of S shows a completely different behaviour at large E_T as compared to that of KP and Ftacnick *et al.* More recently, Gupta and Satz [11] have obtained a fairly good fit for the E_T dependence of S by accounting for the effects arising from the modification of the structure functions in nuclei and by considering the parton scattering in nuclear matter.

As far as the parametrization of thermodynamical variables for the purposes of calculating the suppression rate is concerned it has been done differently by different authors. For example, (i) Blaizot and Ollitrault [6] parametrize the entropy density by using the form

$$s(t_0, r) = s_0(1 - r^2/R^2)^{a'},$$

where R is the projectile radius and a', a free parameter. The predictions made on the basis of this model however could not be compared with experiments as the data were not available at that time (ii) Karsch and Petronzio [7] used the form for the temperature function,

$$T(r) = T(0)(1 - r^2/R^2)^{b/3},$$

where the parameter b is fixed as 1/3 (iii) Recently, Gupta and Satz [11] and earlier

Karsch and Satz [10] have used the energy density in the form

$$\varepsilon(r) = \varepsilon_0(1 - r^2/R^2)^{2/3},$$

and they have shown the possibility of a bump structure around $2\,\mathrm{GeV/fm^3}$ in the energy dependence of S.

It may be noted that in all the above works the dependence of s, T or of ε on r is considered to be of the same power-type function, namely $(1 - r^2/R^2)^a$. This form, no doubt, is well-suited for the description of a collision of a large and a small nucleus, but somehow characterizes the sharp-boundaries for the deconfined phase and as such do not leave any scope for the description of a mixed phase in the peripheral region of the deconfined volume. On the other hand, in a realistic situation while the space-evolution of the deconfined region cannot be as steep as described by the above functional forms of s, T or ε, one should also account for the temporal evolution of QGP. For these regions we resort to choose an exponential fall for the temperature profile function not only in the transverse but also in the longitudinal direction. Also, we account for the time-dependence of T in accordance with the scaling law [12, 14].

In all works, except [9], the dependence of S on p_{T} is sought in a nontransparent manner in spite of the fact that $\mathbf{p}_{\mathrm{T}}$ is the conjugate momentum of the coordinate $\mathbf{r}$ defined in the transverse plane of the cylindrical interaction-volume. Further, the computed results show a sensitive dependence on the critical plasma size, R_{c}, or, in other words, on the time period during which the QGP survives. In a way this latter type dependence of R_{c} again is the manifestation of the functional form chosen for the parametrization.

3. Present model

In this section, we consider the hydrodynamical evolution model for QGP in both space and time. We use [15, 16] the concept of temperature density normalized to unity over the plasma volume. Further the survival probability, S, will be computed separately as the functions of p_{L}, E_L, p_{T} and E_{T}. However, the experimental results are available only as a function of p_{T} or E_{T}.

For cylindrical symmetry, the temperature function, in general, can be written as [17]

$$T(\mathbf{r}, t) = T_0\, T(r) f(z, t), \tag{2}$$

where various functions are chosen as

$$T(r) = \exp(-br/R); \quad f(z, t) = \exp(-\eta z) t^{-1/3}, \tag{3}$$

where the parameter η is the measure of the steepness of the fall of temperature along z-direction. Further as mentioned earlier the t-dependence of T is set in accordance with the scaling law and overall normalization constant T_0 can be determined from [18]

$$\int\int T(\mathbf{r}, t)\,\mathrm{d}^3 r\,\mathrm{d}t = 1. \tag{4}$$

as

$$T_0 = (\alpha^2 \eta/6\pi)[(t_f^{2/3} - t_i^{2/3})\sinh(\eta l/2)\{1 - (1 + \alpha R_{\mathrm{c}})\exp(-\alpha R_{\mathrm{c}})\}]^{-1}, \tag{5}$$

where $\alpha = b/R$, and R is the radius of the projectile nucleus. Note that the form of $T(\mathbf{r}, t)$ in (2) clearly accounts for the fall of temperature with both space and time variables. The cylindrical plasma volume can be characterized by the critical radius R_c, the length l of the region with the origin at $z = 0$. The critical temperature, T_c equal to $T(R_c)$, is also related to the QGP life time t_f through [12] $T_i^3 t_i = T_c^3 t_f$.

