PRAMANA
— journal of
physics

© Printed in India

Vol. 44, No. 2,
February 1995
pp. 133–144

Wormholes in higher dimensions with Gauss–Bonnet terms

B C PAUL[†], A BEESHAM[*] and S MUKHERJEE[†*]
[†]Physics Department, North Bengal University, District Darjeeling, 734 430, India
[*]Department of Applied Mathematics, University of Zululand, Kwa-dlangezwa, South Africa

MS received 17 September 1994

Abstract. A class of wormhole solutions permitted in a theory with Gauss–Bonnet terms in the gravitational action in higher dimensions have been studied. The case of de-Sitter type instantons, with a compact inner space, are of particular interest here. Some of the configurations, when continued analytically to the Lorentzian metric lead to the standard inflationary universe. Some multiple-sphere configurations of the type studied by Myers have also been noted. The Euclidean action for the solutions has been calculated and the relevance of the solutions in the quantum creation of the universe has been considered.

Keywords. Wormholes; higher dimensional cosmology; Gauss-Bonnet terms; quantum cosmology.

PACS Nos **04·20; 04·60; 98·80**

1. Introduction

Model building in higher dimensions was initiated by Kaluza and Klein (KK) [1] who tried to unify gravity with electromagnetic interaction by introducing an extra dimension. The approach has been revived and considerably generalized after realizing that many interesting theories of particle interactions need more than four dimensions for their formulation. It is considered essential to check if consistent cosmological solutions, which can accommodate these theories, are also allowed. Attempts have been made to build cosmological models [2] in higher dimensions which may undergo a spontaneous compactification leading to a product space $M^4 \times M^d$, with M^d describing the compact 'inner' space. The advent of superstrings [3] saw a spurt in model building activities in higher dimensions. Zweibach [4] suggested that the string corrections to Einstein action up to first order in the slope parameter α' and fourth power of momenta should be a term of the type α' (GB), where the Gauss-Bonnet term is given by $(GB) = R_{ABCD}R^{ABCD} - 4R_{AB}R^{AB} + R^2$, with R_{ABCD}, R_{AB}, R denoting the Riemann tensor, Ricci tensor and Ricci scalar respectively. Some attempts [5, 6] have already been made to build cosmological models including the Gauss-Bonnet terms in the action. Bailin *et al* [6] found that the GB terms permit an approximate stable Friedmann solution with a slowly varying compact internal space which is unstable with the $R_{ABCD}R^{ABCD}$ term only. Lorentz-Petzold [7] obtained cosmological solutions with the GB terms in the presence of matter. A consistent scenario of the higher-dimensional early universe and its subsequent evolution into a 4-dimensional FRW stage remains to be fully developed. The problem is seen clearly in the standard KK models, in which the dilaton field in the effective 4-dimensional theory, has negative kinetic energy and an energy spectrum unbounded from below. With the Einstein-Hilbert action, one may conformally rescale the 4-dimensional metric which converts the dilaton into a self interacting scalar field, minimally coupled to gravity and the ground state becomes stable against its fluctuations. However, the method does not work if GB-terms are present. Thus, a theory with GB-terms with geometric fields (one or more inflatons) is an intrinsically unstable theory. However, we have discussed elsewhere [8] a cosmological model with an action in which R and the GB terms have opposite signs, contrary to what one obtains in the small slope expansion of string theory. However, with appropriate compactification, one can recover at a large time the usual Einstein equations [8]. The calculations showed the rich structure of the theories with quadratic curvature terms which may be useful in model building.

A related exercise is to consider quantum cosmology in higher dimensions. Here, a provision has to be made for considering possible topological fluctuations. In the Euclidean path-integral formulation of quantum cosmology, a special role is supposed to be played by wormholes, which are described often by instanton-like configurations. These are the solutions of the D-dimensional Euclidean field equations, connecting two given $(D-1)$ surfaces. It has been suggested by Baum [9], Hawking [10] and Coleman [11] that inclusion of these wormholes may provide a mechanism for the vanishing of cosmological constant. Although it is not clear if this mechanism is really self-consistent, it is nevertheless important to identify and understand explicitly wormhole solutions in as wide a class of theories as possible. Instanton solutions have been noted already in a number of higher-dimensional theories. Gonzalez-Diaz [12], in particular, has studied the solution to a six dimensional Einstein-Gauss- Bonnet theory in $R^1 \times M^5$ manifold and has obtained the Tolman-Hawking wormhole. The purpose of this paper is to study possible instanton configurations in a higher dimensional model $(D \geqslant 6)$ in which the action includes the Gauss-Bonnet terms viz. eq. (1). We will restrict to manifolds of the type $M^4 \times M^d$, where M^d is a compact space of dimension d. We intend to study the role of the GB terms in a simple situation in which the compact space is assumed to have a constant radius, smaller than the outer space scale factor. The effect of a space or time-dependent inflaton field will be taken up elsewhere.

