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Magnetic moments of decuplet baryons in an independent particle potential model

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Abstract. Magnetic moments of decuplet baryons have been calculated in a relativistic independent quark model with a phenomenological potential in equally mixed scalar-vector harmonic form. Such a model has been succeessful in describing wide ranging hadronic phenomena in mesonic and baryonic sectors. Using the solutions of the constituent quark orbitals with the model parameters taken from its earlier applications, the magnetic moments of decuplet baryons Δ^{++} and Ω^- have been obtained which are in good agreement with the available experimental data. However, the agreement is found to be much better when the magnetic moment ratios such as $\mu_{\Delta^{++}}/\mu_p$ and $\mu_{\Omega^-}/\mu_\Lambda$ are considered. Model predictions for the magnetic moments of other decuplet baryons together with the charge radii have also been calculated which may be verified in future experiments.

Keywords. Magnetic moments; decuplet baryons; independent quark; potential model.

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1. Introduction

Recently in two experimental measurements such as the pion-bremstrahlung analysis [1] and the E756 collaboration [2], the magnetic moments of Δ^{++} and Ω^- have been reported. Study of magnetic moments of the decuplet baryons is important so far as their structure is concerned. The above-mentioned experimental measurements [1, 2] in fact have generated a fresh interest in their investigation resulting in several publications on different theoretical models [3–6]. In one such recent work [3], it has also been claimed that it is a challenge for every hadronic model to get consistent values for the magnetic moments of Δ^{++} and Ω^-.

However, magnetic moment of hadrons being a low energy attribute cannot be studied from the first principle applications of quantum chromodynamics (QCD) [7], despite its being the correct theory for hadrons. One resorts to phenomenological models of hadrons incorporating the basic ingredients of QCD. Thus, we investigate here the magnetic moments of decuplet baryons in a phenomenological potential model of relativistic and independent quarks, where the binding between the confined quarks is described by a potential having an equal admixture of vector and scalar parts of the form [8]

$$U(r) = (1/2)(1 + \gamma^0)\, V(r), \tag{1.1}$$

with a harmonic form for $V(r)$ as

$$V(r) = ar^2 + V_0; \quad a > 0. \tag{1.2}$$

With such a potential, the Dirac equation derivable from a zeroth order quark-Lagrangian-density

$$\mathscr{L}^0_q(r) = (-i/2)\bar{q}(r)\gamma^\mu \overleftrightarrow{\partial}_\mu q(r) - \bar{q}(r)[m_q + U(r)]q(r) \tag{1.3}$$

becomes solvable to yield the static quark orbital solutions for positive frequency as [8]

$$\psi_\xi(\mathbf{r}) = \begin{pmatrix} ig_\xi(r)/r \\ (\boldsymbol{\sigma}\cdot\hat{r}) f_\xi(r)/r \end{pmatrix} U_\xi(\hat{r}); \tag{1.4}$$

where $\xi = (l, j, m)$ represents the set of Dirac quantum numbers specifying the corresponding confined eigen modes and $U_\xi(\hat{r})$ represents the spin angular wave function described as

$$U_{l,j,m}(\hat{r}) = \sum_{m_l, m_s} \langle l, m_l \tfrac{1}{2} m_s | j, m\rangle \, Y_l^{m_l}(\hat{r}), \tag{1.5}$$

with

$$E'_q = (E_q - V_0/2), m'_q = (m_q + V_0/2), \lambda_q = (E'_q + m'_q) = (E_q + m_q),$$
$$r_{0q} = (a\lambda_q)^{-1/4} \tag{1.6}$$

one obtains the solutions to the reduced radial equations for $f_\xi(r)$ and $g_\xi(r)$ leading to the bound state conditions for the independent quarks as [8]

$$(\lambda_q/a)^{1/2}(E'_q - m'_q) = (4n + 2l - 1), \tag{1.7}$$

which in turn gives the zeroth order binding energy for quarks inside the hadron for different eigen modes.

However, in the present calculation to estimate the magnetic moments of the decuplet baryons in their ground states, one requires only the lowest eigen modes in the positive frequency sector which when explicitly written become

$$\psi_{1S1/2}(\mathbf{r}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} ig_q(r)/r \\ (\boldsymbol{\sigma}\cdot\hat{r}) f_q(r)/r \end{pmatrix} \chi_\uparrow. \tag{1.8}$$

with the spinors as

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1.9}$$

Further, the reduced radial parts in the upper and the lower components of the solution for a particular flavor q when evaluated become

$$g_q(r) = N_q (r/r_{0q}) \exp(-r^2/2r_{0q}^2)$$

and

$$f_q(r) = -\frac{N_q}{\lambda_q r_{0q}} (r/r_{0q})^2 \exp(-r^2/2r_{0q}^2) \tag{1.10}$$

with the normalization constant calculated as

$$N_q = \left[\frac{8\lambda_q}{\sqrt{\pi} r_{0q}} \cdot \frac{1}{(3E'_q + m'_q)}\right]^{1/2}. \tag{1.11}$$

