PRAMANA
— journal of
physics

© Printed in India

Vol. 43, No. 4,
October 1994
pp. 289–305

A new formalism for the statistical dynamics of vector spin systems

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MS received 12 April 1994; revised 28 June 1994

Abstract. The relaxational dynamics of a classical vector Heisenberg spin system is studied using the Fokker–Planck equation. To calculate the eigenvalues of the Fokker–Planck operator, a new approach is introduced. In this connection, a number space repesentation is introduced, which enables us to visualize the eigenvalue structure of the Fokker–Planck operator. The mean field approximation is derived and a systematic method to improve the mean field approximation is presented.

Keywords. Relaxational dynamics; vector Heisenberg model; Fokker–Planck equation; mean field approximation.

PACS Nos 05·70; 75·10; 76·20

1. Introduction

In a previous paper [1] we introduced a new formalism to study the relaxational dynamics of a planar ferromagnet. This formalism was based on the Fokker–Planck equation for the time-dependent joint probability distribution of the spins in the system. Unlike most recent work [2, 3] on relaxational dynamics, particularly at the critical point, our work deals with fixed length spins. Our method mainly consists in finding out the eigenvalues of the Fokker–Planck operator, which are the relaxation rates in the system. To achieve this, we construct a convenient representation, which is somewhat analogous to the number space representation of second quantization. In this representation a new conservation principle becomes immediately obvious and certain, hitherto unnoticed, features of the relaxation eigenvalue spectrum of planar spin system are brought forth easily. Furthermore the formalism offers a systematic method of doing a perturbation expansion, renormalization group analysis and systematic improvement over the mean field approximation in problems involving dynamics of fixed length spins.

The purpose of the present paper is to extend this formalism to vector spin systems. The formalism for vector spin systems is considerably more complicated than that for planar spins due to additional degrees of freedom, namely azimuthal angles at each site. The paper is organized as follows. In § 2, we develop the Fokker–Planck equation for a system of classical spins interacting via Heisenberg Hamiltonian. This development is a generalization of the earlier work of Kubo and Hashitsume [4] on the relaxation of a single moment. In § 3 we construct a representation for the Fokker–Planck operator which is diagonal in the rotational diffusion operator. This operator is independent for different spins and the interaction term acts to couple diffusion on different sites leading to a band structure for relaxation eigenvalues. It is shown in § 4 that the mean field approximation ignores all the bands except the lowest one. This observation then allows us to construct approximations which are systematic improvements over the mean field approximation. In § 5, the first such approximation is constructed. This section is of a demonstrational nature as the detailed analysis of the improved approximation and its physical consequences are not presented. We conclude with some remarks on the potentiality of the present work for further numerical studies.

2. Fokker–Planck equation for spin system

We consider a lattice of classical spins $\mathbf{S}_i$ of unit magnitude, interacting according to the Heisenberg model, with the Hamiltonian

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j - \mathbf{H}_0\cdot\sum_i \mathbf{S}_i, \tag{2.1}$$

where J_{ij}'s denote short-ranged exchange interactions and $\mathbf{H}_0$ is an external field, which we shall take to be along z-axis. The relaxational dynamics of the system is described through a set of Langevin equations for each spin, which are straightforward generalizations of the Landau–Lifshitz equation [5] for a single magnetic moment in interaction with heat bath. These are

$$\frac{\mathrm{d}\mathbf{S}_i}{\mathrm{d}t} = \gamma[\mathbf{h}_i + \mathbf{H}'_i(t)] \times \mathbf{S}_i - \gamma\eta(\mathbf{h}_i \times \mathbf{S}_i) \times \mathbf{S}_i \tag{2.2}$$

where

$$\mathbf{h}_i = \sum_j J_{ij}\mathbf{S}_j + \mathbf{H}_0 \tag{2.3}$$

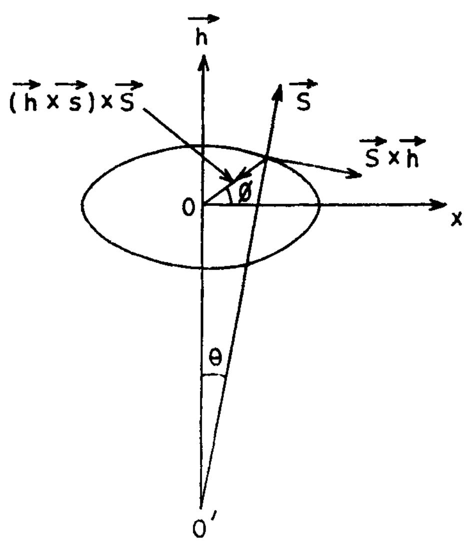

Figure 1. The figure denotes the direction of forces occurring in equation (2.2).

where γ is the gyromagnetic ratio, η the viscosity coefficient and $\mathbf{H}_i'(t)$ the random magnetic field at site i. The physical meaning of the various terms of (2.2) is as follows (see figure 1). The first term $\gamma \mathbf{h}_i \times \mathbf{S}_i$ describes the precession of the spin $\mathbf{S}_i$ about the local field $\mathbf{h}_i$ and this term conserves the energy $\mathbf{h}_i \cdot \mathbf{S}_i$. The remaining terms describe the effect of heat bath. As shown in figure 1 the frictional term $\gamma\eta(\mathbf{h}_i \times \mathbf{S}_i) \times \mathbf{S}_i$ acts to align the spin vector $\mathbf{S}_i$ along the direction of the local field $\mathbf{h}_i$, and is dissipative in nature. Finally the Brownian random force term is $\mathbf{S}_i \times \mathbf{H}_i'(t)$. Note both these terms are perpendicular to $\mathbf{S}_i$, so that the magnitude of the vector is preserved. $\mathbf{H}_i'(t)$ are taken to be Gaussian white noise, with the following usual properties for its components, $H_{i\alpha}'(t)$,

$$\langle H_{i\alpha}'(t) \rangle = 0, \tag{2.4a}$$

$$\gamma^2 \langle H_{i\alpha}'(t_1) H_{j\beta}'(t_2) \rangle = \delta_{ij}\delta_{\alpha\beta}\left[\frac{1}{\tau_{\parallel}}\delta_{\alpha z} + (1-\delta_{\alpha z})\frac{1}{\tau_{\perp}}\right]. \tag{2.4b}$$

Now we follow the treatment of Kubo and Hashitsume [4] for the single moment, in writing down the Liouville equation for time-dependent distribution function $\rho(\mathbf{S}_1, \ldots, \mathbf{S}_N, t)$. This is given by

$$\frac{\partial \rho}{\partial t}(\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_N, t) = -\sum_{i\alpha} \frac{\partial}{\partial S_{i\alpha}}(\dot{S}_{i\alpha}\rho), \tag{2.5}$$

where $\dot{S}_{i\alpha}$ denotes the time derivative of $S_{i\alpha}$. Using (2.2), (2.5) can be written as

$$\frac{\partial \rho}{\partial t} = -i\gamma \sum_i [\mathbf{h}_i + \mathbf{H}_i'(t)] \cdot \mathbf{L}_i \rho - i\gamma\eta \sum_i \mathbf{h}_i \cdot (\mathbf{L}_i \times \mathbf{S}_i)\rho, \tag{2.6}$$

where $\mathbf{L}_i$'s denote angular-momentum-like operators given by

$$\mathbf{L}_j = -i\mathbf{S}_j \times \frac{\partial}{\partial \mathbf{S}_j}. \tag{2.7}$$