After using the expression for T_0 from (5) the Fourier transforms of (2) with respect to z- and r-variables respectively lead to the p_L- and p_T-dependences of T as

$$|T(p_L)| = \frac{\eta}{\sinh(\eta l/2)(\eta^2 + p_L^2)^{1/2}} [\sinh^2(\eta l/2)\cdot\cos^2(p_L \cdot l/2) + \cosh^2(\eta l/2)\cdot\sin^2(p_L \cdot l/2)]^{1/2}, \tag{6}$$

$$|T(p_T)| = \frac{\alpha^2}{(\exp(\alpha R_c) - 1 - \alpha R_c)(\alpha^2 + p_T^2)} [R_c^2(\alpha^2 + p_T^2) + 2\alpha R_c + 2\exp(\alpha R_c)\cdot\{\cosh(\alpha R_c) - (1 + \alpha R_c)\cos(p_T R_c) - p_T R_c \sin(p_T R_c)\}]^{1/2}. \tag{7}$$

It may be mentioned that in obtaining (7) we have assumed that $i\mathbf{p}_T\cdot\mathbf{r} = ip_T r\cos\varphi \simeq ip_T r$, i.e. $\varphi = 0$. In Appendix we outline a method to estimate the corrections arising due to this assumption. This assumption seems to be reasonable for the order of energies involved in the problem. Thus, S as a function of p_L or p_T can be studied using (1).

To obtain E_T-dependence of S we now use the expression for the energy density $\varepsilon(r)$ as

$$\varepsilon(\mathbf{r}, t) = (\alpha_s \pi^2/15)\, T^4(\mathbf{r}, t) \tag{8}$$

where [7] $\alpha_s = N^2 - 1 + (7/4)Nn_f = 18\cdot5$. In fact, for $SU(N)$, $N = 3$ and n_f (the number of quark flavours) is taken to be 2. Further, in view of (4) the relation (8) should also involve a dimensional constant which may be useful for determining the absolute values of the transverse energy. However, in the present work, we restrict ourselves to the computation of the ratio E/E_{max}, in which such a constant will cancel out. Thus in the same spirit as (2), the form of $\varepsilon(\mathbf{r}, t)$ is taken to be

$$\varepsilon(\mathbf{r}, t) = \varepsilon_0 \exp(-4\alpha r)\exp(-4\eta z)t^{-4/3}, \tag{9}$$

where $\varepsilon_0 (= (\alpha_s \pi^2/15)\, T_0^4)$ is now treated as a new normalization constant and determined from

$$\iint \varepsilon(\mathbf{r}, t) \mathrm{d}^3 r \mathrm{d}t = 1, \tag{10}$$

as

$$\varepsilon_0 = (16\alpha^2\eta/3\pi)[(t_f^{-1/3} - t_i^{-1/3})\sinh(2\eta l)\{1 - (1 + 4\alpha R_c)\exp(-4\alpha R_c)\}]^{-1}. \tag{11}$$

As before, the Fourier transforms of $\varepsilon(\mathbf{r}, t)$ with respect to z- and r-variables give rise to

$$\varepsilon(p_L) = \frac{4\eta}{\sinh(2\eta l)(16\eta^2 + p_L^2)^{1/2}} [\sinh^2(2\eta l)\cos^2(p_L \cdot l/2) + \cosh^2(2\eta l)\sin^2(p_L \cdot l/2)]^{1/2}, \tag{12}$$

and

$$\varepsilon(p_T) = \frac{16\alpha^2}{\{\exp(4\alpha R_c) - (1 + 4\alpha R_c)\}(16\alpha^2 + p_T^2)} [R_c^2(16\alpha^2 + p_T^2) + 8\alpha R_c +$$
$$2\exp(4\alpha R_c)\cdot\{\cosh(4\alpha R_c) - (1 + 4\alpha R_c)\cos(p_T R_c) - p_T R_c \sin(p_T R_c)\}]^{1/2}, \quad (13)$$

respectively. Once again, here $\varphi \approx 0$ is assumed for the angular integral in obtaining (13). It can be seen that for both $\varepsilon(p_L)$ and $\varepsilon(p_T)$ the maximum occurs at $p_L = 0$ and $p_T = 0$, respectively. Therefore, for comparison with experiments, we compute $X_1 = \varepsilon_L/\varepsilon_{L\max}$ and $X_2 = \varepsilon_T/\varepsilon_{T\max}$, where $\varepsilon_L = \varepsilon(p_L)$; $\varepsilon_{L\max} = \varepsilon_L(p_L = 0)$; $\varepsilon_T = \varepsilon(p_T)$ and $\varepsilon_{T\max} = \varepsilon(p_T = 0)$. Further note that the expression for total energy (E), $E = \pi R^2 l\varepsilon$, is used throughout this work. Other simplifying points used in the computation of S as a function of X_1 and X_2, will be mentioned in the next section.