2. Field equations

We consider an Euclidean action which includes a cosmological constant, the GB terms and a matter field term:

$$I_E = \int \sqrt{g} d^D x [C_0 + C_1 R + C_2 (\mathrm{GB}) + L_m], \tag{1}$$

where $A, B, \ldots$ are $D(= 1 + 3 + d)$ dimensional indices; C_0, C_1 and C_2 are dimensional constants. We consider the minisuperspace metric

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + a^2(\tau)\mathrm{d}\Omega_3^2 + b^2(\tau)\mathrm{d}\Omega_d^2, \tag{2}$$

with $\mathrm{d}\Omega_3^2 = \dfrac{\mathrm{d}r^2}{1 - k_3 r^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)$ and $\mathrm{d}\Omega_d^2$ is its d-dimensional generalization. We shall choose $k_3 = k_d = 1$, and assume that the internal space is compact, $b = b_0$. The solutions are thus relevant for tunneling processes of a FRW universe with additional d compact dimensions.

The energy momentum tensor is written as

$$T_{AB} = [\rho, p g_{ij}, p' g_{mn}], \tag{3}$$

with $i,j = 1,2,3$ and $m,n = 4,\ldots,d$. The field equations for the metric (2) becomes $[\Lambda = C_0/2]$

$$-C_1\left[\frac{d(d-1)}{2}\frac{1}{b_0^2} - 3\left(\frac{\dot a^2}{a^2} - \frac{1}{a^2}\right)\right]$$
$$= \Lambda - \rho + C_2\left[\frac{d(d-1)(d-2)(d-3)}{2b_0^4} - \frac{6d(d-1)}{b_0^2}\left(\frac{\dot a^2}{a^2} - \frac{1}{a^2}\right)\right] \tag{4a}$$

$$-C_1\left[\frac{d(d-1)}{2b_0^2} - 2\frac{\ddot a}{a} - \left(\frac{\dot a^2}{a^2} - \frac{1}{a^2}\right)\right] = \Lambda + p$$
$$+ C_2\left[\frac{d(d-1)(d-2)(d-3)}{2b_0^4} - \frac{2d(d-1)}{b_0^2}\left(\frac{\dot a^2}{a^2} - \frac{1}{a^2}\right) - \frac{4d(d-1)}{b_0^2}\frac{\ddot a}{a}\right] \tag{4b}$$

$$-C_1\left[\frac{(d-1)(d-2)}{2b_0^2} - 3\left(\frac{\dot a^2}{a^2} - \frac{1}{a^2}\right) - 3\frac{\ddot a}{a}\right]$$
$$= \Lambda + p' + C_2\left[12\frac{\ddot a}{a}\left(\frac{\dot a^2}{a^2} - \frac{1}{a^2}\right) + \frac{(d-1)(d-2)(d-3)(d-4)}{2b_0^4}\right.$$
$$\left. - \frac{6(d-1)(d-2)}{b_0^2}\frac{\ddot a}{a} - \frac{6(d-1)(d-2)}{b_0^2}\left(\frac{\dot a^2}{a^2} - \frac{1}{a^2}\right)\right]. \tag{4c}$$

The conservation equation for ρ, p and p' is given by

$$\frac{\mathrm{d}}{\mathrm{d}\tau}[\Omega_3\Omega_d\rho] + p\Omega_d\frac{\mathrm{d}\Omega_3}{\mathrm{d}\tau} + p'\Omega_3\frac{\mathrm{d}\Omega_d}{\mathrm{d}\tau} = 0 \tag{5}$$

where $\Omega_3 = 2\pi^2 a^3$ and $\Omega_d = \dfrac{2\pi^{(d+1)/2}}{\Gamma((d+1)/2)} b_0^d$. The scalar curvature is given by

$$R = -6\left[\frac{\ddot a}{a} + \left(\frac{\dot a^2}{a^2} - \frac{1}{a^2}\right)\right] + \frac{d(d-1)}{b_0^2}. \tag{6}$$

The model considered above involves essentially a large number of parameters C_0, C_1, C_2, d, b_0^2. We would, therefore, look for relations among the parameters so that instanton solutions of the type discussed in our earlier paper [13] can be obtained. The approach followed here is to leave C_1 and C_2 undetermined (but $C_1 < 0$, $C_2 > 0$) and look for required values of C_0, b_0^2 and d (or constraints on their values) and also to determine the relevant equation of state for the matter part. A number of interesting solutions are given in the next section which may be useful in quantum cosmology.

3. Wormhole solutions

Solutions of the type

$$\dot a^2 = 1 - \mu a^2 - \sum_{n=1}^{N}\frac{\nu_n}{a^{2n}} \tag{7}$$

can now be obtained following the prescription of [13]. We consider three cases,

$N=0$, $N=1$ and $N=2$.