Our next task is to average over the random fields $\mathbf{H}_i'(t)$. This can be done using the interaction representation, for which purpose we write (2.6) as

$$\frac{\partial \rho}{\partial t} = -i[\mathscr{L}_0 + \mathscr{L}'(t)]\rho \tag{2.8}$$

$$\mathscr{L}_0 = \gamma \sum_i [\mathbf{h}_i \cdot \mathbf{L}_i + \eta \mathbf{h}_i \cdot (\mathbf{L}_i \times \mathbf{S}_i)] \tag{2.9}$$

and

$$\mathscr{L}'(t) = \gamma \sum_i \mathbf{H}_i'(t) \cdot \mathbf{L}_i. \tag{2.10}$$

Equation (2.8) is now solved in the interaction representation, the exact formal solution being

$$\rho(t) = \exp[-i\mathscr{L}_0 t] \exp_T\left[-i\int_0^t \mathrm{d}t' \sum_j \mathbf{H}_j'(t') \cdot \mathbf{L}_{Ij}(t')\right]\rho(0), \tag{2.11}$$

where $\exp_T$ denotes the usual time-ordered exponential and

$$\mathbf{L}_{Ij} = \exp[i\mathscr{L}_0 t]\mathbf{L}_j \exp[-i\mathscr{L}_0 t] \tag{2.12}$$

Using the Gaussian properties given in (2.4) and denoting the average value of $\rho(t)$ by $P(t)$, one averages (2.11) to find

$$P(t) = \exp[-i\mathscr{L}_0 t]\exp_T\left[-\frac{1}{2}\int_0^t \mathrm{d}t' \sum_k \left\{\frac{1}{\tau_\parallel}(L^z_{Ik}(t'))^2 + \frac{1}{\tau_\perp}((L^x_{Ik}(t'))^2 + (L^y_{Ik}(t'))^2)\right\}\right]\rho(0) \tag{2.13}$$

$$= \exp\left[-i\mathscr{L}_0 t - \frac{t}{2}\sum_k\left\{\frac{1}{\tau_\parallel}(L^z_k)^2 + \frac{1}{\tau_\perp}(L^x_k)^2 + \frac{1}{\tau_\perp}(L^y_k)^2\right\}\right]\rho(0). \tag{2.14}$$

The last step is permissible as the two operators in the exponential are commuting and L^2 and L_z^2 are time independent.

From (2.14), the Fokker–Planck equation follows as

$$\frac{\partial P}{\partial t} = -WP \tag{2.15}$$

with

$$W = i\gamma\sum_j[\mathbf{h}_j\cdot\mathbf{L}_j + \eta\mathbf{h}_j\cdot(\mathbf{L}_j\times\mathbf{S}_j)] + \frac{1}{2}\sum_j\left[\frac{1}{\tau_\parallel}(L^z_j)^2 + \frac{1}{\tau_\perp}(L^x_j)^2 + \frac{1}{\tau_\perp}(L^y_j)^2\right] \tag{2.16}$$

The first term in the Fokker–Planck operator W is the Liouville term, while the second one corresponds to rotational diffusion of spins. It is straightforward to verify that the steady state solution of (2.15) is the equilibrium distribution function, $P_{\mathrm{eq}}(\mathbf{S}_i)$, given by

$$P_{\mathrm{eq}}(\mathbf{S}_i) = Z^{-1}\exp\left[\frac{\beta}{2}\sum_{ij}J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j + \beta\mathbf{H}_0\cdot\sum_i\mathbf{S}_i\right] \tag{2.17}$$

provided the following Einstein relation holds

$$\frac{1}{\tau_\perp} = 2\gamma\eta k_B T. \tag{2.18}$$

Further Z is the equilibrium partition function for the system of (2.1). Note that only $\tau_\perp$ enters the Einstein relation. This is due to the fact that $\tau_\parallel$ controls the precessional motion (azimuthal angle) about the local field direction, while $\tau_\perp$ controls the relaxation towards the local axis (θ-angle). Clearly only the latter is of relevance for establishing thermal equilibrium. If we let $f(\{\mathbf{S}_i\}, t; \{\mathbf{S}'_i\}, 0)$ denote the conditional probability distribution, which is the solution of (2.15) with initial condition

$$f(\{\mathbf{S}_i\}, t; \{\mathbf{S}'_i\}, 0) = \prod_i \delta(\mathbf{S}_i - \mathbf{S}'_i) \tag{2.19}$$

then we can write the correlation function between any two operators $A_1(\{\mathbf{S}_i\})$ and

$A_2(\{\mathbf{S}_i\})$ as

$$C_{A_1,A_2}(t) = \int \prod_i \mathrm{d}\mathbf{S}_i \mathrm{d}\mathbf{S}'_i A_1(\{\mathbf{S}_i\}) f(\{\mathbf{S}_i\}, t; \{\mathbf{S}'_i\}, 0) \times A_2(\{\mathbf{S}'_i\}) p_{\mathrm{eq}}(\{\mathbf{S}'_i\}). \tag{2.20}$$

This correlation function can be recast in a Heisenberg-like representation by noting that formally the conditional probability distribution function can be written as

$$f(\{\mathbf{S}_i\}, t; \{\mathbf{S}'_i\}, 0) = \exp(-tW) \prod_i \delta(\mathbf{S}_i - \mathbf{S}'_i).$$

Further we define inner products and adjoints by the following equations

$$(g_1, g_2) = \int \mathrm{d}\Omega g_1^*(\Omega) g_2(\Omega)$$

$$(g_1, Ag_2) = \int \mathrm{d}\Omega (A^+ g_1, g_2),$$

where g's represent functions of angular variables Ω corresponding to vector $\mathbf{S}$ and the integration is over the solid angle. Now

$$\begin{aligned} C_{A_1 A_2}(t) &= \int\int \prod_i \mathrm{d}\mathbf{S}_i \mathrm{d}\mathbf{S}'_i A_1(\{\mathbf{S}_i\}) \mathrm{e}^{-tW} \prod_i \delta(\mathbf{S}_i - \mathbf{S}'_i) A_2(\{\mathbf{S}'_i\}) P_{\mathrm{eq}}(\{\mathbf{S}'_i\}) \\ &= \int \prod_i \mathrm{d}S_i \mathrm{e}^{-tW^+} A_1(\{\mathbf{S}_i\}) A_2(\{\mathbf{S}_i\}) P_{\mathrm{eq}}(\{\mathbf{S}_i\}) \\ &= \langle A_1(t) A_2 \rangle, \end{aligned} \tag{2.21}$$

where $\langle\ \rangle$ denotes the average over the equilibrium distribution function and

$$A(t) = \exp(-tW^+)A. \tag{2.22}$$

Further note

$$\frac{\mathrm{d}A}{\mathrm{d}t} = -W^+ A. \tag{2.23}$$

At this point, it is useful to display the full form of W^+, which is

$$\begin{aligned} W^+ = &-i\gamma H_0 \sum_k L_k^z - i\gamma \sum_{j,k} J_{jk} \mathbf{S}_j \cdot \mathbf{L}_k + \frac{1}{2\tau_\perp} \sum_{j,k} \frac{i\beta}{2} J_{jk} (\mathbf{S}_k \times \mathbf{S}_j) \cdot (\mathbf{L}_k - \mathbf{L}_j) \\ &+ \frac{1}{2\tau_\parallel} \sum_k (L_k^z)^2 + \frac{1}{2\tau_\perp} \sum_k \{(L_k^x)^2 + (L_k^y)^2\} \end{aligned} \tag{2.24}$$