4. Comparison of results

The predictions made on the basis of the model discussed in § 3, are compared here with the results of NA38 experiments [4, 5] and also with the work of other authors for the oxygen-uranium (O–U) central collisions at 200 GeV/nucleon.

The predicted results for S as a function of p_L and X_1 are shown in figures (1) and (2), respectively for [7] $l = 2$ fm and for two values of the parameter η, namely $\eta = 0{\cdot}9\,\text{fm}^{-1}$ (solid curve) and $\eta = 1{\cdot}1\,\text{fm}^{-1}$ (dashed curve). In general, it can be seen that the J/ψ suppression decreases with p_L and increases with X_1. However, for comparison of either of these predictions, neither the experimental results are available nor these quantities are calculated earlier to the best of our knowledge except for some discussions mainly of qualitative nature (see, for example, [10], [15] and [19]). No doubt there are indications [19] of similar longitudinal and transverse momentum distributions of O–U events in NA38 experiments but somehow the suppression rate either as a function of p_L or as a function of X_1 has not been extracted from these data so far.

The calculated results for S as a function of p_L and X_2 are shown (solid curves) in

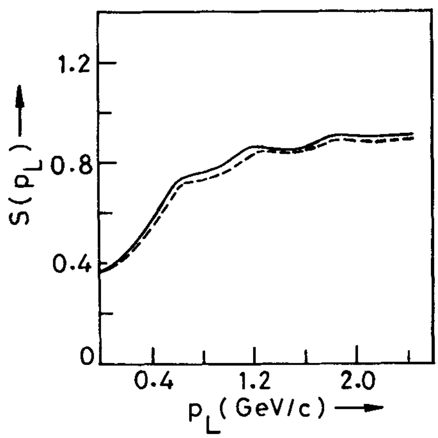

Figure 1. Survival probability as a function of p_L: solid and dashed curves correspond to two values of η, namely $\eta = 0{\cdot}9\,\text{fm}^{-1}$ and $\eta = 1{\cdot}1\,\text{fm}^{-1}$ respectively.

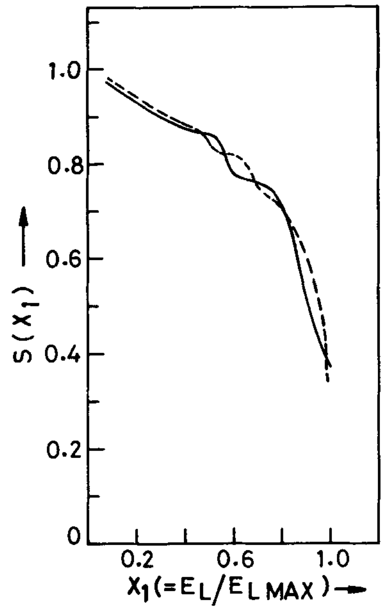

Figure 2. Survival probability as a function of $X_1(=E_L/E_{L\max})$. For the explanation of solid and dashed curve, see the caption for figure 1.