(A) *For* $N=0$:

We neglect the matter contribution and get a de Sitter instanton given by

$$a=\frac{1}{\sqrt{\mu}}\sin\sqrt{\mu}\tau \tag{8}$$

where μ is related to the D-dimensional cosmological constant, viz.

$$\Lambda=C_0/2=-3\mu C_1-\tfrac{1}{2}d(d-1)C_1b_0^{-2}-\tfrac{1}{2}d(d-1)(d-2)(d-3)C_2b_0^{-4}$$
$$-6d(d-1)\mu C_2b_0^{-2} \tag{9}$$

Equation (4c) now gives a quadratic equation for μ, viz

$$A\mu^2+B\mu+C=0, \tag{10}$$

with

$$A=12C_2 \tag{11a}$$

$$B=3[C_1+2(d-1)(d-4)C_2b_0^{-2}] \tag{11b}$$

$$C=-(d-1)b_0^{-2}[C_1+2(d-2)(d-3)C_2b_0^{-2}] \tag{11c}$$

For an acceptable instanton solution, we should satisfy (9) and (10) simultaneously. For a given d, this relates Λ and μ with b_0^2. The relation can be described conveniently by dividing the range of b_0^2 values into several regions. The allowed values for Λ and μ can be determined by studying the discriminant of (10).

Region 1. When $B>0$ and $C<0$, we have $0<b_0^2\leqslant 2(d-1)(d-4)C_2/|C_1|$, this allowspositive as well as negative values for μ. For a given b_0^2, two values each of $\mu(\mu_+,\mu_-)$ and $\Lambda(\Lambda_+,\Lambda_-)$ are allowed. The positive root (μ_+) here corresponds to a negative Λ_+ and the negative root (μ_-) corresponds to a positive Λ_-. We have shown these results for $D=10$, 11 and 26 in figures 1–3. We note that when b_0^2 is small, both Λ_- and $|\Lambda_+|$ are very large. With an increase in b_0^2, μ_- increases and the corresponding Λ_- decreases. In the other branch, as b_0^2 increases, μ_+ decreases and the corresponding Λ_+ moves towards zero. Note that the solutions with $\mu>0$ are physically interesting.

Region 2. Here, $B<0$ and $C<0$. This corresponds to $2(d-1)(d-4)C_2/|C_1|<b_0^2<2(d-2)(d-3)C_2/|C_1|$ and also leads to $\mu>0$ and $\mu<0$. From figures 1, 2 and 3, it is observed that with an increase in b_0^2, Λ_- still decreases; the corresponding μ_- increases from the negative side. However, in the other branch, with an increase of b_0^2, Λ_+ eventually becomes positive.

A special case occurs when $C=0$, $B<0$, so that $b_0^2=2(d-2)(d-3)C_2/|C_1|$. This gives for $d>3$,