It is also of interest to write down the equations of motion for $\mathbf{S}_i$, using (2.24). These are

$$\frac{\mathrm{d}S_i^z(t)}{\mathrm{d}t} = \gamma \sum_j J_{ij} (\mathbf{S}_j \times \mathbf{S}_i)_z - \frac{1}{2\tau_\perp} \sum_j \beta J_{ij} [(\mathbf{S}_j \times \mathbf{S}_i) \times \mathbf{S}_i]_z - \frac{1}{\tau_\perp} S_i^z(t) \tag{2.25a}$$

$$\frac{dS_i^x(t)}{dt} = \gamma H_0 S_i^y(t) - \frac{1}{2\tau_\perp}\sum_j \beta J_{ij}[(\mathbf{S}_j \times \mathbf{S}_i) \times \mathbf{S}_i]_x + \gamma \sum_j J_{ij}(\mathbf{S}_j \times \mathbf{S}_i)_x - \frac{1}{2}\left(\frac{1}{\tau_\parallel} + \frac{1}{\tau_\perp}\right) S_i^x(t) \tag{2.25b}$$

$$\frac{dS_i^y(t)}{dt} = -\gamma H_0 S_i^x(t) - \frac{1}{2\tau_\perp}\sum_j \beta J_{ij}[(\mathbf{S}_j \times \mathbf{S}_i) \times \mathbf{S}_i]_y + \gamma \sum_j J_{ij}(\mathbf{S}_j \times \mathbf{S}_i)_y - \frac{1}{2}\left(\frac{1}{\tau_\parallel} + \frac{1}{\tau_\perp}\right) S_i^y(t) \tag{2.25c}$$

In deriving (2.25) we have made use of the following relations

$$L_k^\beta S_k^\alpha = (-i)\theta_{\alpha\beta\gamma} S_k^\gamma \tag{2.26a}$$

and

$$L_k^2 S_k^\alpha = 2S_k^\alpha \tag{2.26b}$$

Though (2.25a–c) are in Heisenberg representations, in the present context their physical meaning is obtained only when we take averages of over the equilibrium distribution function.

This dynamics in which the random noise has been averaged out differs from the Langevin equation essentially by the replacement of the random field term by the diffusion term.

3. Number space representation for Fokker–Planck operator

Since we are dealing with only periodic functions of angles at each site, we can introduce a basis of a complete set of spherical harmonics at each site. A typical member of the basis is denoted as

$$|l_1, m_1; l_2, m_2; \ldots\rangle = \prod_k Y_{l_k, m_k}(\theta_k, \phi_k) \tag{3.1}$$

where k runs over all the spins.

The utility of this basis lies in two points. First, the diffusion operator is diagonal in this basis. Second, the operator W^+ contains products of vector operators and angular momentum operators which can cause transitions in which l and m at a site change only by ± 1 or 0. Just to set the notation, we record here the operation of basic operators in this basis

$$L_k^2 |\{l_k, m_k\}\rangle = l_k(l_k + 1)|\{l_k, m_k\}\rangle \tag{3.2}$$

$$L_k^z |\{l_k, m_k\}\rangle = m_k |\{l_k, m_k\}\rangle \tag{3.3}$$

$$L_k^\pm |\{l_k, m_k\}\rangle = B_\pm(l_k, m_k)|\ldots l_k, m_k \pm 1, \ldots\rangle \tag{3.4}$$

$$S_k^z |\{l_k, m_k\}\rangle = A_{0+}(l_k, m_k)|\ldots l_k + 1, m_k \ldots\rangle + A_{0-}(l_k, m_k)|\ldots l_k - 1, m_k, \ldots\rangle \tag{3.5}$$

$$S_k^+ |\{l_k, m_k\}\rangle = A_{++}(l_k, m_k)|\ldots l_k + 1, m_k + 1, \ldots\rangle + A_{+-}(l_k, m_k)|\ldots l_k - 1, m_k + 1, \ldots\rangle \tag{3.6a}$$

$$S_k^- |\{l_k, m_k\}\rangle = A_{-+}(l_k, m_k)|\ldots l_k + 1, m_k - 1, \ldots\rangle$$
$$+ A_{--}(l_k, m_k)|\ldots l_k - 1, m_k - 1, \ldots\rangle \tag{3.6b}$$

where $S^\pm = S^x \pm iS^y$ and $L^\pm$ are similarly defined. The coefficients $\{B_\pm\}$ and $\{A\}$ are collected together in Appendix-1 for ready reference. Using (3.2) to (3.6) it is straightforward to write the action of W^+ on a typical member of the basis. To see this we explicitly work out the action of the second term of W^+ ((2.24)), i.e.

$$\sum_{k,p} J_{k,p}\mathbf{S}_p \cdot \mathbf{L}_k |\{l_k, m_k\}\rangle = \sum_{k,p} J_{k,p}[S_p^z L_k^z + \tfrac{1}{2}S_p^- L_k^+ + \tfrac{1}{2}S_p^+ L_k^-]|\{l_k, m_k\}\rangle$$

$$= \sum_{k,p} J_{k,p}\Bigg[m_k \sum_{\sigma = \pm 1} A_{0\sigma}(l_p, m_p)|\ldots l_p \pm \sigma, m_p, \ldots\rangle$$
$$+ \frac{1}{2}B_+(l_k, m_k) \sum_{\sigma = \pm 1} A_{-\sigma}(l_p, m_p)|\ldots l_k, m_k + 1, \ldots, l_p \pm \sigma, m_p - 1, \ldots$$
$$+ \frac{1}{2}B_-(l_k, m_k) \sum_{\sigma = \pm 1} A_{+\sigma}(l_p, m_p)|\ldots l_k, m_k - 1, \ldots, l_p \pm \sigma, m_p - 1, \ldots\rangle \Bigg] \tag{3.7}$$

First, note that in the transitions caused by this term the total azimuthal quantum number, i.e. $C = \sum_k m_k$, does not change. An examination of the other terms in W^+ shows that this is not just the property of the second term, but that the entire W^+ conserves C.

