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Perturbative and nonperturbative quark masses in structure functions

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Abstract. Flavour symmetry breaking is introduced in the approximate formula for quark mass reported earlier. We use SLAC-MIT as well as NMC data to estimate the up, down and strange quark masses from structure functions. Using the approximate Q^2 evolutions of Altarelli–Parisi equations, we also estimate its perturbative and nonperturbative components.

Keywords. Quark mass; structure function; quantum chromodynamics.

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1. Introduction

Some time back [1], we derived an approximate formula for the quark mass which occurred in the ξ-formalism [2], and studied its phenomenological application to the SLAC-MIT data [3]. To that end, we used the approximate QCD model of $F_2^{ep}(x, Q^2)$ as given by Buras and Gaemers [4] for numerical analysis. In that work, we studied the sensitivity of our results with QCD scale parameter Λ, gluon distribution, intrinsic charm component of the proton wave function [5, 6]; and higher twist (HT) effects [7–12] in the structure function.

One of the limitations of the earlier work was that we could only measure the average quark mass in the explored Q^2 range under study, but not the quark mass of individual flavour. Nor could we study the Q^2 running of the quark mass. The reasons are understandable: we did not have suitable method of introducing flavour symmetry breaking in the model, neither did we explore the asymptotic behaviour of the quark mass formula so that $\mathrm{Lt}_{Q^2 \to \infty} m(Q^2)$ vanishes.

In recent years we reported [13, 14] approximate solutions of Altarelli–Parisi equations [15] valid for intermediate x and it was tested for considerable high $Q^2 : Q^2 < 90\,\mathrm{GeV}^2$ [16].

It is therefore meaningful to study the mass problem as well with these approximate solutions rather than with the moment methods of Buras Gaemers [4]. The advantage of the approximate method over the moment method is that for each value of x, we require input information only for larger values of x, and not over the full range from 0 to 1. This is important in practice, because structure functions are poorly known at small values of x.

While addressing to the mass problem one encounters three kinds of quark mass: current, dynamical and constituent quark mass [17–22]. In QCD, current quark mass is perturbative while the dynamical quark mass is non-perturbative [19]. In the present method, since quark mass is expressed in terms of structure function $F_2(x,Q^2)$, the non-perturbative input for $F_2(x,Q_0^2)$ generates dynamical quark mass while the rest, the perturbative piece. The present paper will deal with this aspect of the problem as well.

Section 2 discusses the theory while § 3 deals with results and discussion.

2. Formalism

2.1 *Formula for quark mass*

In an earlier communication [1] we derived an approximate formula for quark mass from ξ formalism [2]. In this formalism the structure function with target and quark mass correction has the form

$$\bar{F}_2(x,Q^2)=\frac{x^2V^3F_2(\xi,Q^2)}{\xi^2}+\frac{6M^2x^3V^4}{Q^2}\int_\xi^1\frac{\mathrm{d}x'F_2(x',Q^2)}{x'^2}+\frac{12M^4x^4V^5}{Q^4}\int_\xi^1\mathrm{d}x'\int_{x'}^1\frac{\mathrm{d}x''F_2(x'',Q^2)}{x''^2} \tag{2.1}$$

where

$$\xi=\frac{x[1+(1+4mq^2/Q^2)^{1/2}]}{1+(1+(4M^2x^2/Q^2))^{1/2}} \tag{2.2}$$

and

$$V=\left(1+\frac{4M^2x^2}{Q^2}\right)^{-1/2}. \tag{2.3}$$

Here $m_q=m_q(x,Q^2)$ is the momentum dependent running quark mass, M is the target mass while $\bar{F}_2$ is the experimental structure function incorporating the target and quark mass effects.

Taking form $O\left(\frac{1}{Q^2}\right)$ only, (2.1) is reduced to [1]

$$\bar{F}_2(x,Q^2)=F_2(x,Q^2)\left[1+\frac{1}{Q^2}\left\{-2(m_q^2+2M^2x^2)-(m_q^2-x^2M^2)\frac{\tilde{f}(x,Q^2)}{F_2(x,Q^2)}+6M^2x^3\frac{F_2'(x,Q^2)}{F_2(x,Q^2)}\right\}\right] \tag{2.4}$$

which yields

$$m_q^2(x,Q^2)=\left[\frac{Q^2}{2+[\tilde{f}(x,Q^2)/F_2(x,Q^2)]}\right]\left(\frac{F_2(x,Q^2)-\bar{F}_2(x,Q^2)}{F_2(x,Q^2)}\right)-\frac{[M^2x^2(4-[\tilde{f}(x,Q^2)/F_2(x,Q^2)])-6M^2x^3(F_2'(x,Q^2)/F_2(x,Q^2))]}{2+f(x,Q^2)/F_2(x,Q^2))}. \tag{2.5}$$