$$b_0^2=2(d-2)(d-3)C_2/|C_1| \quad \text{and} \quad \mu=b_0^{-2}. \tag{12}$$

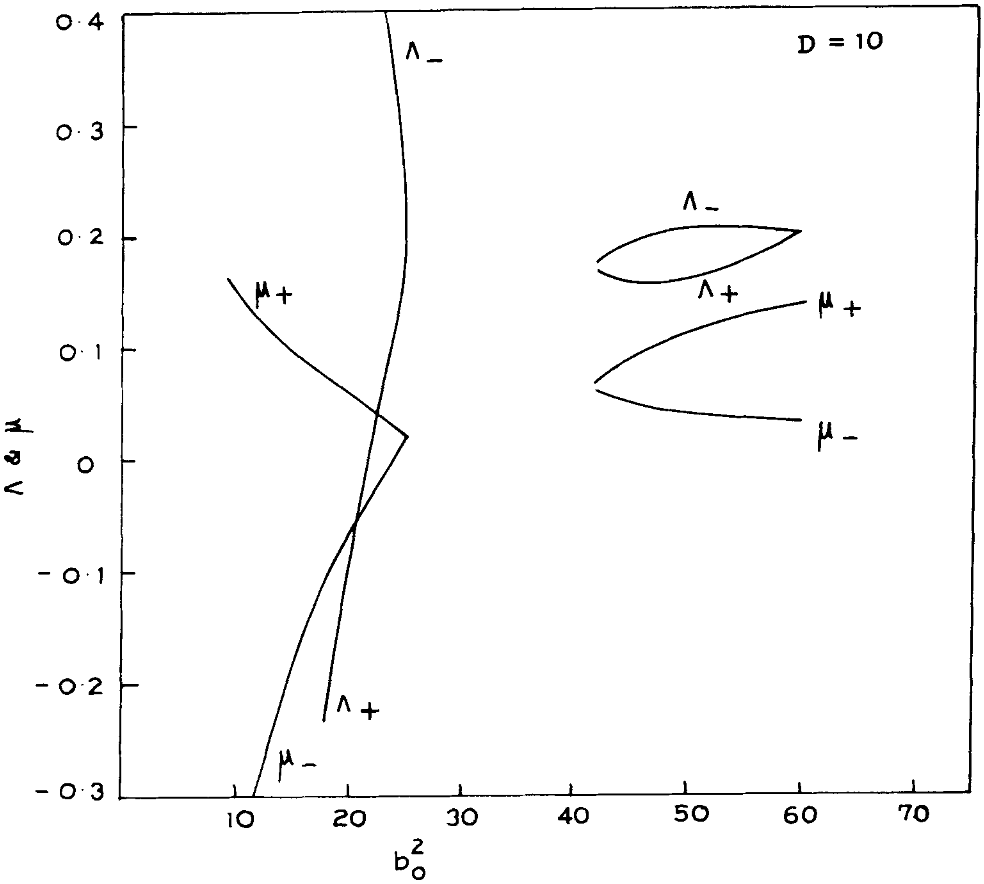

Figure 1. Permitted values of Λ and μ in units of C_1^2/C_2 and $|C_1|/C_2$ respectively for various b_0^2 (in units of $C_2/|C_1|$) for $D = 10$. Λ_+, Λ_- corresponds to μ_+, μ_- respectively.

The higher dimensional cosmological constant Λ, which permits the solution, may be written as

$$\Lambda = C_2 b_0^{-4}[(d^2 - 5d)[6 + \tfrac{1}{2}d(d-1)] + 36 - 3d(d-1)], \tag{13}$$

which shows that $\Lambda > 0$ only for $d \geqslant 6$. For $d = 6$, $b_0^2 = 24C_2/|C_1|$, and $\mu = b_0^{-2}$.

Region 3. For $C > 0$, $B^2 \geqslant 4AC$, we have $2(d-2)(d-3)C_2/|C_1| < b_0^2 \leqslant \frac{2}{3}[(3d-8)(d-1) - 4((d-1)(d-4))^{1/2}]C_2/|C_1|$. In this range μ_+, μ_- both are positive and they are equal $[= -(B/2A)]$ at the boundary $b_0^2 = \frac{2}{3}[(3d-8)(d-1) - 4((d-1)(d-4))^{1/2}]\, C_2/|C_1|$.

Region 4. For $B^2 < 4AC$, $C > 0$, we have $\alpha_- < b_0^2 < \alpha_+$ where $\alpha_- = \frac{2}{3}[(3d-8)(d-1) - 4((d-1)(d-4))^{1/2}]C_2/|C_1|$ and $\alpha_+ = \frac{2}{3}[(3d-8)(d-1) + 4((d-1)(d-4))^{1/2}]C_2/|C_1|$. The quadratic eq. (10) now leads to imaginary values for μ and Λ. Thus one

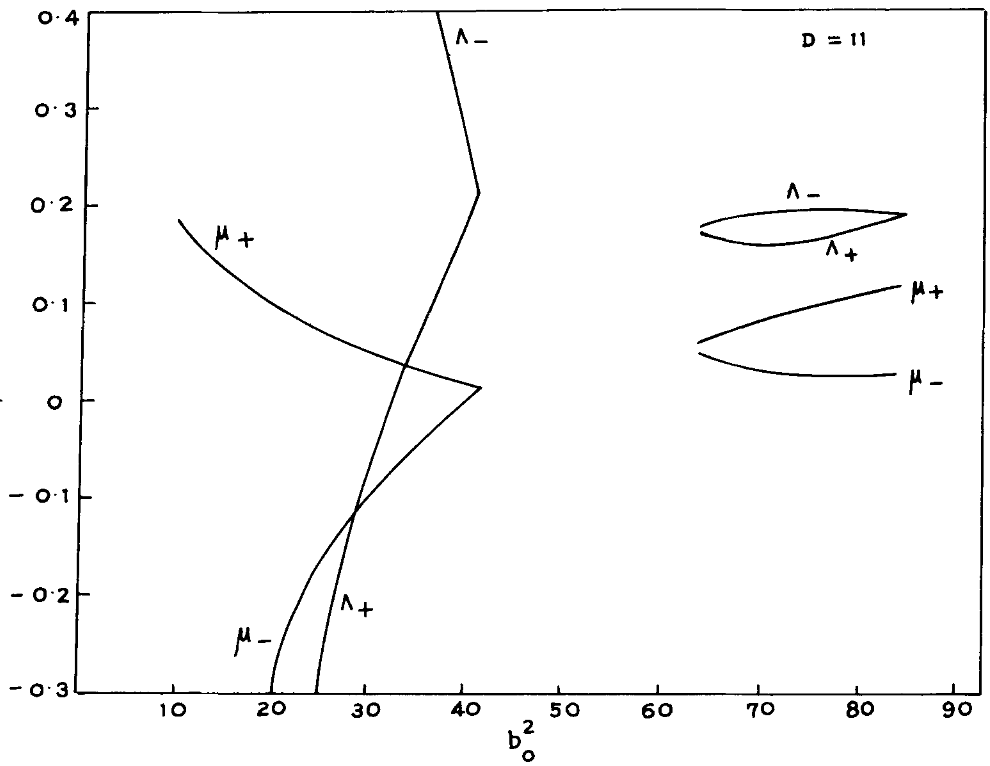

Figure 2. Permitted values of Λ and μ in units of C_1^2/C_2 and $|C_1|/C_2$ respectively for various b_0^2 (in units of $C_2/|C_1|$) for $D = 11$. Λ_+, Λ_- corresponds to μ_+, μ_- respectively.

cannot get an instanton solution for this range of values of b_0^2. In the context of this model, the region is a forbidden zone for instanton solutions.