In its earlier applications, the present model with chiral symmetry restoration has been proved to be a simple and successful alternative to the cloudy bag model (CBM). It has also been successful in explaining the leptonic decays of light neutral vector mesons [9] and radiative transitions between vector and pseudoscalar mesons [10] as well as in estimating the pseudoscalar decay constants [11] and the top quark mass from the weak mixing between B_d^0 and $\bar{B}_d^0$ mesons [12]. Most recently, it has also been able to estimate correctly the electromagnetic polarizabilities of proton [13]. This model was also used earlier to study several low energy phenomena in baryonic sector such as mass spectrum [14], weak electric form factors [15] and magnetic moments [16] of octet baryons as well as their charge radii and electromagnetic form factors [8]. Also in the light meson sector, the model was applied successfully to estimate $(q\bar{q})$ pion mass in consistency with that of PCAC-pion [17] and $(\varrho - \pi)$ as well as $(\varrho - \omega)$ mass splittings [18]. In view of these successes of the model in explaining varieties of hadronic phenomena, we apply it here to estimate the magnetic moments of decuplet baryons to see if it can meet the challenge described in ref. [3].

We organize the present article as follows. In § 2, we estimate the magnetic moments of decuplet baryons. In § 3, we discuss our results and compare them with other model calculations and experimental results.

2. Magnetic moments of decuplet baryons

The magnetic moment vector $\boldsymbol{\mu}_q$ of a confined quark inside the baryon core is expressed in terms of the electromagnetic current in a conventional manner as

$$\boldsymbol{\mu}_q = (1/2) \int \mathrm{d}\mathbf{r}(\mathbf{r} \times \mathbf{J}_{\mathrm{em}}), \tag{2.1}$$

where the current $\mathbf{J}_{\mathrm{em}}$ when expressed in terms of confined quark wave functions of the present potential model becomes,

$$\mathbf{J}_{\mathrm{cm}} = e\, e_q \bar{\psi}_q(\mathbf{r}) \boldsymbol{\gamma} \psi_q(\mathbf{r})$$

which when substituted in the expression (2.1) yields

$$\boldsymbol{\mu}_q = (1/2) e\, e_q \int \mathrm{d}\mathbf{r}\, \psi_q^{\dagger}(\mathbf{r})(\mathbf{r} \times \boldsymbol{\alpha}) \psi_q(\mathbf{r}) \tag{2.2}$$

where e and e_q are the magnitudes of proton charge and the electric charge of quark (in units of proton charge) respectively. However, with the conventional identification of $(\boldsymbol{\mu}_q)_z$ to be the magnetic moment of the quark, we obtain

$$\mu_q = (1/2) e\, e_q \int \mathrm{d}\mathbf{r}\, \psi_q^{\dagger}(\mathbf{r})(\mathbf{r} \times \boldsymbol{\alpha})_z \psi_q(\mathbf{r}), \tag{2.3}$$

where $\psi_q(\mathbf{r})$ is the ground state wave function of quarks. Thus, using eqs (1.8)–(1.11) the expression for the magnetic moment of quark of flavor q becomes

$$\mu_q = \frac{4 M_P e_q}{(3E_q' + m_q')} nm. \tag{2.4}$$

with M_P as the mass of the proton. Next, with the usual assumptions of quark addivitity, the quark core contribution towards the magnetic moments of the decuplet baryons can be obtained from the defining relation described as

$$\mu_B^0 = \left\langle B_{3/2} \left| \sum_q \mu_q \sigma_{qz} \right| B_{3/2} \right\rangle \tag{2.5}$$

with μ_q's as the individual quark magnetic moments as described in (2.4). However, a consideration of the normalized decuplet baryon state along with their appropriate spin, flavor and isospin configurations yields (2.5) to

$$\mu_B^0 = \sum_q \mu_q, \tag{2.6}$$

which results effectively to the fact that the magnetic moment of a decuplet baryon as the sum of their respective individual quark magnetic moments, as expected. Thus, with (2.4), it yields to an explicit expression for the magnetic moment of a baryon in the decuplet as

$$\mu_B^0 = 4M_P \sum_q \frac{e_q}{(3E_q' + m_q')}. \tag{2.7}$$

However, we may note here that the estimation of the magnetic moment in (2.7) considers only the independent motion of the quarks inside the baryon core and such a consideration does not lead to a state of definite total momentum as it should represent the physical state of the baryon. Hence, appropriate correction due to the centre-of-mass-motion must be taken into consideration. In doing so we would follow the same prescription as adopted in ref. [8] so as to obtain the centre-of-mass-motion corrected expression for the magnetic moment of a decuplet baryon as

$$\mu_B = \left[2\mu_B^0 + \frac{Q_B}{3}\left(\frac{M_P}{M_B}\right)(1 - \delta_B^2)/(1 + \delta_B^2) \right]. \tag{2.8}$$