In (2·4) or (2·5), $\tilde{f}(x,Q^2)$ and $F_2'(x,Q^2)$ can be explicitly obtained from the structure function $F_2(x, Q^2)$ since

$$F_2(\xi, Q^2) \simeq F_2(x, Q^2) - (m_q^2 - x^2 M^2)\frac{\tilde{f}(x, Q^2)}{Q^2} \tag{2.6}$$

and

$$F_2'(x, Q^2) = \int_x^1 \mathrm{d}x' \frac{F_2(x', Q^2)}{x'^2}, \tag{2.7}$$

where

$$\xi = x + \frac{x}{Q^2}(m_q^2 - x^2 M^2). \tag{2.8}$$

We note that in the limit $M \to 0$ and $m_q \to 0$, $\bar{F}_2(x, Q^2)$ and $F_2(x, Q^2)$ are identical.

We take the absolute value of the RHS of (2.5) so that $m_q^2 > 0$ for $0 < x < 1$ and $0 < Q^2 < \infty$.

If we include the higher twist (HT) term [8] with the following form

$$F_2^{\mathrm{QCD}}(x, Q^2) = F_2^{\mathrm{LT}}(x, Q^2)\left(1 + \frac{\mu_4^2 x^\alpha}{(1 - x)Q^2}\right) \tag{2.9}$$

with $\alpha = 3{\cdot}1 \pm 0{\cdot}05$ and $\mu_4^2 = 1{\cdot}7 \pm 0{\cdot}05\,\mathrm{GeV}^2$, then m_q^2 of (2.5) is modified to

$$m_q^2 \to m_q^2 + \frac{\mu_4^2 x^\alpha}{(1 - x)} \bigg/ \left(2 + \frac{\tilde{f}(x, Q^2)}{F_2(x, Q^2)}\right). \tag{2.10}$$

In (2.9) F_2^{LT} denotes the leading twist contribution.

More recently, HT effects have been reported in tabular form [11, 12] instead of the explicit model dependent form (2.9). The parametrization then has the general form

$$F_2^{\mathrm{QCD}}(x, Q^2) = F_2^{\mathrm{LT}}(x, Q^2)\left(1 + \frac{C_i(x)}{Q^2}\right) \tag{2.11}$$

where the coefficients $C_i(x)$ is x-dependent numericals. For small $x(x < 0{\cdot}40)$ they are small ($\sim 0{\cdot}05$) while for large $x(x > 0{\cdot}40)$ they increase as expected in models like (2·9). Equation (2.10) is then modified to

$$m_q^2 \to m_q^2 + C_i(x) \bigg/ \left(2 + \frac{\tilde{f}(x, Q^2)}{F_2(x, Q^2)}\right) \tag{2.12}$$

where $C_i(x)$ for hydrogen target of ref. [12] are given in table 2.

In our analysis, we will test both (2.10) and (2.12). Let us now derive the asymptotic condition on $\bar{F}_2(x, Q^2)/F_2(x, Q^2)$ as $Q^2 \to \infty$. As $m_q^2 \to 0$ asymptotically, (2.5) predicts

$$\mathop{\mathrm{Lt}}_{Q^2 \to \infty} \frac{\bar{F}_2(x, Q^2)}{F_2(x, Q^2)} = 1 - \frac{1}{Q^2}\left[M^2 x^2\left(4 - \frac{\tilde{f}(x, Q^2)}{F_2(x, Q^2)}\right) - 6M^2 x^3 \frac{F_2'(x, Q^2)}{F_2(x, Q^2)}\right]. \tag{2.13}$$

Table 1. Dynamical quark mass.