Region 5. $B^2 - 4AC > 0$, $C \geqslant 0$, which gives $b_0^2 \geqslant \alpha_+$. However, we may put an upper bound on b_0^2, viz. $b_0^2 < 2d(d-1)C_2/|C_1|$ which follows from the condition that the 4-dimensional Newton's Gravitational constant $G_4 > 0$. In this range also, we have two branches of Λ, whose values (Λ_+, Λ_-), and the corresponding values of $\mu(\mu_+, \mu_-)$ are all positive. From figures 1–3 we see that with an increase of b_0^2, μ_- increases but the corresponding Λ_- first decreases, attains a minimum value and then increases. In the other branch, when μ_- decreases, Λ_- increases, attains a maximum and thereafter decreases. In both the cases, for $b_0^2 = 2d(d-1)C_2/|C_1|$, Λ_+ and Λ_- attain the same numerical value

$$\Lambda_+ = \Lambda_- = \frac{d^2 - 3d - 6}{8d(d-1)} \frac{C_1^2}{C_2}. \tag{14}$$

We note that for a given value of Λ, we have in general two different values of b_0^2 excepting for the following ranges of Λ (in units of C_1^2/C_2) values: (i) (0·155 – 0·170) and (0·170 – 0·205) for $d = 6$, (ii) (0·155 – 0·170) and (0·170 – 0·195) in $d = 7$ and (iii)

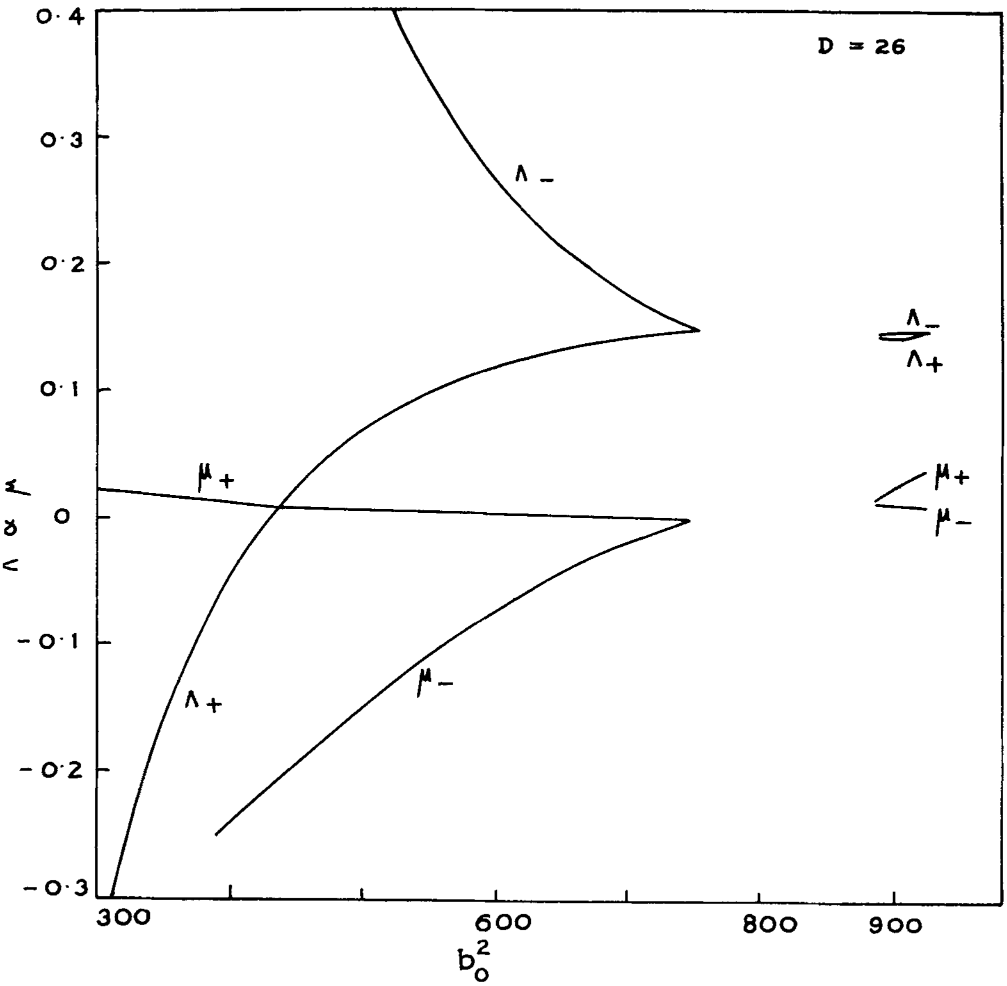

Figure 3. Permitted values of Λ and μ in units of C_1^2/C_2 and $|C_1|/C_2$ respectively for various b_0^2 (in units of $C_2/|C_1|$) for $D=26$. Λ_+, Λ_- corresponds to μ_+, μ_- respectively.