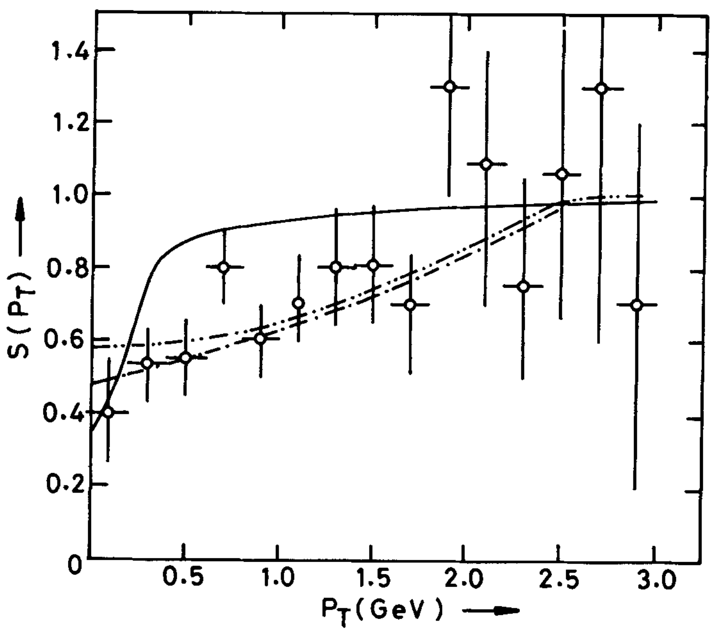

Figure 3. Survival probability as a function of p_T: Solid curve represents the results of present model for $b=0{\cdot}7$ and $t_f=3{\cdot}0$ fm/c. Double dot-dashed and dot-dashed curves represent the results of KP [7] and Ftacnik *et al* [8] (corresponding to the dressing-up times of J/ψ and χ equal to 3·0 fm/c) respectively. Experimental points are from Baglin *et al* [4].

figures 3 and 4 respectively for $b = 0{\cdot}7$ along with the experimental [4, 5] points and the theoretical results of other authors for the sake of comparison. From figure 3 it can be seen that our results show a stronger suppression and a faster saturation with p_T as compared to that of KP [7] (double dot-dashed curve) and of Ftacnik *et al* [8] (dot-dashed curve). In view of large error bars in the data for p_T-dependence it is however difficult to draw any definite conclusion at this stage regarding the merit of a theoretical model.

In figure 4, we compare our results for S vs X_2 not only with the experiments [5] but also with the results obtained in [6] (as cited in [8]), [7], [11], [9]. No doubt this piece of data is more stable as compared to that of S vs. p_T (cf. figure 3), however the large experimental errors at low E_T in this case have forced to normalize almost all theoretical calculations with experiments. For example, KP, Ftacnik *et al* and GS

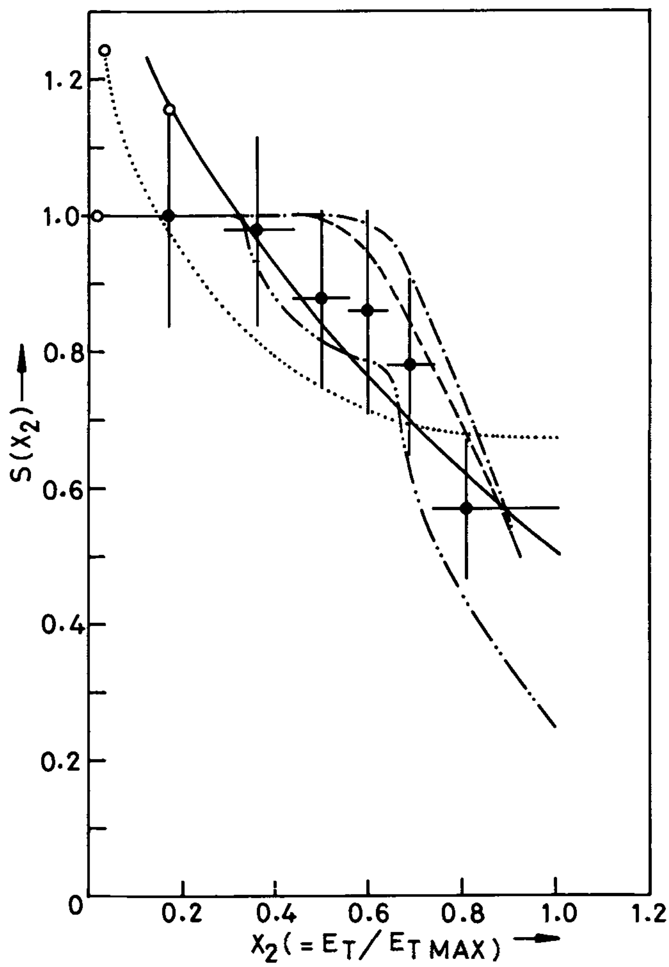

Figure 4. Survival probability as a function of $X_2(=E_T/E_{T_{max}})$. Solid curve represents the results of present model for $b = 0{\cdot}7$ and $t_f = 3{\cdot}0$ fm/c. Dashed and dot-dashed curves are the results of KP [7] and Blaizot and Ollitrault [6] (as cited in [8]), respectively. Dotted and double dot-dashed curves represent the results of [9] and [11] respectively. Experimental points are from Baglin *et al* [5].

normalize their results at $S = 1$ (open circle) whereas GM normalize somewhere at $S = 1{\cdot}25$. We have also normalized our results at $S = 1{\cdot}16$, i.e. at the point which corresponds to the maximum error in S for the first data point.