We shall now write the action of W^+ on a basis function in a compact form

$$W^+|\{l_k, m_k\}\rangle = -i\gamma H_0 \sum_k m_k |\{l_k, m_k\}\rangle$$
$$- i\gamma \sum_{k,p} \sum_{\substack{\sigma_p, \mu_p \\ \mu_k}} J_{k,p} g(l_k, m_k, \mu_k; l_p, m_p, \sigma_p, \mu_p)|\ldots l_k, m_k + \mu_k, \ldots, l_p + \sigma_p, m_p + \mu_p, \ldots\rangle$$
$$+ \frac{1}{\tau_\perp}\sum_{k,p} \beta J_{k,p} \sum_{\substack{\sigma_k, \mu_k, \\ \mu_p, \sigma_p}} f(l_k, m_k, \sigma_k, \mu_k; l_p, m_p, \sigma_p, \mu_p)|\ldots l_k + \sigma_k, m_k + \mu_k, \ldots, l_p + \sigma_p, m_p + \mu_p, \ldots\rangle$$
$$+ \frac{1}{2}\sum_k \left\{ \frac{1}{\tau_\perp} l_k(l_k + 1) + \left(\frac{1}{\tau_\parallel} - \frac{1}{\tau_\perp} \right) m_k^2 |\{l_k, m_k\}\rangle \right. \tag{3.8}$$

where σ_k's take values ± 1 only, whereas μ_k's take values $1, 0, -1$, with the constraint $\mu_k + \mu_p = 0$. Note that the first two terms on the right hand side of (3.8) are imaginary. These correspond to the inertial precession of a spin in the external field and the internal field due to the other spins. In most of what follows we shall set $\mathbf{H}_0 = 0$ and drop the spin precession term which contributes an imaginary part to the eigenvalues of W^+. Clearly for the long time relaxation such a term is relatively less important. With $\mathbf{H}_0 = 0$, we can set $\tau_\parallel = \tau_\perp = \tau$. Now the off-diagonal terms come from the friction part of the operator, i.e. the third term on the right hand side of (3.8). Here we have 8 off-diagonal terms corresponding to the possibilities $\mu_k = \mu_p = 0$; $\mu_k = 1$, $\mu_p = -1$; combining with $\sigma_k = \pm 1$, $\sigma_p = \pm 1$. The coefficients f and g are listed in Appendix-1.

4. The mean field approximation

We shall now use the above formalism to calculate the Green's function $G_{10,10}(\mathbf{R}_k, t|\mathbf{R}_j)$ defined as

$$G_{10,10}(\mathbf{R}_k, t|\mathbf{R}_j) = \theta(t)\langle Y_1^0(\theta_k(t), \phi_k(t))\, Y_1^0(\theta_j, \phi_j)\rangle. \tag{4.1}$$

where $\theta(t)$ is the step function.

The equation of motion for this function is

$$\frac{\partial}{\partial t} G_{10,10}(\mathbf{R}_k, t|\mathbf{R}_j) = C_{10,10}(\mathbf{R}_k|\mathbf{R}_j) - \theta(t)\langle [W^+ Y_1^0(\theta_k, \phi_k)]\, Y_1^0(\theta_j, \phi_j)\rangle \tag{4.2}$$

where $C_{10,10}(\mathbf{R}_k|\mathbf{R}_j)$ is the equilibrium correlation function given by

$$C_{10,10}(\mathbf{R}_k|\mathbf{R}_j) = \langle Y_1^0(\theta_k, \phi_k)\, Y_1^0(\theta_j, \phi_j)\rangle. \tag{4.3}$$

Now, setting the external field to zero and ignoring the precessional term, the action of W^+ on $Y_1^0(\theta_k, \phi_k)$ is given by

$$\begin{aligned} W^+|\ldots l_k = 1, m_k = 0, \ldots\rangle &= \frac{1}{\tau}\Big|\ldots l_k = 1, m_k = 0, \ldots\rangle \\ &- \frac{1}{4\tau}\sum_p \beta J_{kp}\Big[\tfrac{4}{3}|\ldots l_p = 1, m_p = 0, \ldots\rangle \\ &+ \frac{2}{\sqrt{15}}\{|\ldots l_k = 2, m_k = -1, \ldots, l_p = 1, m_p = 1, \ldots\rangle \\ &+ |\ldots l_k = 2, m_k = 1, \ldots l_p = 1, m_p = -1, \ldots\rangle\} \\ &+ \frac{4}{3\sqrt{5}}\Big|\ldots l_k = 2, m_k = 0, \ldots l_p = 1, m_p = 0, \ldots\rangle\Big]. \end{aligned} \tag{4.4}$$

The explicit use of the forms of spherical harmonics in (4.4) shows that (4.4) is indeed equivalent to (2.25). Further (4.4) shows that the equation of motion for $G_{10,10}$ involves higher order Green's functions, as is expected in any many-body problem. To set up a systematic scheme of approximations, we note that in the above representation, at each site we have a discrete set of rotational diffusion states with eigenvalues $l(l+1)/\tau$ which are coupled by the interaction term. There are two physical effects of this coupling. First, the site levels broaden into bands which are well separated at high temperatures (because β is small). Second, it causes transitions between bands, by having a pair of coupled sites change their local states (see figure 2). For long times, clearly the lowest band is the most important one. So one can develop a set of approximations, by projecting out the higher bands from the calculations. The simplest

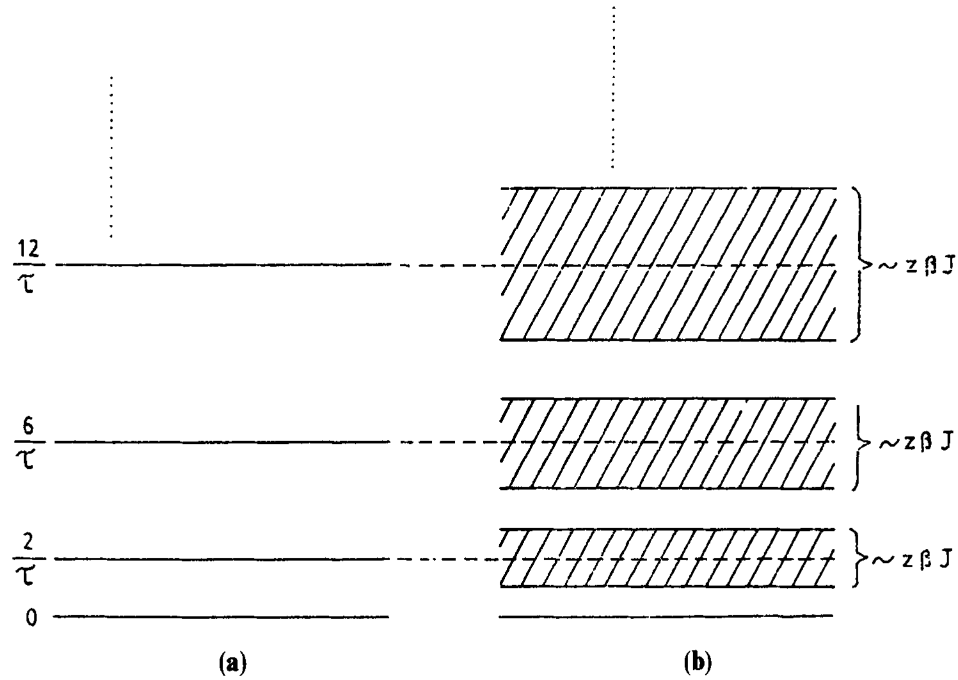

Figure 2. (**a**) The relaxational eigenvalues when $J = 0$. These correspond to relaxation of uncoupled spins due to thermal rotational diffusion. (**b**) Schematic structure of eigenvalues when $J \neq 0$. Now the rotational diffusion at different sites gets coupled leading to formation of bands much like tight-binding approximation.