(0·142 – 0·145) and (0·146 – 0·149) for $d=22$. Within the ranges mentioned, three different values of b_0^2 are permitted. It may be noted that as the number of extra dimensions increases the widths of these ranges decrease. Also, for $\mu > 0$, the D-dimensional cosmological constant lies in the range

$$0 < \Lambda < \frac{d(d-1)}{8(d-2)(d-3)} \frac{C_1^2}{C_2},$$

whereas for $\mu < 0$, we have

$$\frac{d(d-1)}{8(d-2)(d-3)} \frac{C_1^2}{C_2} < \Lambda < \infty.$$

The case $\mu < 0$ does not give instanton configuration, but it may be useful in cosmological model building. We note that even with negative values of D-dimensional Λ, we can have a de Sitter type instanton with a small b_0^2.

The euclidean action for the de Sitter type solution (8) may be written as:

$$I_E^0 = \tfrac{8}{3}\pi^2[C_0 + 12\mu C_1 + d(d-1)b_0^{-2}C_1 + C_2(\mathrm{GB}_0)]\, V_d\mu^{-2} \tag{15}$$

where $\mathrm{GB}_0 = d(d-1)(d-2)(d-3)b_0^{-4} + 24\mu^2 + 24d(d-1)b_0^{-2}\mu$, and V_d is the volume of the extra space. The action is finite.

4. Wormhole solutions with higher N

We have found some approximate wormhole solutions with $N = 1$ and $N = 2$, by including a perfect fluid. These are given below:

(B) *For* $N = 1$

We assume that $b_0 < a$, neglect terms of order a^{-8} and obtain a solution

$$\dot{a}^2 = 1 - \mu a^2 - \frac{v_1}{a^2} \tag{16}$$

where v_1 is given by

$$v_1 = \frac{\rho a^4}{3(C_1 + 2d(d-1)C_2 b_0^{-2})}$$

with

$$\rho = 3p \quad \text{and} \quad p' = 0(a^{-8}) \sim 0. \tag{17}$$

The instanton corresponding to (16) is

$$a = (2\mu)^{-1/2}[1 + \eta\cos\sqrt{\mu}\tau]^{1/2} \tag{18}$$

where $\eta = \sqrt{(1 - 4\mu v_1)}$.

This solution requires the same Λ as in (9) and the pressure $p' = 0$ implies the condition given in (10). Since (9) and (10) give the same parametric relations, the results regarding the behaviour of Λ and μ obtained for the de Sitter instantons will be valid in this case also. However, the action here is not finite, as we will see in the next section.

An interesting case is obtained by putting $\mu = 0$, viz.

$$\dot{a}^2 = 1 - \frac{v_1}{a^2}. \tag{19}$$

This gives $b_0^2 = 2(d-2)(d-3)C_2/|C_1|$ and $\Lambda = \dfrac{d(d-1)C_1^2}{8(d-2)(d-3)C_2}$. The instanton corresponding to (19) is described by

$$a^2 = \tau^2 + v_1 \tag{20}$$

so that $a \sim \tau$, as $\tau \to \infty$. This when continued to Lorentzian metric gives an asymptotic flat space.

C. For $N = 2$

We find a few interesting solutions of the type

$$\dot{a}^2 = 1 - \mu a^2 - \frac{\nu_2}{a^4} \tag{21}$$

for specific values of b_0^2. The first solution corresponds to the stiff matter equation of state in 4-dimensions:

$$\rho = p = 3\left(C_1 + \frac{2d(d-1)}{b_0^2} C_2\right)\frac{\nu_2}{a^6} \tag{22}$$

$\Lambda = C_0/2 = \dfrac{(d^2 + 2d - 9)C_1^2}{8d(d-2)C_2}$ and $p' = 0$ when terms of order a^{-12} are neglected. The solution requires $b_0^2 = 2d(d-2)C_2/|C_1|$ and $\mu = \dfrac{1}{4d}\dfrac{|C_1|}{C_2}$. This is similar to the instanton solution obtained by Lee [14] for a complex scalar field in Einstein gravity in 4-dimensions.

Lastly, we note that for $D = 6$ i.e., with $d = 2$, and $C_1 = 0$, eq. (21) reduces to a simple form:

$$\dot{a}^2 = 1 - \nu_2 a^{-4} \tag{23}$$

with

$$\rho = p = 12 C_2 \nu_2 b_0^{-2} a^{-6}, p' = 0. \tag{24}$$

The wormhole corresponding to eq. (23) is similar to that obtained by Giddings and Strominger [16] in 4-dimensions in the presence of an antisymmetric tensor field $B_{\mu\nu}$.