As far as the overall trend of the calculated results is concerned, the results of [6] and [7] show somewhat fast suppression of J/ψ whereas those of [11], [9] and ours show some sort of saturation for large E_T. In fact, the results of [9] have the tendency to attain a constant value of S with E_T (something like $S = 0{\cdot}68$), those of [11] and ours show somewhat faster suppression of J/ψ for large E_T. However, this latter suppression is not as prominent as shown by the results of [7] and [6]. In a way our results show a balancing trend not only with experiments but also with other theoretical results. It may be mentioned that the data used in [11] are those from Pappilon's work [20] which is free from horizontal error bars whereas we have taken the experimental results from [5] and normalized them for unity at $E_{T_{max}} = 70\,\text{GeV}$. Further for transcribing the theoretical results of [9] and [11] for the comparison in figure 4 we have normalized their results also for $\varepsilon_T/\varepsilon_{T\max} = 1$.

Regarding the computation of S as a function of X_1, it is noticed that the expression for X_1 (cf. eq. (12)) attains a much simpler form of the type $p_L = 4\eta(1 - X_1^2)^{1/2}/X_1$, as the square of the sine and cosine hyperbolic functions in (12) becomes large and almost equal for the values of l and η used in the calculations. Similarly, for S vs. X_2 (cf. figure 4) we first compute X_2 as a function of p_T from (13) and for these same values of p_T we compute $|T(p_T)|$ from (7) and subsequently obtain S from (1).

5. Discussion and Summary

Within the framework of the Björken hydrodynamical model we have tried to understand the space- and time-evolution of QGP by considering an exponential fall of the temperature in both longitudinal and transverse directions. The tail of this fall is expected to account for the possibility of a mixed phase in the peripheral region of QGP volume. In view of the agreement of the present results (cf. figures (3) and (4)) with experiments it may be emphasized that such a possibility cannot be ruled out. Perhaps it requires further fine-tuning of the data.

The predictions are made for the J/ψ suppression as a function of p_L and E_L (cf. figures 1 and 2). The presently available data [19] for the longitudinal momentum distribution however are not sufficient for a meaningful comparison of these predicted results. It may be mentioned that further experimental work from the point of view of extracting the information on this aspect is desirable. In fact, the studies of both p_L and p_T-dependences of S will be helpful in a complete understanding of this phenomenon of J/ψ suppression.

Finally, we discuss the role of dressing-up time of QGP in these calculations. In fact, such a dependence mainly arises through the computation of R_c (cr. $T_c = T(R_c)$). The results for S vs p_L and S vs. X_1 are found to be independent of such a parameter. While our results presented in figures 3 and 4 correspond to $t_f = 3\,\text{fm/c}$, the results for $S(X_2)$ vs. X_2 are found to be more sensitive than those for $S(p_T)$ vs. p_T with respect to the value of t_f. As a matter of fact similar observations are also made in earlier [6, 8] calculations. To some extent the normalization of the theoretical results with experiments can be attributed to this unknown parameter.

Acknowledgements

The authors are greatly benefited from the discussions with Professor H Satz at CERN and they wish to express their gratitude to him. Authors also wish to thank Dr S K Gupta of BARC (Bombay) for bringing the work of R A Salmeron to their notice. Thanks are also due to Dr Y K Mathur for several helpful discussions. RSK acknowledges the award of UGC Research Scientist Scheme during the course of this work.