Quark flavour	Data	Inputs: Gluck *et al* ($Q_0^2 = \text{GeV}^2$)	ELHQ $Q_0^2 = \text{GeV}^2$	EHQ $Q_0^2 = \text{GeV}^2$
m_u	SLAC-MIT	0.126 ± 0.01 GeV	0.366 ± 0.02 GeV	0.410 ± 0.01 GeV
	NMC	0.410 ± 0.12 GeV	0.467 ± 0.13 GeV	0.456 ± 0.21 GeV
m_d	SLAC-MIT	0.434 ± 0.03 GeV	0.373 ± 0.01 GeV	0.381 ± 0.11 GeV
	NMC	0.544 ± 0.11 GeV	0.436 ± 0.12 GeV	0.564 ± 0.15 GeV

2.2 Q^2 *evolution of structure functions*

As mentioned in the introduction we will use the approximate form of structure function [13]

$$F_2(x,t) = \frac{5}{18} F_2^{\rm S}(x,t_0)\left(\frac{t}{t_0}\right)^{H^{\rm S}(x)} + \frac{3}{18} F_2^{\rm NS}(x,t_0)\left(\frac{t}{t_0}\right)^{H^{\rm NS}(x)} \tag{2.14}$$

with

$$H^{\rm S}(x) = \frac{4}{25}[3 + 4\ln(1-x) + 2I^{\rm S}(x) + 6g(x)] \tag{2.15}$$

$$H^{\rm NS}(x) = \frac{4}{25}[3 + 4\ln(1-x) + 2I^{\rm NS}(x)] \tag{2.16}$$

where $I^{\rm S}(x)$, $I^{\rm NS}(x)$ and $g(x)$ are the integrals [13] defined as

$$I^{\rm S}(x) = \int_x^1 \frac{{\rm d}w}{(1-w)}\left\{(1+w^2)\frac{F_2^{\rm S}(x/w,t_0)}{F_2^{\rm S}(x,t_0)} - 2\right\} \tag{2.17}$$

$$I^{\rm NS}(x) = \int_x^1 \frac{{\rm d}w}{(1-w)}\left\{(1+w^2)\frac{F_2^{\rm NS}(x/w,t_0)}{F_2^{\rm NS}(x,t_0)} - 2\right\} \tag{2.18}$$

$$g(x) = \int_x^1 \left\{{\rm d}w(w^2 + (1-w)^2)\frac{G(x/w,t_0)}{F_2^{\rm S}(x,t_0)}\right\} \tag{2.19}$$

We evaluate the integrals using Taylor approximation method reported earlier [23, 14].

2.3 *Flavour symmetry breaking*

In order to evaluate quark mass of definite flavour in (2.5) or (2.10) we have to identify m_q with the definite flavour. Similarly, the contribution to $F_2(x,Q^2)$ of an individual flavour, $F_2(x,Q^2)$, $F_2'(x,Q^2)$ and $\tilde{f}(x,Q^2)$ needs to be identified. We therefore write the standard form of structure function neglecting the heavy flavour contents as

$$F^{ep}(x,Q^2) = F^u(x,Q^2) + F^d(x,Q^2) + F^{\rm S}(x,Q^2) \tag{2.20}$$

where

$$F^u(x,Q^2) = \tfrac{4}{9}[u_v(x,Q^2) + 2u_s(x,Q^2)], \tag{2.21}$$

$$F^d(x,Q^2) = \tfrac{1}{9}[d_v(x,Q^2) + 2d_s(x,Q^2)], \tag{2.22}$$

$$F^S(x,Q^2) = \tfrac{2}{9}S(x,Q^2). \tag{2.23}$$

Using (2.20)–(2.23) one obtains $F_2'(x,Q^2)$ and $\tilde{f}(x,Q^2)$ for individual flavour.

In order to obtain $\bar{F}_2(x,Q^2)$ for individual flavours, we use the flavour contents of F_2 as defined in the quark model.

To proceed further we need information on input structure function. There are several inputs [24–26] in the literature. Each of them has the generic form

$$x_{q_v}(x,Q_0^2) = \sum_{i=1}^{n} a_i^q x^{p_i^q}(1-x^{\alpha_i^q})^{l_i^q}, \quad (q = u,d,s) \tag{2.24}$$

Numerical values of a_i^q, p_i^q, α_i^q and l_i^q are given in Appendix A.