Another solution is obtained for

$$\rho = p = p' \tag{25}$$

with

$$b_0 = (2d(d-2)C_2/|C_1|)^{1/2},$$

$$\mu = \frac{(d-1)|C_1|}{2d(d-2)C_2}, \tag{26}$$

$$\Lambda = \frac{C_0}{2} = \frac{d^3 - 19d + 18}{8d(d-2)^2}\frac{C_1^2}{C_2} \tag{27}$$

This type of wormhole configuration was noted by Myers [15] in higher dimensions without the GB term.

5. Action for wormholes with $N = 1, 2$

The wormholes discussed above involve coordinate singularities. There is, however, no curvature singularity and one has to consider a regular extension of the metric beyond the extremal surfaces $a = a_{\min}$ and $a = a_{\max}$. One may follow Halliwell and Myers [17] and consider sewing together a sequence of Euclidean manifolds of the type (7). A multitude of tunneling processes can be described by these configurations, although the calculations have technical difficulties excepting for $2+1$ dimensions [17]. The actions of the Euclidean solutions (16) and (21) covered within the coordinate patch $a_{\min}$ to $a_{\max}$ can be determined analytically. For the solution (16), the Euclidean action is given by:

$$I_E^1 = 2\pi^2 V_d A\mu^{-1/2}\left[((1+\eta)/(18\mu))^{1/2}\left[\frac{2}{\mu}E\left(\frac{\pi}{2},q\right)+\frac{2v_1}{1+\eta}F\left(\frac{\pi}{2},q\right)\right]\right]$$
$$+2\pi^2 V_d\left(C_1+\frac{2d(d-1)}{b_0^2}C_2\right)v_1(2/(1+\eta))^{1/2}F\left(\frac{\pi}{2},q\right) \tag{28}$$

where $F(E)$ are the elliptic integrals of first (second) kind,

$$\eta = (1-4\mu v_1)^{1/2}$$
$$q = (2\eta/(1+\eta))^{1/2}, \quad v_1 > 0,$$
$$A = 0 \text{ when } \mu = 0 \text{ and}$$
$$= d(d-1)b_0^{-2}[C_1+(d-2)(d-3)C_2 b_0^{-2}] - \frac{12d(d-1)C_1}{(d-2)(d-3)b_0^2}$$
$$+\frac{d^4-6d^3-13d^2+66d-48}{4(d-2)^2(d-3)^2}\frac{C_1^2}{C_2} \quad \text{where} \quad \mu \neq 0 \tag{29}$$

The Euclidean action corresponding to the wormhole given by (21) is

$$I_E^2 = \frac{4\pi^2 A V_d}{3\mu^{3/2}}\left[\frac{2\gamma}{(\alpha-\gamma)^{1/2}}F\left(\frac{\pi}{2},\psi\right)+2\frac{\alpha}{\beta}(\alpha-\gamma)^{1/2}E\left(\frac{\pi}{2},\psi\right)\right]+\frac{\pi^2 B V_d}{(\mu)^{1/2}}\left[\frac{2}{\alpha\gamma(\alpha-\gamma)}\right.$$
$$\left.(\gamma-\beta)\Pi\left(\frac{\pi}{2},\frac{\gamma}{\beta}\psi^2,\psi\right)+\beta F\left(\frac{\pi}{2},\psi\right)\right] \tag{30}$$

where F, E, Π are the elliptic integrals of first, second and third kinds respectively, $\psi = \left[\frac{\alpha-\beta}{\alpha-\gamma}\right]^{1/2}$, and A and B are given below for the equation of states (22) and (25):

(i) for $\rho = p$ and $p' = 0$:

$$A = \frac{3(d-1)C_1^2}{d^2(d-2)C_2}, \quad B = \frac{3(3d-4)}{d^2(d-2)^2}C_1 v_2. \tag{31}$$

When $D = 6$, $C_1 = 0$ $(\mu = 0)$, the Euclidean action is given by

$$I_E^2 = 6\pi^3 C_2 v_2 V_d / b_0^2. \tag{32}$$

The action is finite:

(ii) for $\rho = p = p'$:

$$A = \frac{9d^2-15d+6}{d^2(d-2)^2}\frac{C_1}{C_2}, \quad B = \frac{3(5d-4)}{d(d-2)}C_1 v_2, \tag{33}$$

we note also that $v_2 > 0$ and α, β and γ are the roots of the polynomial

$$-x^3 + \mu^{-1}x^2 + v_2\mu^{-1} = 0, \quad (x = a^2)$$

and $\alpha > \beta > 0 > \gamma$.