of such approximations is clearly the one in which we just keep the lowest band. At the next level one can include a group of higher bands to which the lowest band couples directly and see their influence on the relaxation eigenvalues of the lowest band. Now we show that the simplest approximation discussed above leads to an analog of the mean field theory discussed by Edwards and Anderson [6] in the context of the planar spin glass. In this approximation, we drop the third, fourth and fifth terms on the right hand side of (4.4) as these correspond to a local excitation level of $\tau K = \Sigma_k l_k(l_k + 1) = 8$. Now the equation of motion for $G_{10,10}$ can be written as

$$\frac{\partial}{\partial t} G_{10,10}(\mathbf{R}_k, t|\mathbf{R}_j) = -\theta(t)\langle [W^+ Y_1^0(\theta_k, \phi_k)]\, Y_1^0(\theta_j, \phi_j)\rangle + C_{10,10}(\mathbf{R}_k|\mathbf{R}_j)$$

$$= -\frac{1}{\tau}\sum_p \left(\delta_{kj} - \frac{\beta J_{kj}}{3}\right) G_{10,10}(\mathbf{R}_p, t|\mathbf{R}_j) + C_{10,10}(\mathbf{R}_k|\mathbf{R}_j)$$

which for a translationally invariant system yields the following solution

$$G_{10,10}(R_k, t|R_j) = \frac{1}{N}\sum_q C_{10,10}(q)\exp - \frac{t}{\tau}\left(1 - \frac{\beta J(q)}{3}\right)\exp i\mathbf{q}\cdot(\mathbf{R}_k - \mathbf{R}_j) \tag{4.5}$$

where $C_{10,10}(q)$ and $J(q)$ denote the spatial Fourier transforms of $C_{10,10}(R_k|R_j)$ and

J_{kj} respectively. These are written as

$$C_{10,10}(q) = \sum_k \exp[iq\cdot(R_k - R_j)]C_{10,10}(R_k|R_j)$$

$$J(q) = \sum_j J_{kj}\exp[iq\cdot(R_k - R_j)]$$

eq. (4.5) is the analogue of Edwards–Anderson mean field result for vector spins. Note that this result has been derived here not by any ad hoc decoupling procedure, but by a systematic projection procedure. This procedure makes the nature of the result clear in the following way. It is seen that EA results amount to keeping the lowest band of relaxation eigenvalues, which clearly is good at long enough times. The correction to these results arise by considering the higher band of eigenvalues. These bands are certainly important at shorter times, and moreover since the couplings cause interband transitions, they also renormalize the lowest band and thus affecting the long time behaviour. The next section describes the procedure for improving the mean field result.

5. Beyond mean field theory

In this section we discuss a systematic procedure to incorporate the effects of the coupling of higher bands on the lowest lying band, which is the one responsible for long-time behaviour.

Now, we define a set of Green's functions,

$$\begin{aligned} G_{10,10}(R_k,t|R_j) &= \langle Y_1^0(\theta_k(t),\phi_k(t))\,Y_1^0(\theta_j,\phi_j)\rangle, \quad t>0 \\ &= 0, \quad t<0 \end{aligned} \tag{5.1}$$

$$\begin{aligned} G_{l_1m_1,l_2m_2,\ldots}(R_1,R_2,\ldots,t|R_j) &= \left\langle \prod_k Y_{l_k}^{m_k}(\theta_k(t),\phi_k(t))\,Y_1^0(\theta_j,\phi_j)\right\rangle, \quad t>0 \\ &= 0, \quad t<0 \end{aligned} \tag{5.2}$$

We also define their spatial Fourier transform and time-Laplace transform by the following relations

$$G_{10,10}(\mathbf{q},t) = \sum_k \exp[i\mathbf{q}\cdot(\mathbf{R}_j - \mathbf{R}_k)]G_{10,10}(\mathbf{R}_k,t|\mathbf{R}_j) \tag{5.3}$$

$$\tilde{G}_{10,10}(\mathbf{q},z) = \int_0^\infty \exp[izt]G_{10,10}(\mathbf{q},t)\mathrm{d}t, \quad \mathrm{Im}\, z>0 \tag{5.4}$$

$$\begin{aligned} \tilde{G}_{l_1m_1,l_2m_2,\ldots}(\mathbf{q}_1,\mathbf{q}_2,\ldots,z) = \sum_{\mathbf{R}_1,\mathbf{R}_2,\ldots}\int_0^\infty \exp[izt]\mathrm{d}t\exp\left[i\sum_k \mathbf{q}_k\cdot(\mathbf{R}_j - \mathbf{R}_k)\right] \\ \times G_{l_1m_1,l_2m_2,\ldots}(\mathbf{R}_1,\mathbf{R}_2,\ldots,t|\mathbf{R}_j) \end{aligned} \tag{5.5}$$

A point about notation: if any of the m's is negative, we denote it by a bar over it, i.e. $G_{1,-1}$ is written $G_{1\bar{1}}$. The equations of motion of the G's also involve the equilibrium correlation functions, which we denote as

$$C_{l_1m_1,l_2m_2,..}(\mathbf{R}_1,\mathbf{R}_2,..)=\left\langle \prod_k Y_{l_k}^{m_k}(\theta_k,\phi_k)\right\rangle_0. \tag{5.6}$$

The Fourier transform of the correlation functions are defined in the same way as the Green's functions.

