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Effect of two body core on the charge form factor of ^{3}He including three body force and meson exchange current

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Abstract. We compare the effect of two body core on the charge form factor of ^{3}He by the hyperspherical harmonics expansion method using various 2BF potentials with the inclusion of three body force. We also include the meson exchange current contribution to the CFF for the same potentials in addition to the 3BF. The results indicate that the combined effect of 3BF and MEC (i) moves $q^2_{\rm min}$ (the first diffraction minimum) appreciably to the left, amount of shift depends on the 2BF at $r_{12} \sim 0{\cdot}7$ fm and (ii) enhances $F_{\rm max}$ (the height of the secondary maximum of CFF) by an appreciable amount, the increment in general increases with the repulsive core of 2BF ($r \leqslant 0{\cdot}1$ fm).

Keywords. Charge form factor; three body force; hyperspherical harmonics expansion.

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Experimentally observed charge form factor (CFF) of the trinucleon system has long eluded complete theoretical understanding. The observed first diffraction minimum ($q^2_{\rm min}$) for ^{3}He occurs at $q^2_{\rm min} = 11{\cdot}8\,{\rm fm}^{-2}$ (q being the momentum transfer) and the magnitude of the secondary maximum ($F_{\rm max}$) is about 6×10^{-3} at $q^2 = 15{\cdot}65\,{\rm fm}^{-2}$ [1–3]. Calculations with standard two body force (2BF) produce $q^2_{\rm min}$ around $16\,{\rm fm}^{-2}$ and $F_{\rm max} \sim 10^{-3}$ [4–12]. A Fourier transform of experimental CFF with correction for finite size of the proton gives the point proton density, $\rho(r)$, which shows a pronounced central depression. Although it might be thought that a strongly repulsive 2BF core could produce a central hole in $\rho(r)$, it is immediately apparent that while such a core will produce a central hole for the equilateral triangle configuration (ETC), it will have no effect on the collinear configuration (CC), no matter how strong the repulsive core is. This is also borne out in theoretical calculations. A preference for ETC is provided by the nuclear three body force (3BF), which is attractive for ETC and repulsive for CC. However a 3BF without sufficiently strong repulsive 2BF core cannot produce a central hole and therefore fails to reproduce experimental CFF. Inclusion of standard 3BF [13–17] increases $F_{\rm max}$ slightly while $q^2_{\rm min}$ moves further out. Thus the effect of 3BF on CFF is marginal. However binding energy (BE) enhances by an appreciable amount, accounting for nearly the whole of the missing value in 2BF calculations.

In 1974 Kloet and Tjon [18] included meson exchange currents (MEC) contribution to the CFF of ^{3}He using standard 2BF and found that $q^2_{\rm min}$ moves closer to the experimental value and $F_{\rm max}$ increased to a value of about $2{\cdot}5 \times 10^{-3}$ at $q^2 = 16{\cdot}5\,{\rm fm}^{-2}$. The contribution of the MEC at large momentum transfer is strongly influenced by the nature of the wave function at very short separations, which in turn is determined by the softness of the 2BF core.

Thus the nature of the 2BF core is expected to play an important role in the behaviour of the CFF at large momentum transfers. This motivates us to study the behaviour of CFF for various 2BF with varying core softness when 3BF and MEC are included. We have adopted the hyperspherical harmonics expansion (HHE) method [10] to solve the trinucleon system. The ground state is represented by the space totally symmetric S-state for simplicity. Appropriate to this restriction, we choose only S-projected central (spin dependent or not) potentials. We concentrate our attention on two such potentials, viz Afnan–Tang S3 potential [19] and Malfleit–Tjon MTV potential [20]. In addition we have investigated softer potentials like (in increasing order of softness) Eikemeir–Hackenbroich S4 potential [21], Gogny-Pires–deTourreil (GPDT) potential [22], Volkov [23] and Baker potentials [24]. For the 3BF we choose the form given by Fujita–Miyazawa (FM-3BF) for the two pion exchange 3BF [25].

The exchange contribution to the CFF of ^{3}He is given by [18]

$$F_{ch}^{cz} = \frac{-g^2}{2(4\pi\sqrt{M})^3}[F_v(q^2) + G_v(q^2) + 3F_s(q^2) + 3G_s(q^2)]K(q^2) \tag{1}$$

where the function $K(q^2)$ is defined as,

$$K(q^2) = \int \mathrm{d}\mathbf{p}_1 \mathrm{d}\mathbf{p}'_1 \mathrm{d}\mathbf{q}_1 \phi_0(p_1, q_1)\phi_0(p'_1, q'_1) \times \frac{q^2 - 2(\mathbf{p}_1 - \mathbf{p}'_1)\cdot\mathbf{q}\sqrt{M}}{\left[(\mathbf{p}_1 - \mathbf{p}'_1)\sqrt{M} - \frac{\mathbf{q}}{2}\right]^2 + \mu^2} \tag{2}$$

where $\phi_0(p, q)$ is the momentum space wave function for the S-state of trinucleon, $\mathbf{p}_1, \mathbf{q}_1$ are relative momenta and $q' = |\mathbf{q}_1 - (\mathbf{q}_1 - (\mathbf{q}/2\sqrt{3M}))|$.