2.4 *Determination of* $\tilde{f}(x,Q^2)$

Let us illustrate it with explicit evaluation of $\tilde{f}_{u_v}(x,Q^2)$:
From (2.6)

$$\tilde{f}_{u_v}(x,Q^2) = \frac{Q^2}{(mq^2 - x^2M^2)}[U_v(x,Q^2) - U_v(\xi,Q^2)] \tag{2.25}$$

where ξ is defined in (2.8).

Determination of $\tilde{f}_{u_v}(x,Q^2)$, therefore needs $u_v(x,Q^2)$ as well as $u_v(\xi,Q^2)$.

To that end we use the evolution

$$u_v(x,Q^2) = u_v(x,Q_0^2)\left(\frac{t}{t_0}\right)^{H^{NS}}(x) \tag{2.26}$$

where $H^{NS}(x)$ is given by (2.16). $I^{NS}(x)$ occurred in (2.16) and defined in (2.18) should correspond to single quark species, and has approximate analytical form [16]

$$I_{u_v}^{NS}(x) = 2x(1-x)\frac{u_v'(x)}{u_v(x)} - 2(1-x) - x(1-x)^2\frac{u_v'(x)}{u_v(x)} + \frac{(1-x)^2}{2} + \frac{x(1-x)^3}{3}\frac{u_v'(x)}{u_v(x)} \tag{2.27}$$

By using (2.27) $I_{u_v}^{NS}(x)$ can then be explicitly calculated once $u_v(x)$ is known from (2.24).

By making the replacement $x \to \xi$ as defined in eq. (2.8) $I^{NS}(\xi)$ and $\ln(1-\xi)$ occurred in $H_{u_v}^{NS}(\xi)$ then take the forms

$$I^{NS}(\xi) = I^{NS}(x) + \frac{m_q^2 - x^2M^2}{Q^2}R^{NS}(x) \tag{2.28}$$

$$\ln(1-\xi) = \ln(1-x) - x\frac{(m_q^2 - x^2M^2)}{(1-x)Q^2} \tag{2.29}$$

where

$$R^{\rm NS}(x) = 2x - \frac{x(1-x)}{2} + \frac{\sum_i a_i x^{p_i}(1-x^{\alpha_i})^{l_i}}{\sum_j a_j x^{p_j}(1-x^{\alpha_j})^{l_j}} \left[p_i(1-x)\left\{(p_i - p_j)M(x) - \frac{\alpha_i l_i x^{\alpha_i}}{(1-x^{\alpha_i})}M(x) \frac{x}{1-x}N(x) + \frac{\alpha_j l_j x^{\alpha_j}}{(1-x^{\alpha_j})}M(x)\right\} - l_i\alpha_i x^{\alpha_i}\frac{(1-x)}{(1-x^{\alpha_i})} \left\{(\alpha_i + p_i - p_j)M(x) - \frac{(l_i-1)\alpha_i x^{\alpha_i}}{(1-x^{\alpha_i})}M(x) - \frac{x}{1-x}N(x) + \frac{\alpha_j l_j x^{\alpha_j}}{(1-x^{\alpha_j})}M(x)\right\}\right]. \tag{2.30}$$

$$M(x) = 2 - (1-x) + \frac{(1-x)^2}{3} \tag{2.31}$$

and

$$N(x) = 2 - 2(1-x) + (1-x)^2. \tag{2.32}$$

These result in

$$H^{\rm NS}_{u_v}(\xi) = H^{\rm NS}_{u_v}(x) - \frac{(m_q^2 - x^2M^2)}{Q^2}\left[\frac{16x}{25(1-x)} - 8R^{\rm NS}(x)\right] \tag{2.33}$$

so that

$$\left(\frac{t}{t_0}\right)^{H^{\rm NS}_{u_v}(\xi)} \approx \left(\frac{t}{t_0}\right)^{H^{\rm NS}_{u_v}(x)}\left[1 - \frac{(m_q^2 - x^2M^2)}{Q^2}\left(\frac{16x}{25(1-x)} - 8R^{\rm NS}(x)\right)\right]. \tag{2.34}$$

This yields

$$u_v(\xi, Q^2) = u_v(x, Q^2) - \sum_i a_i x^{p_i}(1-x^{\alpha_i})^{l_i}\left(\frac{t}{t_0}\right)^{H^{\rm NS}_{u_v}(x)}\frac{(m_q^2 - x^2M^2)}{Q^2} \times\left[\left\{\frac{16x}{25(1-x)} - 8R^{\rm NS}(x)\right\}\ln\frac{t}{t_0} + (p_i + l_i\alpha_i x^{\alpha_i})\right]. \tag{2.35}$$