We now write the equation of motion of $G_{10,10}(\mathbf{R}_k,t|\mathbf{R}_j)$ for $t>0$,

$$\begin{aligned}\frac{\partial}{\partial t}G_{10,10}(\mathbf{R}_k,t|\mathbf{R}_j) &= -\langle[W^+ Y_1^0(\theta_k(t),\phi_k(t))]\, Y_1^0(\theta_j,\phi_j)\rangle \\ &= -\frac{1}{\tau}G_{10,10}(\mathbf{R}_k,t|\mathbf{R}_j)+\frac{\beta}{\tau}\sum_p J_{kp}\left[\frac{1}{3}G_{10,10}(\mathbf{R}_p,t|\mathbf{R}_j)\right.\\ &\quad+\frac{1}{2\sqrt{15}}\{G_{2\bar{1},11}(\mathbf{R}_k,\mathbf{R}_p,t|\mathbf{R}_j)+G_{21,1\bar{1}}(\mathbf{R}_k,\mathbf{R}_p,t|\mathbf{R}_j)\}\\ &\quad\left.-\frac{1}{3\sqrt{5}}G_{20,10}(\mathbf{R}_k,\mathbf{R}_p,t|\mathbf{R}_j)\right].\end{aligned} \tag{5.7}$$

Taking its Fourier and Laplace transform yields

$$\begin{aligned}\tilde{G}_{10,10}(\mathbf{q},z)=\frac{1}{(r_q-iz)}&\left[C_{10,10}(\mathbf{q})+\frac{\beta}{N\tau}\sum_{q_1}J(q_1)\left[\frac{1}{2\sqrt{15}}\{\tilde{G}_{2\bar{1},11}(\mathbf{q}-\mathbf{q}_1,\mathbf{q}_1;z)\right.\right.\\ &\left.\left.+\tilde{G}_{21,1\bar{1}}(\mathbf{q}-\mathbf{q}_1,\mathbf{q}_1;z)\}-\frac{1}{3\sqrt{5}}\tilde{G}_{20,10}(\mathbf{q}-\mathbf{q}_1,\mathbf{q}_1;z)\right]\right]\end{aligned} \tag{5.8}$$

where

$$r_q=\frac{1}{\tau}\left(1-\frac{\beta J(q)}{3}\right). \tag{5.9}$$

To go beyond mean-field theory, we now write the equations of motion for $G_{21,1\bar{1}}$, $G_{2\bar{1},11}$ and $G_{20,10}$ which follow by considering the action of W^+ on $|\ldots l_k=2, m_k=1,\ldots l_p=1, m_p=-1,\ldots\rangle$, $|\ldots l_k=2, m_k=-1,\ldots l_p=1, m_p=1,\ldots\rangle$ and $|\ldots l_k=2, m_k=0,\ldots l_p=1, m_p=0,\ldots\rangle$.

The action of W^+ on $|\ldots l_k=2, m_k=1,\ldots l_p=1, m_p=-1,\ldots\rangle$ state is given by the following equation

$$\begin{aligned}W^+\left|\ldots l_k=2, m_k=1,\ldots l_p=1, m_p=-1,\ldots\right\rangle &= \frac{4}{\tau}\left|\ldots l_k=2,\right.\\ &\qquad m_k=1,\ldots l_p=1, m_p=-1,\ldots\rangle\\ +\frac{\beta}{4\tau}\left[\sum_{j\neq k}J_{pj}\left\{\frac{2}{3\sqrt{5}}\right|\right.&\ldots l_k=2, m_k=1,\ldots l_j=1, m_j=-1,\ldots l_p=2,\\ &\qquad m_p=0..\rangle\end{aligned}$$

$$-\frac{2}{3}\Big|\ldots l_k=2,m_k=1,\ldots l_j=1,m_j=-1\ldots\rangle-\frac{2\sqrt{2}}{\sqrt{15}}\Big|\ldots l_k=2,$$

$$m_k=1,\ldots l_j=1,m_j=1,\ldots l_p=2,m_p=-2\ldots\rangle$$

$$+\frac{2}{\sqrt{15}}\Big|\ldots l_k=2,m_k=1,\ldots l_j=1,m_j=0,\ldots l_p=2,m_p=-1,\ldots\rangle$$

$$-\frac{4}{3\sqrt{5}}\Big|\ldots l_k=2,m_k=1,\ldots l_j=1,m=-1,\ldots l_p=2,m_p=0,\ldots\rangle$$

$$-\frac{2}{3\sqrt{5}}\Big|\ldots l_k=2,m_k=1,\ldots l_j=1,m_j=-1,\ldots\rangle\Big\}+J_{kp}\Big\{\frac{-4}{\sqrt{35}}\Big|\ldots l_k=3,$$

$$m_k=2,\ldots l_p=2,m_p=-2,\ldots\rangle$$

$$+\frac{2}{5\sqrt{7}}\Big|\ldots l_k=3,m_k=0,\ldots l_p=2,m_p=0,\ldots\rangle-\frac{2}{\sqrt{35}}\Big|\ldots l_k=3,m_k=0,\ldots\rangle$$

$$-\frac{2}{5\sqrt{3}}\Big|\ldots l_k=1,m_k=0,\ldots l_p=2,m_p=0,\ldots\rangle+\frac{2}{\sqrt{15}}\Big|\ldots l_k=1,m_k=0,\ldots\rangle$$

$$+\frac{6}{5\sqrt{7}}\Big|\ldots l_k=3,m_k=0,\ldots l_p=2,m_p=0,\ldots\rangle+\frac{6}{\sqrt{35}}\Big|\ldots l_k=3,m_k=0,\ldots\rangle$$

$$-\frac{4}{5\sqrt{7}}\Big|\ldots l_k=1,m_k=0,\ldots l_p=2,m_p=0,\ldots\rangle+\frac{4}{\sqrt{35}}\Big|\ldots l_k=1,m_k=0,\ldots\rangle$$

$$+\frac{6}{5}\sqrt{\frac{2}{7}}\Big|\ldots l_k=3,m_k=1,\ldots l_p=2,m_p=-1,\ldots\rangle-\frac{2}{5}\Big|\ldots l_k=1,m_k=1,\ldots$$

$$\ldots l_p=2,m_p=-1,\ldots\rangle$$

$$+\frac{2}{5}\sqrt{\frac{2}{7}}\Big|\ldots l_k=3,m_k=1,\ldots l_p=2,m_p=-1,\ldots\rangle-\frac{4}{5}\Big|\ldots l_k=1,m_k=1,\ldots$$

$$\ldots l_p=2,m_p=-1,\ldots\rangle$$

$$-\frac{4}{\sqrt{35}}\Big|\ldots l_k=3,m_k=2,\ldots l_p=2,m_p=-2,\ldots\rangle\Big\}\Big]. \tag{5.10}$$

From this equation, it is clear that the equation of motion for $G_{21,1\bar{1}}$ will introduce a number of higher order Green's functions, and to make things mathematically manageable, we must introduce a procedure to truncate these equations. The different terms occurring here correspond to different bands whose mean position is approximately given by the diagonal expectation value K of that term. The mean field approximation was obtained by retaining terms up to $K=2$. The next order approximation is obtained by keeping terms with $K\leqslant 8$. Retaining such terms reduces (5.10) to

$$W^+|\ldots l_k=2,m_k=1,\ldots,l_p=1,m_p=-1,\ldots\rangle=\frac{4}{\tau}\Big|\ldots l_k=2,m_k=1,\ldots,l_p=1,$$

$$m_p = -1,\ldots\rangle + \frac{\beta}{4\tau}\left[-\frac{2}{3}\left(1+\frac{1}{\sqrt{5}}\right)\sum_{j\neq k} J_{pj}|\ldots l_k = 2, m_k = 1,\ldots\right.$$

$$\ldots l_j = 1, m_j = -1,\ldots\rangle + J_{kp}\left\{ -\frac{2}{5}\left(\frac{2}{\sqrt{7}}+\frac{1}{\sqrt{3}}\right)\right|\ldots l_k = 1, m_k = 0,\ldots$$

$$\ldots l_p = 2, m_p = 0,\ldots\rangle + \frac{2}{\sqrt{5}}\left(\frac{1}{3}+\frac{2}{\sqrt{7}}\right)\Big|\ldots l_k = 1, m_k = 0,\ldots\rangle$$

$$\left.\left. -\frac{6}{5}\right|\ldots l_k = 1, m_k = 1,\ldots l_p = 2, m_p = -1,\ldots\rangle\right\}\Bigg]. \tag{5.11}$$