Here the quantity δ_B^2 in terms of which the centre-of-mass-motion correction is effected is defined in accordance with ref. [8] as,

$$\delta_B^2 = \langle M_B^2 / E_B^2 \rangle = [1 - \langle \mathbf{P}_B^2 \rangle / E_B^2] \tag{2.9}$$

with the centre-of-mass momentum $\mathbf{P}_B$ obtained through the expression

$$\langle \mathbf{P}_B^2 \rangle = \sum_q \langle p_q^2 \rangle = \sum_q \frac{(11\,E_q' + m_q')(E_q'^2 - m_q'^2)}{6(3E_q' + m_q')}. \tag{2.10}$$

We must note here that M_B and Q_B in (2.8) are respectively the observed mass and the charge (in units of proton charge) of the baryons. The total energy of the baryon E_B is obtained through the relativistic mass energy relation as $(\langle \mathbf{P}_B^2 \rangle + M_B^2)^{1/2}$.

One can also estimate the charge radii of the decuplet baryons within the framework of the present potential model at the quark core level, from its defining relation as

$$\langle \mathbf{r}^2 \rangle_B = \left\langle B_{3/2} \left| \sum_q e_q \langle \mathbf{r}_q^2 \rangle \right| B_{3/2} \right\rangle \tag{2.11}$$

which with the normalised decuplet baryon state $|B_{3/2}\rangle$ and the reduced orbital solutions of the model becomes

$$\langle \mathbf{r}^2 \rangle_B^0 = \frac{3}{2}\sum_q e_q \frac{(11\,E_q' + m_q')}{(3\,E_q' + m_q')(E_q'^2 - m_q'^2)}. \tag{2.12}$$

Again, with the prescription of ref. [8], the centre-of-mass-motion corrected expression for the charge radius of a baryon in the decuplet becomes

$$\langle \mathbf{r}^2 \rangle_B' = 3[\langle \mathbf{r}^2 \rangle_B^0 - Q_B \langle \mathbf{R}^2 \rangle]/(2 + \delta_B^2) \tag{2.13}$$

where $\langle \mathbf{R}^2 \rangle$ is the mean square centre-of-mass radius and the expression for it in terms of the model parameters is written as

$$\langle \mathbf{R}^2 \rangle = \frac{3}{2E_B^2}\sum_q E_q^2 \frac{(11\,E_q' + m_q')}{(3\,E_q' + m_q')(E_q'^2 - m_q'^2)}. \tag{2.14}$$

3. Results and discussion

We now estimate the magnetic moments of decuplet baryons using the expression obtained in the earlier section. We must point out here that there exists only a couple of data points in this sector corresponding to the magnetic moments measured for Δ^{++} and Ω^- and a satisfactory reproduction of these two measured quantities in the present model with as many as five parameters (such as a, V_0, m_u, m_d and m_s) may not appear to be a big achievement for the model. Therefore instead of searching afresh the appropriate values for the model parameters to fit the measured magnetic moments of Δ^{++} and Ω^-, we prefer to take the same set of parameters with which the model in its earlier applications has successfully described wide ranging phenomena in the mesonic and baryonic sectors [8–17]. In that case we would be having effectively no free parameters in the present calculation which on reproducing the measured magnetic moments of Δ^{++} and Ω^- to a reasonable extent can add to the credibility of the model. Hence, we take the model parameters such as the flavor independent potential parameters and the quark masses from the earlier applications of the model [8–17] as

$$(a, V_0) = (0{\cdot}017166\,\text{GeV}^3, -0{\cdot}1375\,\text{GeV})$$

and

$$m_u' = m_d' = 0{\cdot}01\,\text{GeV}; m_s' = 0{\cdot}247\,\text{GeV}. \tag{3.1}$$

Then the binding energies of the quarks in their lowest eigen mode; which are relevant to the present calculation and are derivable from the solution of the bound state condition in (1.7), become

$$E_u' = E_d' = 0{\cdot}54\,\text{GeV}; \quad E_s' = 0{\cdot}65975\,\text{GeV}. \tag{3.2}$$

Thus with these parameters and expressions (2.7) and (2.8) of the earlier section, we estimate the uncorrected and the corrected magnetic moments for all the decuplet baryons and report them in table 1, where we observe a reasonable agreement with the available experimental data [1, 2]. In fact for Δ^{++}; the centre-of-mass-motion corrected estimation is a little higher than the corresponding measured value.

Table 1. Estimated centre-of-mass-motion uncorrected and corrected magnetic moments (in nm) of decuplet baryons in the present investigation compared with the experiment and other theoretical estimations.