M and μ are the nucleon (939 MeV) and pion (139·6 MeV) masses respectively. F_v and F_s are respectively the isovector and isoscalar charge form factor of the nucleons, while G_v and G_s are its isovector and isoscalar magnetic form factors and are given in [26]. g is the πNN coupling constant.

Introducing Fourier transforms to express $K(q^2)$ in terms of the configuration space wave function $\psi(\mathbf{x}, \mathbf{y})$ obtained by the HHE method and doing the angular integrations analytically, we have

$$K(q^2) = \frac{4\pi^2 q}{M^{3/2}} \int \sum_{KK'} \mathcal{N}_K \mathcal{N}_{K'} u_K(r) u_{K'}(r) r^{-5/2\,(2)}P_{2K}^{00}(\phi)^{(2)}P_{2K'}^{00}(\phi) \times j_0\left(\frac{qy}{2\sqrt{3}}\right) j_1\left(\frac{xq}{2}\right) \frac{\exp(-\mu x)}{x} x^2 y^2 \mathrm{d}x\mathrm{d}y \tag{3}$$

where $r = (x^2 + y^2)^{1/2}$ and $\tan\phi = x/y$ are the hyperspherical variables. The other quantities as well as the HHE method have been explained in other technical reports [10, 13]. The hyperradial wave function $u_K(r)$ is obtained by solving a set of coupled differential equations resulting from the HHE method. The contribution of MEC is then obtained by numerically evaluating the double integral (3).

The FM-3BF has a strong singularity (which goes as r^{-6} for $r \to 0$) and is attractive for the equilateral triangle configuration. Unless regularized for very short separations, this will lead to unphysical results. Hence we introduce a phenomenological cut-off parameter (x_0), such that the 3BF is not allowed to diverge for $r < x_0$. For small values of x_0, the strongly attractive 3BF induces nodes near origin (NNO) in the ground state hyperradial wave function, which are manifestations of the unphysical nature of the 3BF interaction for such values of x_0. Hence we choose x_0 for each given 2BF by a criterion similar to that used by Das *et al* [13], namely x_0 should be such that no unphysical nodes appear in the hyperradial wave function, but F_{max} has the largest value. This highlights the effect of 3BF on F_{max} and may result in a somewhat larger value of F_{max}. However explicit calculations using the regularized FM-3BF ($C_p = 0{\cdot}9$, $x_0 = 0{\cdot}34$ fm) as well as Tucson–Melbourne (TM-3BF) [27] with $\Lambda^2 = 25$ and Brazil (BR-3BF) [28] with $\Lambda^2 = 25$ three body forces in addition to S3-2BF, produce respectively the values 1·580, 1·584 and 1·582 for F_{max} and 15·54 fm^{-2}, 15·85 fm^{-2} and 15·73 fm^{-2} for q^2_{min}. This demonstrates that the overall effect of the regularized FM-3BF is the same as other standard 3BF.

Inclusion of 3BF enhances calculated binding energy (BE) from 5·789 MeV to 6·485 MeV for the S3 potential and from 7·369 MeV to 8·361 MeV for the MTV potential. Calculated CFF's have been demonstrated in figures 1 and 2. In figure 1 we plot CFF of ^{3}He as a function of q^2 for 2BF (S3 and MTV) with and without MEC contribution. In figure 2, the same quantities are plotted when FM-3BF is added to the chosen 2BF. From these figures we see that with the inclusion of 3BF without MEC, q^2_{min} moves further out for both MTV and S3 potentials, while increment of F_{max} is marginal in both cases. However the effect of MEC is substantial for both. For

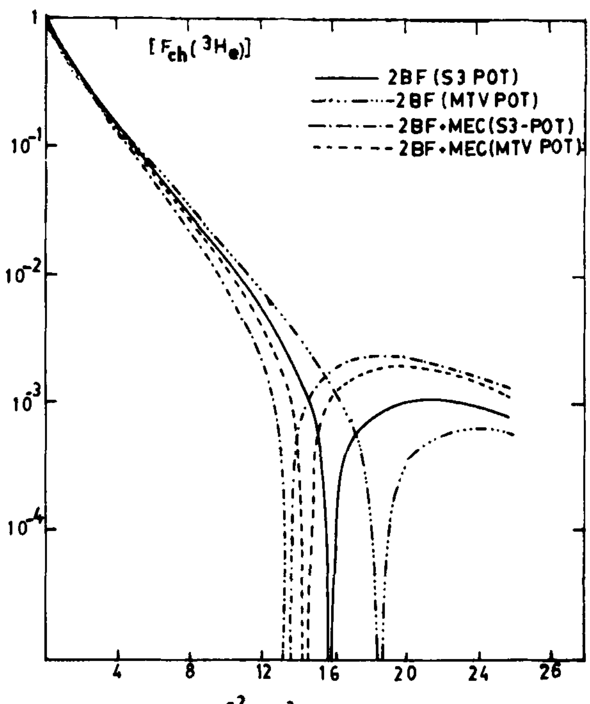

Figure 1. Absolute value of charge form factor of ^{3}He for 2BF with and without exchange correction.