Using this truncated equation, one obtains the following equation for $\tilde{G}_{21,1\bar{1}}(\mathbf{q}_1, \mathbf{q}_2; z)$,

$$\tilde{G}_{21,1\bar{1}}(\mathbf{q}_1,\mathbf{q}_2;z) = \frac{1}{-iz + r(q_1) + \frac{3}{\tau} + \frac{\beta}{6\tau}\left(1-\frac{1}{\sqrt{5}}\right)J(q_1)}\Bigg[C_{21,1\bar{1}}(\mathbf{q}_1,\mathbf{q}_2)$$

$$-\frac{\beta}{2}J(q_2)\left\{\left(\frac{2}{\sqrt{7}}+\frac{1}{\sqrt{3}}\right)\left(-\frac{1}{5}\tilde{G}_{20,10}(\mathbf{q}_1,\mathbf{q}_2;z) + \frac{1}{\sqrt{5}}\tilde{G}_{10,10}(\mathbf{q}_1;z)\right)\right.$$

$$\left. -\frac{3}{5}\tilde{G}_{2\bar{1},11}(\mathbf{q}_1,\mathbf{q}_2;z)\right\}\Bigg]. \tag{5.12}$$

Similarly, one can obtain the following equations for $\tilde{G}_{2\bar{1},11}(\mathbf{q}_1, \mathbf{q}_2; z)$ and $\tilde{G}_{20,10}(\mathbf{q}_1, \mathbf{q}_2; z)$

$$\tilde{G}_{2\bar{1},11}(\mathbf{q}_1,\mathbf{q}_2;z) = \frac{1}{-iz + r_{(q_1)} + \frac{3}{\tau} + \frac{\beta}{6\tau}\left(1-\frac{1}{\sqrt{5}}\right)J(q_1)}\Bigg[C_{2\bar{1},11}(\mathbf{q}_1,\mathbf{q}_2)$$

$$-\frac{\beta}{2\tau}J(q_2)\left\{-\frac{1}{5}\left(\frac{1}{\sqrt{3}}+\frac{2}{\sqrt{7}}\right)\tilde{G}_{20,10}(\mathbf{q}_1,\mathbf{q}_2;z) - \frac{3}{5}G_{21,1\bar{1}}(\mathbf{q}_1,\mathbf{q}_2;z)\right.$$

$$+\frac{1}{\sqrt{5}}\left(\frac{1}{\sqrt{3}}+\frac{2}{\sqrt{7}}\right)\tilde{G}_{10,10}(\mathbf{q}_1;z)\Bigg] \tag{5.13}$$

and

$$\tilde{G}_{20,10}(\mathbf{q}_1,\mathbf{q}_2;z) = \frac{1}{-iz + r_{(q_1)} + r_{(q_2)} + \frac{2}{\tau} - \frac{7}{15}\beta J(q_2)}\Bigg[C_{20,10}(\mathbf{q}_1,\mathbf{q}_2)$$

$$-\frac{\beta}{2\tau}J(q_2)\left\{-\frac{2}{5}\tilde{G}_{10,10}(\mathbf{q}_1;z) - \frac{3}{5\sqrt{7}}\tilde{G}_{21,1\bar{1}}(\mathbf{q}_1,\mathbf{q}_2;z) + \tilde{G}_{2\bar{1},11}(\mathbf{q}_1,\mathbf{q}_2;z)\right\}\Bigg] \tag{5.14}$$

Since this is a closed set of equations, one can obtain an expression for $G_{10,10}(\mathbf{q};z)$ to be

$$\tilde{G}_{10,10}(\mathbf{q};z) = \frac{A(\mathbf{q},z)}{\left[r_q - iz - \left\{\frac{\beta}{2N}\sum_{q_1} J_{(q_1)}\frac{1}{\sqrt{15}}\left(a_3 b_3 + \frac{\Gamma_2}{5}a_1 b_1 d_2 a_2 - \frac{\Gamma_2 a_1}{5} + \frac{3}{5}\Gamma_2 b_3 a_3\right) - \frac{2d_2 a_2}{3\sqrt{5}}\right\}\right]} \tag{5.15}$$

with

$$A(\mathbf{q};z) = C_{10,10}(\mathbf{q}) + \frac{1}{\sqrt{15}}\left(b_3 d_3 + \frac{\Gamma_2}{5}a_1 b_1 d_2 + \frac{3}{5}\Gamma_2 b_3 d_3 - \frac{2d_2}{\sqrt{3}}\right) \times C_{20,10}(\mathbf{q}_1,\mathbf{q}_2)$$
$$+ \frac{1}{\sqrt{15}}\left(b_3 + d_3 b_3 b_2 + \frac{\Gamma_2}{5}a_1 b_1 d_2 b_2 + \frac{3}{5}\Gamma_2 b_3 + \frac{3}{5}\Gamma_2 d_3 b_2 b_3 - \frac{2d_2 b_2}{\sqrt{3}}\right) \times C_{21,1\bar{1}}(\mathbf{q}_1,\mathbf{q}_2)$$
$$+ \frac{1}{\sqrt{15}}\left(d_3 b_3 b_2 + b_1 + \frac{2}{5}a_1 b_1 d_2 b_2 + \frac{3}{5}\Gamma_2 d_3 b_2 b_3 - \frac{2d_2 b_2}{\sqrt{3}}\right) \times C_{2\bar{1},11}(\mathbf{q}_1,\mathbf{q}_2)$$

$$a_1 = \frac{2}{\sqrt{7}} + \frac{1}{\sqrt{3}}, \quad \Gamma_2 = \frac{\beta J(q_2)}{\tau}$$

$$b_1(\mathbf{q};z) = \frac{1}{-iz + r_{(q_1)} + \frac{3}{\tau} + \frac{\beta}{6\tau}\left(1 - \frac{1}{\sqrt{5}}\right)J(\mathbf{q}_1)}$$

$$d(\mathbf{q};z) = \frac{1}{-iz + r_{(q_1)} + r_{(q_2)} + \frac{2}{\tau} - \frac{7}{15}\frac{\beta J(q_2)}{\tau}}$$

$$d_2(\mathbf{q};z) = \frac{d(\mathbf{q};z)}{1 - \frac{6a_1 b_1 d\Gamma_2^2}{25\sqrt{7}\left(1 - \frac{3}{5}b_1\Gamma_2\right)}}$$