Using it in (2.2), we then obtain

$$\tilde{f}_{u_v}(x, Q^2) = \sum_i a_i x^{p_i}(1-x^{\alpha_i})^{l_i}\left(\frac{t}{t_0}\right)^{H^{\rm NS}_{u_v}(x)} \times\left\{\left(\frac{16x}{25(1-x)} - 8R^{\rm NS}(x)\right)\ln\frac{t}{t_0} + (p_i + l_i x^{\alpha_i}\alpha_i)\right\}. \tag{2.36}$$

For the sea components, $H^{\rm NS}(x)$ occurring in (2.26) is to be replaced by $H^{\rm S}(x)$ given in eq. (2.15) and $\tilde{f}_{us}(x, Q^2)$ follows from (2.37) with replacement $H^{\rm NS}_{u_v}(x) \to H^{\rm S}_{us}(x)$.

2.5 Perturbative and non-perturbative quark masses

Elias and Scadron [19] pursues a theoretical model where the total quark mass is the sum of a flavour dependent current mass and a flavour independent dynamical mass, away from the chiral limit

$$m(Q^2) = m_{\text{current}}(Q^2) + m_{\text{dyn}}(Q^2). \tag{2.37}$$

In the chiral limit $m_{\text{current}}(Q^2) \to 0$, $m_{\text{dyn}}(Q^2)$ is identified with the constituent quark mass. As the current quark mass is perturbative while the dynamical quark mass is non-perturbative [19] in QCD, we identify the quark mass at $Q^2 = Q_0^2$ to be of dynamical origin. It will contain two pieces, the flavour dependent piece coming from input $F_2(x, Q_0^2)$ and the flavour independent one from HT term. The Q^2 dependence of the dynamical mass is solely due to HT so that without HT, it corresponds to the constituent mass. The perturbative quark mass $m_{\text{current}}(Q^2)$ can then be obtained from (2.37). We note that $m_{\text{dyn}}(x, Q^2)$ will critically depend on the inputs used. We will therefore test the sensitivity of our results with the inputs of Glück *et al* [24], ELHQ [25] and EHQ [26] as representative samples.

If the additivity relation (2.37) is extrapolated down to $Q^2 \to Q_0^2$, $m_{\text{current}}(Q^2)$ should tend to zero, on the other hand, as $Q^2 \to \infty$, $m_{\text{current}}(Q^2)$ should also have a vanishing limit [27] in QCD:

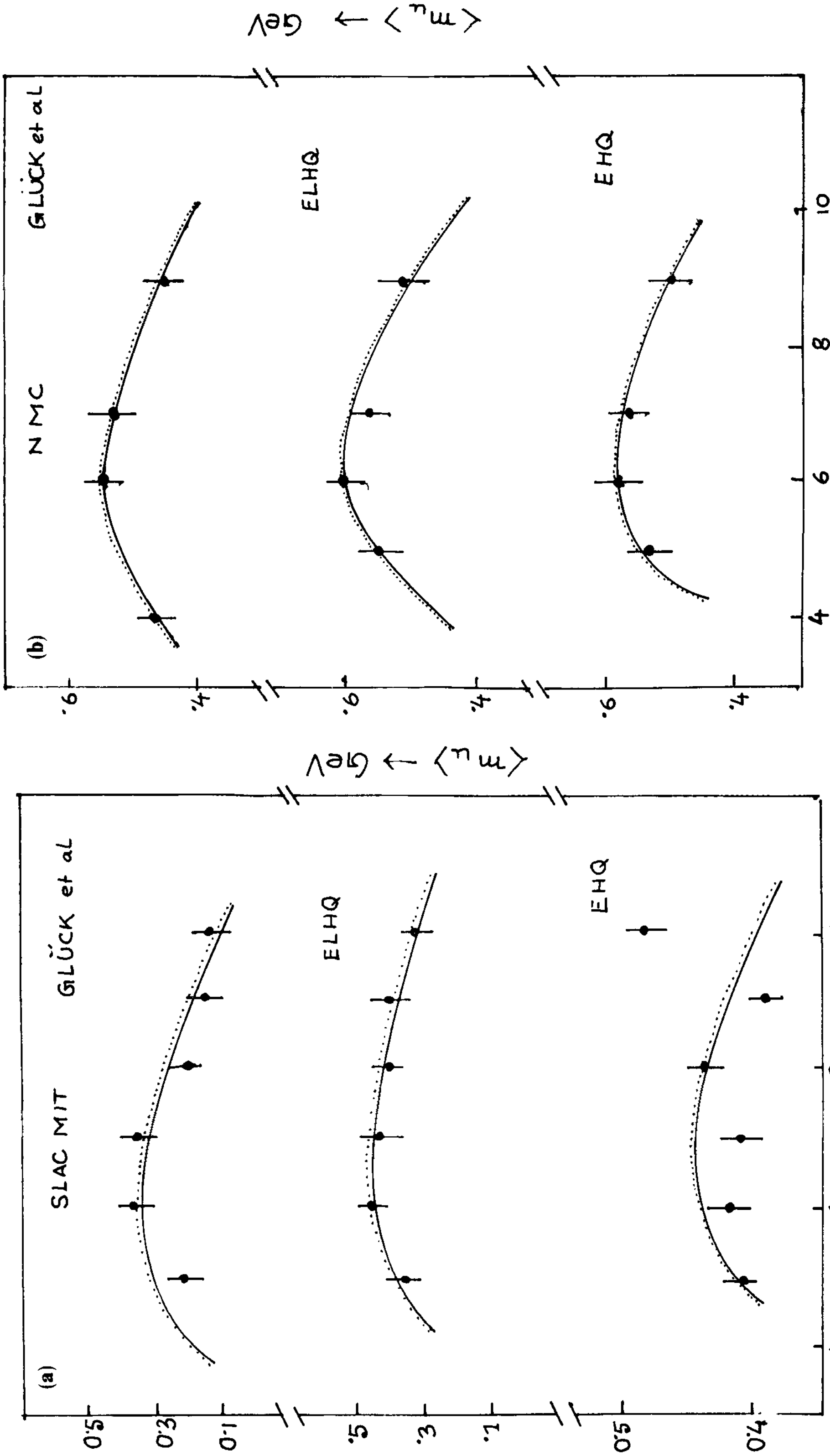
$$m_{\text{current}}(Q^2) = \frac{\hat{m}}{(1/2 \ln Q^2/\Lambda^2)^{-\gamma_1/\beta_1}} \tag{2.38}$$

where $\hat{m}$ is the scale invariant mass, $\gamma_1 = 2$ for three colours and $\beta_1 = -11/2 + n_{f/3}$, n_f being the number of flavours. Equations (2.37) and (2.38) then determine the behaviour of $m_{\text{current}}(Q^2)$ down to $Q^2 = Q_0^2$. In the next section we will discuss its behaviour graphically.

3. Results and discussion

In the present work, we report our analysis for up, down and strange quark. In figure 1(a–d) we present our results (denoted by solid points with bars) for x-averaged m_u and m_d using representative data samples of SLAC-MIT [3] (figure 1(a), 1(c)) and NMC [28] (figure 1(b), 1(d)) respectively, using the inputs of Glück *et al* [24], ELHQ [25] and EHQ [26]. To obtain these representative results, we need $\bar{F}_u$ and $\bar{F}_d$, the structure function contributions due to u and d quarks respectively. This can be estimated in a quark parton model as described in appendix B. In table 1, we record the values of dynamical quark masses (m_u and m_d) with the three inputs; (Glück *et al* [24], ELHQ [25] and EHQ [26]. In figure 2(a–d) we plot $m_{\text{cur}}(Q^2)$ vs Q^2 using SLAC-MIT [3] and NMC [28] data. In figure 3(a–b) we also plot $\bar{F}^u(x, Q^2)/F^u(x, Q^2)$ as Q^2 using (2.13) for $x = 0{\cdot}225$ and $0{\cdot}5$ respectively. Using SLAC-MIT [3] and NMC [28] data, similar curve can be drawn for d quark as well.

We have also made similar analysis for S-quark. The results are found to be much more sensitive to the values of x as well as inputs. Its magnitude is also relatively high. As an illustration with $x \sim 0{\cdot}18$ from NMC data and inputs of Glück *et al* [24] ELHQ [25] and EHQ [26] yields n_s the dynamical s quark to be 0·75, 0·87, 0·9 respectively. This is rather an unfortunate feature of the present formalism.