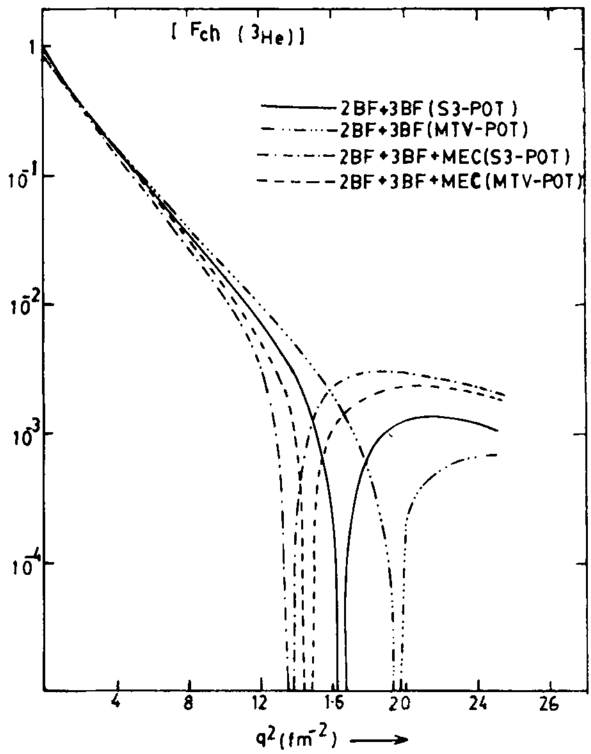

Figure 2. Absolute value of charge form factor of ^3He including 3BF with and without exchange correction.

S3 potential, values of q^2_{min} and F_{max} are $13{\cdot}36\,fm^{-2}$ and $2{\cdot}24 \times 10^{-3}$ respectively for MEC with 2BF alone. The corresponding values for MTV potential are $14{\cdot}38\,fm^{-2}$ and $1{\cdot}85 \times 10^{-3}$ respectively. For the MEC contribution in addition to 2BF plus 3BF, q^2_{min} and F_{max} take the values $13{\cdot}61\,fm^{-2}$ and $2{\cdot}98 \times 10^{-3}$ for S3 potential and $14{\cdot}77\,fm^{-2}$ and $2{\cdot}37 \times 10^{-3}$ for MTV potential. The MTV has a stronger repulsive core for $r_{ij} < 0{\cdot}1$ fm than S3 potential and we find the enhancement effects on q^2_{min} and F_{max} are larger in the direction of the experimental values for the MTV potential than the other. However the results including 3BF and MEC for MTV are worse than those for S3, since the CFF obtained with 2BF alone is far worse for MTV than S3 potential. From the calculations with other 2BF (Baker, Volkav, GPDT and S4) it was seen that enhancement effects on F_{max} due to the inclusion of 3BF and MEC generally increases with decreasing softness of the 2BF. Enhancement of BE due to the inclusion of 3BF over 2BF is fairly large and increases with decreasing softness of the 2BF core. The enhancement of q^2_{min} due to MEC contribution appears to be uncorrelated with 2BF core softness, if we consider the strengths of 2BF at $\sim 0{\cdot}1$ fm, but it decreases as the value of 2BF at $r_{ij} \sim 0{\cdot}7$ fm increases. This may reflect the fact that $K(q^2)$ (eq. (3)) contains an oscillatory factor (through the spherical Bessel functions) and for a large value of $q(\sim 4\,fm^{-1})$ only small values of x and y should contribute. At very small values of x and y, the wave function itself decreases rapidly and the major contribution to $K(q^2)$ comes from intermediate values of x and y. As the attractive 2BF increases at such values of $x(=r_{12})$ and y which contribute dominantly to $K(q^2)$, the three body wave function $\psi(x,y)$ increases and the contribution of MEC to CFF increases.

Two body core and charge form factor

From the foregoing discussions we can draw the following conclusions.

(i) Inclusion of 3BF enhances BE, the enhancement being generally larger for stronger repulsive 2BF core ($r_{ij} \sim 0{\cdot}1$ fm).
(ii) Inclusion of 3BF increases F_{max} slightly but q^2_{min} moves in general outwards. Thus inclusion of 3BF makes the agreement of CFF worse.
(iii) Inclusion of MEC moves q^2_{min} inwards by appreciable amounts, the amount of shift appears to depend on the 2BF at $r_{12} \sim 0{\cdot}7$ fm.
(iv) Inclusion of MEC enhances F_{max} by an appreciable amount, the increment in general increases with the strength of repulsive core of 2BF ($r_{ij} \sim 0{\cdot}1$ fm).

In view of the above observations it may be concluded that the effects of the 3BF and MEC will be largest towards explaining the experimental data if the 2BF has (i) a very strong repulsive core at $r_{ij} \sim 0{\cdot}1$ fm and (ii) is strongly attractive at $r_{ij} \sim 0{\cdot}7$ fm.

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