Appendix

Here, we outline a method for computing the corrections arising from the angular integral in (7) and (13). For the exponential radial function this integral appears in the form

$$I_{r\varphi}=\int_0^{R_c}\exp(-\alpha r)r\mathrm{d}r\int_0^{2\pi}\exp(ip_{\mathrm{T}}r\cos\varphi)\mathrm{d}\varphi.$$

For small φ, one can however use $\cos\varphi\approx 1-\varphi^2/2$ and obtain the corrections arising from the φ^2-term in terms of error functions. Alternatively, one can use the formula involving the Bessel function as

$$\int_0^{2\pi}\exp(ip_{\mathrm{T}}r\cos\varphi)\mathrm{d}\varphi=2\pi J_0(p_{\mathrm{T}}r)$$

with

$$J_0(z)=\sum_{k=0}^{\infty}\frac{(-1)^k z^{2k}}{2^{2k}(k!)^2},$$

and obtain directly $|T(p_{\mathrm{T}})|$ as

$$|T(p_{\mathrm{T}})|=\left|1+\sum_{k=1}^{\infty}\frac{(-1)^k p_{\mathrm{T}}^{2k}}{2^{2k}(k!)^2}\cdot\left(\frac{M_k}{M_0}\right)\right|,$$

where

$$M_k=-[\alpha R_c+2k+1]\frac{R_c^{2k}}{\alpha^2}\exp(-\alpha R_c)+\frac{2k(2k+1)}{\alpha^2}M_{k-1},$$

with

$$M_0=-(R_c/\alpha)\exp(-\alpha R_c)+\alpha^{-2}(1-\exp(-\alpha R_c)).$$

A similar result can be derived for $\varepsilon(p_{\mathrm{T}})$ as well.

References

[1] L Van Hove, *Phys. Lett.* **B118**, 138 (1982); CERN Preprint No. TH-5069/88 (June)
[2] L McLerran, *Rev. Mod. Phys.* **58**, 1021 (1986)
[3] T Matsui and H Satz, *Phys. Lett.* **B178**, 416 (1986)
[4] C Baglin *et al*, *Phys. Lett.* **B262**, 362 (1991)
C Baglin *et al*, *Phys. Lett.* **B251**, 465 (1990)
[5] C Baglin *et al*, *Phys. Lett.* **B255**, 459 (1991)
[6] J P Blaizot and J Y Ollitrault, *Phys. Lett.* **B199** 499 (1987)

[7] F Karsch and R Petronzio, *Phys. Lett.* **B212**, 255 (1988);
F Karsch and R Petronzio, *Z. Phys.* **C37**, 627 (1988)
[8] J Ftacnik, P Lichard, N Pisutova and J Pisut, *Z. Phys.* **C42**, 139 (1989)
[9] M Gazdzicki and S Mrowczynski, *Z. Phys.* **C49**, 569 (1991)
[10] F Karsch and H Satz, *Z. Phys.* **C51**, 209 (1991);
P V Ruuskanen and H Satz, *Z. Phys.* **C37**, 623 (1988)
[11] S Gupta and H Satz, *Phys. Lett.* **B383**, 439 (1992)
[12] J D Bjorken, *Phys. Rev.* **D27**, 140 (1983)
[13] Here T is dimensionless. See § 3
[14] G Baym, B L Friman, J P Blaizot, M Soyeur and W Czyz, *Nucl. Phys.* **A407**, 541 (1983)
[15] S K Karn and R S Kaushal, *Proc. Nucl. Phys. Symp.* (Bombay) **B35**, 378 (1992)
[16] F Cooper, G Frye and E Schonberg, *Phys. Rev.* **D11**, 192 (1975)
[17] Note that the angular function ϕ, given by $\phi(\varphi) = \cos m\varphi/\sqrt{2\pi}$, is dropped all through out as it contributes only for $m = 0$ in the present normalization of $T(\mathbf{r}, t)$. Also, this will not affect the latter calculations as we shall be computing the ratio $\varepsilon/\varepsilon_{max}$, only.
[18] It may be mentioned that the normalization of $T(\mathbf{r}, t)$ to unity in eq. (4) ensures the dimensionless character of $|T(p_L)|$ and $|T(p_T)|$ (cf. eqs. (6) and (7)) and subsequently that of S in eq. (1).
[19] See, for example, the data cited in R A Salmeron, "The suppression of the J/ψ in high energy nucleus–nucleus collisions: Experimental data and Models", Invited talk at 24th Recontres de Moriond, Les Arcs, France, 12–18 March, 1989; R A Salmeron, in "Quark-Gluon Plasma" edited by B Sinha *et al* (Springer Verlag, 1990) p. 44
[20] S Papillon, *Dependance en impulsion de la production du J/ψ dans les interactions p–U, ^{16}O–U et ^{32}S–U a 200 GeV/c par nucléon.* These, Universite de Paris-Sud, Orsay IPNO-T, 91-03, March 1991