6. Discussion

We have presented above a number of wormhole solutions in a theory based on an action which includes the Gauss-Bonnet combination of quadratic terms in curvatures. The model is different from what one gets from the string theory in the small slope limit, as we have chosen C_1 and C_2 with opposite signs. The resulting theory has some striking features. If one tries to build a quantum cosmological model starting from this classical action, in dimensions $D = 4 + d$, the instanton-like configurations mentioned above will be relevant. Of particular interest in this connection are de-Sitter type instanton configurations. The model makes some definite predictions for the cosmological tunneling probabilities. We note that the action for the de-Sitter instantons increases rapidly as b_0^2 increases from zero up to $b_0^2 = \frac{2}{3}[(d-1)(3d-8) - 4((d-1)(d-4))^{1/2}]C_2/|C_1|$. Obviously the universe is unlikely to nucleate with $b_0^2 \leqslant \frac{2}{3}[(d-1)(3d-8) - 4((d-1)(d-4))^{1/2}]C_2/|C_1|$. Again for the range of values, $\frac{2}{3}[(d-1)(3d-8) - 4((d-1)(d-4))^{1/2}]C_2/|C_1| < b_0^2 < \frac{2}{3}[(d-1)(3d-8) + 4((d-1)(d-4))^{1/2}]C_2/|C_1|$, there is no instanton. A tunneling mechanism is, therefore, more likely in region 5. While the probability for cosmic tunneling may or may not be testable by any experiment now, there is one prediction which in principle can be tested. If the universe is actually higher dimensional, with d extra dimensions of radius b_0, this model predicts that b_0^2 cannot have a value lying between $\frac{2}{3}[(d-1)(3d-8) - 4((d-1)(d-4))^{1/2}]C_2/|C_1|$ and $\frac{2}{3}[(d-1)(3d-8) + 4((d-1)(d-4))^{1/2}]C_2/|C_1|$. This of course, presupposes that the nucleation is described by a de-Sitter instanton. The Myers-type wormholes mentioned above have infinite action and a possible application of these solutions to quantum cosmology is still uncertain.

Acknowledgements

One of the authors (SM) would like to thank Inter-University Centre for Astronomy and Astrophysics, Pune and the University of Zululand, South Africa for hospitality during the period of this work. SM and AB also acknowledge financial support from FRD (South Africa).

References

[1] T Kaluza, *Sitz. Preuss. Acad. Wiss* **F1**, 966 (1921)
O Klein, *Z. Phys.* **37**, 895 (1926)

[2] P G O Freund and M A Rubin, *Phys. Lett.* **B97**, 233 (1980)
M J Duff, B E W Nilsson and C N Pope, *Nucl. Phys.* **B233**, 433 (1984)
J Schwarz, *Nucl. Phys.* **B226**, 269 (1983)
E Cremmer and J Scherk, *Nucl. Phys.* **B108**, 409 (1976)
S Randjbardaemi, A Salam and J Strathdee, *Phys. Lett.* **B124**, 349 (1983); *Nucl. Phys.* **B214**, 491 (1983)
P S Howe and P C West, *Nucl. Phys.* **B238**, 181 (1984)

[3] M B Green and J H Schwarz, *Phys. Lett.* **B149**, 117 (1984); **B151**, 21 (1985)
[4] B Zweiebach, in *Anomalies, geometry and topology* edited by W A Bardeen and A White (Singapore, World Scientific, 1985); *Phys. Lett.* **B156**, 314 (1985)
[5] Q Shafi and C Wetterich, *Nucl. Phys.* **B289**, 787 (1987)
C Wetterich, *Phys. Lett.* **B113**, 377 (1982)
F S Accetta, M Gleiser, R Holman and E W Kolb, *Nucl. Phys.* **B276**, 501 (1986)
[6] D Bailin and A Love, *Phys. Lett.* **B163**, 135 (1985)
D Bailin, A Love and D Wong, *Phys. Lett.* **B165**, 270 (1985)
[7] D Lorentz-Petzold, *Prog. Theor. Phys.* **78**, 969 (1987); *Mod. Phys. Lett.* **A3**, 827 (1988)
[8] B C Paul and S Mukherjee, *Phys. Rev.* **D42**, 2595 (1990)
[9] E Baum, *Phys. Lett.* **B133**, 185 (1983)
[10] S W Hawking, *Phys. Lett.* **B134**, 403 (1984)
[11] S Coleman, *Nucl. Phys.* **B310**, 643 (1988)
[12] P Gonzalez-Diaz, *Phys. Lett.* **B247**, 251 (1990)
[13] S Mukherjee, B C Paul, N Dadhich and A Kshirsagar, *Phys. Rev.* **D45**, 2772 (1992)
[14] K Lee, *Phys. Rev. Lett.* **61**, 263 (1988)
[15] R C Myers, *Nucl. Phys.* **B323**, 225 (1989)
[16] S Giddings and A Strominger, *Nucl. Phys.* **B306**, 890 (1988); **B307**, 854 (1988)
[17] J J Halliwell and R C Myers, *Phys. Rev.* **D40**, 4011 (1989)