$$b_2(\mathbf{q};z) = \frac{3\Gamma_2 b_1}{5\sqrt{7}\left(1 - \frac{3}{5}b_1\Gamma_2\right)}$$

$$a_2(\mathbf{q};z) = \frac{2}{5}\Gamma_2\left\{1 - \frac{3a_1 b_1\Gamma_2}{\sqrt{35}\left(1 - \frac{3}{5}b_1\Gamma_2\right)}\right\}$$

$$d_3(\mathbf{q};z) = \frac{\Gamma_2 a_1}{5}\left(1 - \frac{3}{5}\Gamma_2 b_1\right) d_2$$

$$b_3(\mathbf{q};z) = \frac{b_1}{1 + \left(\frac{3}{5} b_1 \Gamma_2\right)^2}$$

$$a_3(\mathbf{q};z) = \frac{\Gamma_2 a_1}{\sqrt{5}}\left(1 - \frac{3}{5} b_1 \Gamma_2\right)\left(\frac{d_2 a_2}{\sqrt{5}} - 1\right).$$

These equations have interesting implications which can be fully understood only by their numerical evaluation. They are in the spirit of usual field theory results as corrections occurs by addition of self energy terms to the denominator and to the spectral weight. However, we do not present such results here as in a later paper we shall study these equations numerically and extend the formalism to the critical dynamics.

6. Concluding remarks

To summarize, we have introduced in this paper a new method to deal with the statistical dynamics of vector spin systems. The method builds a series of approximations by systematically restricting the Hilbert space for the Fokker–Planck operator. Since the method is different from the usual field-theoretic methods, we expect that its numerical study will lead to elucidation of the notion of universality in dynamic critical behaviour. The method also reveals interesting qualitative features about the hierarchy in the relaxation spectrum of vector spin systems. This hierarchical feature which we believe has been pointed out for the first time, should provide new methods to implement renormalization group ideas in the studies of the dynamics of vector spin systems.

Appendix 1

$$B_{\pm}(l_k, m_k) = [(l_k \mp m_k)(l_k \pm m_k + 1)]^{1/2}$$

$$A_{0+}(l,m) = \left[\frac{(l+m+1)(l-m+1)}{(2l+1)(2l+3)}\right]^{1/2}$$

$$A_{0-}(l,m) = \left[\frac{(l+m)(l-m)}{(2l+1)(2l+3)}\right]^{1/2}$$

$$A_{++}(l,m) = -\left[\frac{(l+m+1)(l+m+2)}{(2l+1)(2l+3)}\right]^{1/2}$$

$$A_{+-}(l,m) = \left[\frac{(l-m)(l-m-1)}{(2l+1)(2l-1)}\right]^{1/2}$$

$$A_{-+}(l,m) = \left[\frac{(l-m+1)(l-m+2)}{(2l+1)(2l+3)}\right]^{1/2}$$

$$A_{--}(l,m) = \left[\frac{(l+m)(l+m-1)}{(2l+1)(2l-1)}\right]^{1/2}$$

$$
\begin{aligned}
g|\ldots l_k, m_k+\mu_k,\ldots,l_p+\sigma_p,m_p+\mu_p\ldots\rangle
&= m_k\{A_{0+}(l_p,m_p)|\ldots l_k,m_k,\ldots,l_p+1,m_p\ldots\rangle \\
&\quad + A_{0-}(l_p,m_p)|\ldots l_k,m_k,\ldots,l_p-1,m_p\ldots\rangle\} \\
&+ \frac{1}{2}B_+(l_k,m_k)\{A_{-+}(l_p,m_p)|\ldots l_k,m_k+1,\ldots,l_p+1,m_p-1,\ldots\rangle \\
&\quad + A_{--}(l,m_p)|\ldots l_k,m_k+1,\ldots,l_p-1,m_p-1,\ldots\rangle\} \\
&+ \frac{1}{2}B_-(l_k,m_k)\{A_{++}(l_p,m_p)|\ldots l_k,m_k-1,\ldots,l_p+1,m_p+1,\ldots\rangle \\
&\quad + A_{+-}(l_p,m_p)]\ldots l_k,m_k-1,\ldots,l_p-1,m_p+1,\ldots\rangle\}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{\sigma_k,\sigma_p,\mu_k,\mu_p} f|\ldots l_k+\sigma_k,m_k+\mu_k,\ldots,l_p+\sigma_p,m_p+\mu_p\ldots\rangle
&= m_k\Big\{A_{++}(l_k,m_k)\sum_{\sigma_p=\pm}A_{-\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p-1,\ldots, \\
&\qquad\qquad l_k+1,m_k+1,\ldots\rangle \\
&+ A_{+-}(l_k,m_k)\sum_{\sigma_p=\pm}A_{-\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p-1,\ldots,l_k-1,m_k+1,\ldots\rangle \\
&- A_{-+}(l_k,m_k)\sum_{\sigma_p=\pm}A_{+\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p+1,\ldots,l_k+1,m_k-1,\ldots\rangle \\
&- A_{--}(l_k,m_k)\sum_{\sigma_p=\pm}A_{+\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p+1,\ldots,l_k-1,m_k-1,\ldots\rangle\Big\} \\
&+ B_-(l_k,m_k)\Big\{A_{0+}(l_k,m_k-1)\sum_{\sigma_p=\pm}A_{+\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p+1,\ldots \\
&\qquad\qquad \ldots,l_k+1,m_k-1,\ldots\rangle \\
&+ A_{0-}(l_k,m_k-1)\sum_{\sigma_p=\pm}A_{+\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p+1,\ldots,m_k-1\ldots\rangle \\
&- A_{++}(l_k,m_k-1)\sum_{\sigma_p=\pm}A_{0\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p,\ldots,l_k+1,m_k,\ldots\rangle \\
&- A_{+-}(l_k,m_k-1)\sum_{\sigma_p=\pm}A_{0\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p,\ldots,l_k-1,m_k,\ldots\rangle\Big\} \\
&+ B_+(l_k,m_k)\Big\{A_{-+}(l_k,m_k+1)\sum_{\sigma_p=\pm}A_{0\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p,\ldots \\
&\qquad\qquad \ldots l_k+1,m_k,\ldots\rangle \\
&+ A_{--}(l_k,m_k+1)\sum_{\sigma_p=\pm}A_{0\sigma_p}(l_p,m_p)|\ldots l_p+\sigma_p,m_p,\ldots l_k-1,m_k,\ldots\rangle
\end{aligned}
$$

$$- A_{0+}(l_k, m_k + 1) \sum_{\sigma_p = \pm} A_{-\sigma_p}(l_p, m_p)|\ldots l_p + \sigma_p m_p, \ldots l_k + 1, m_k + 1, \ldots\rangle$$

$$\left. - A_{0-}(l_k, m_k + 1) \sum_{\sigma_p = \pm} A_{-\sigma_p}(l_p, m_p)|\ldots l_p + \sigma_p, m_p, \ldots, l_k - 1, m_k - 1, \ldots\rangle \right\}$$

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