μ_B	Present investigation uncorr.	δ_B^2	corr.	Ref. [3]	Ref. [4]	Ref. [5]	Ref. [6]	Ref. [19]	Expt. [1,2]
$\mu_{\Delta^{++}}$	4·60	0·797	5·18	4·76	4·53	6·54	6·09	5·56	$4·52 \pm 0·50$
μ_{Δ^+}	2·30	0·797	2·59	2·38	2·09	—	3·05	2·73	—
μ_{Δ^0}	0·00	0·797	0·00	0·00	−0·36	—	0·00	−0·09	—
μ_{Δ^-}	−2·30	0·797	−2·59	−2·38	−2·80	—	−3·05	−2·92	—
$\mu_{\Sigma^{*+}}$	2·51	0·813	2·79	1·82	2·55	—	3·16	3·09	—
$\mu_{\Sigma^{*0}}$	0·20	0·813	0·23	−0·27	−0·02	—	0·33	0·27	—
$\mu_{\Sigma^{*-}}$	−2·10	0·813	−2·34	−2·36	−2·60	—	−2·50	−2·56	—
$\mu_{\Xi^{*0}}$	0·41	0·827	0·45	−0·60	0·40	—	0·58	0·63	—
$\mu_{\Xi^{*-}}$	−1·89	0·827	−2·09	−2·41	−2·31	—	−2·08	−2·20	—
μ_{Ω}	−1·68	0·839	−1·85	−2·35	−1·98	−2·52	−1·73	−1·83	$-1·94 \pm 0·22$

Table 2. Estimated ratios of the centre-of-mass-motion uncorrected and corrected magnetic moments $\mu_{\Delta^{++}}/\mu_p$ and $\mu_{\Omega^-}/\mu_\Lambda$ in the present model compared with the experiment and theoretical estimations.

Magnetic moment	Present investigation uncorr.	corr.	Ref. [3]	Ref. [4]	Ref. [5]	Ref. [6]	Ref. [19]	Expt. [1,2]
$\mu_{\Delta^{++}}$	4·60	5·18	4·76	4·53	6·54	6·09	5·56	$4·52 \pm 0·50$
$\mu_{\Delta^{++}}/\mu_p$	2·0	1·90	1·69	1·98	2·34	2·18	2·00	$1·62 \pm 0·18$
μ_{Ω^-}	−1·68	−1·85	−2·35	−1·98	−2·52	−1·73	−1·84	$-1·94 \pm 0·22$
$\mu_{\Omega^-}/\mu_\Lambda$	3·0	3·09	3·41	3·73	4·13	3·60	3·00	$3·16 \pm 0·28$

Table 3. Estimated centre-of-mass-motion uncorrected and corrected charge radii $\langle \mathbf{r}^2 \rangle_B^{1/2}$ (in fm) of decuplet baryons in the present potential model.

Baryon B	Charge radius uncorrected	Charge radius corrected
Δ^{++}	1·209	1·02
Δ^{+}	0·855	0·723
Δ^{0}	0·0	0·0
Δ^{-}	−0·855	−0·723
Σ^{*+}	0·894	0·788
Σ^{*-}	−0·814	−0·689
Σ^{*0}	0·262	0·276
Ξ^{*0}	0·370	0·381
Ξ^{*-}	−0·770	−0·651
Ω^{-}	−0·725	−0·608

However, when we compare the estimated ratios of the magnetic moments such as $\mu_{\Delta^{++}}/\mu_p$ and $\mu_{\Omega^-}/\mu_\Lambda$ with the corresponding experimental quantities [1, 2] as presented in table 2, the agreement seems much better. In tables 1 and 2 we also provide a comparison of our results with the estimations of some other models such as NQM [19], Skyrme model [4], CBM [5], lattice simulation [6] and light-front relativistic quark model [3]. We find that the present estimation is more or less comparable with the predictions of ref. [4, 19]. We must add here that we have ignored in this calculation any further additional contributions due to possible residual effects of the virtual pion and kaon cloud surrounding the bare quark core in a physical baryon as conceived in a chiral potential model. We believe that such additional contributions would be quite small only to the extent of fine tuning of model estimations.

Finally we estimate the charge radii for different decuplet baryons, both uncorrected and corrected ones using (2.12) and (2.13) respectively of the earlier section and present the results in table 3. However, since no experimental data in this regard are available, our results stand as model predictions and may be verified in future experiments.

Thus, the present potential model, even at the valency quark core level explains reasonably the experimentally measured magnetic moments and predicts the unobserved ones which may be verified in future experiments. It does so in a much simplified and tractable manner due to the ansatz for the potential as an admixture of equal scalar and vector parts in harmonic form. Nevertheless, we feel the accurate results from the succeeding experiments, expected to be obtained in near future, can further constrain different theoretical models [20].

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