(b)
NMC
GLÜCK et al
ELHQ
EHQ
⟨m_u⟩ → GeV
Q² → GeV²
(a)
SLAC MIT
GLÜCK et al
ELHQ
EHQ
⟨m_u⟩ → GeV
Q² → GeV²

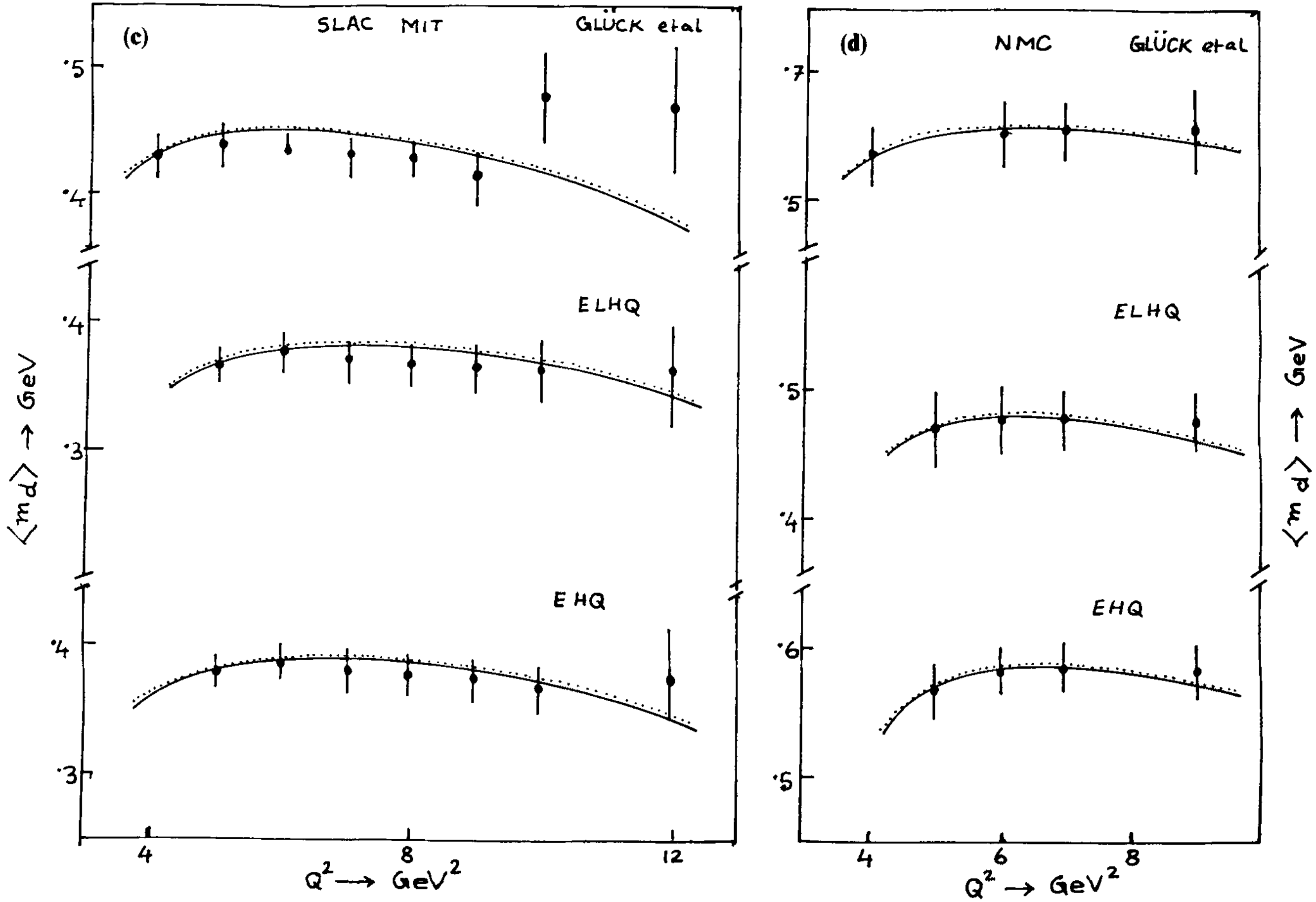
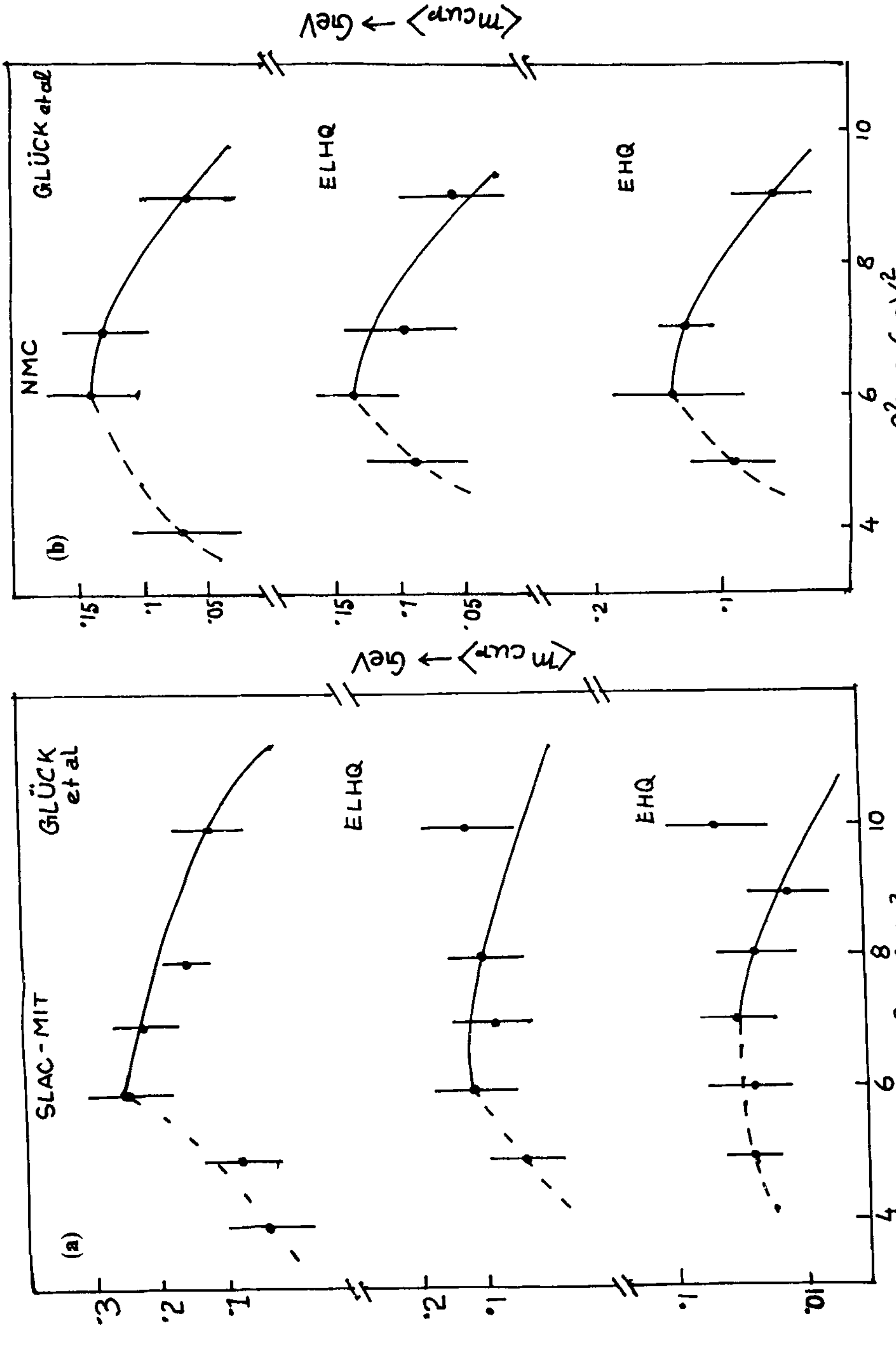


Figure 1(a–d). x averaged $m_u(Q^2)$ vs Q^2 using the inputs of Glück *et al* [24], ELHQ [25] and EHQ [26]. Data taken from SLAC-MIT [3] and NMC [28] respectively. Dotted lines represent HT effect.


(a)
SLAC-MIT
GLÜCK et al
ELHQ
EHQ
Q2 → GeV2
(b)
NMC
GLÜCK et al
ELHQ
EHQ
Q2 → GeV2

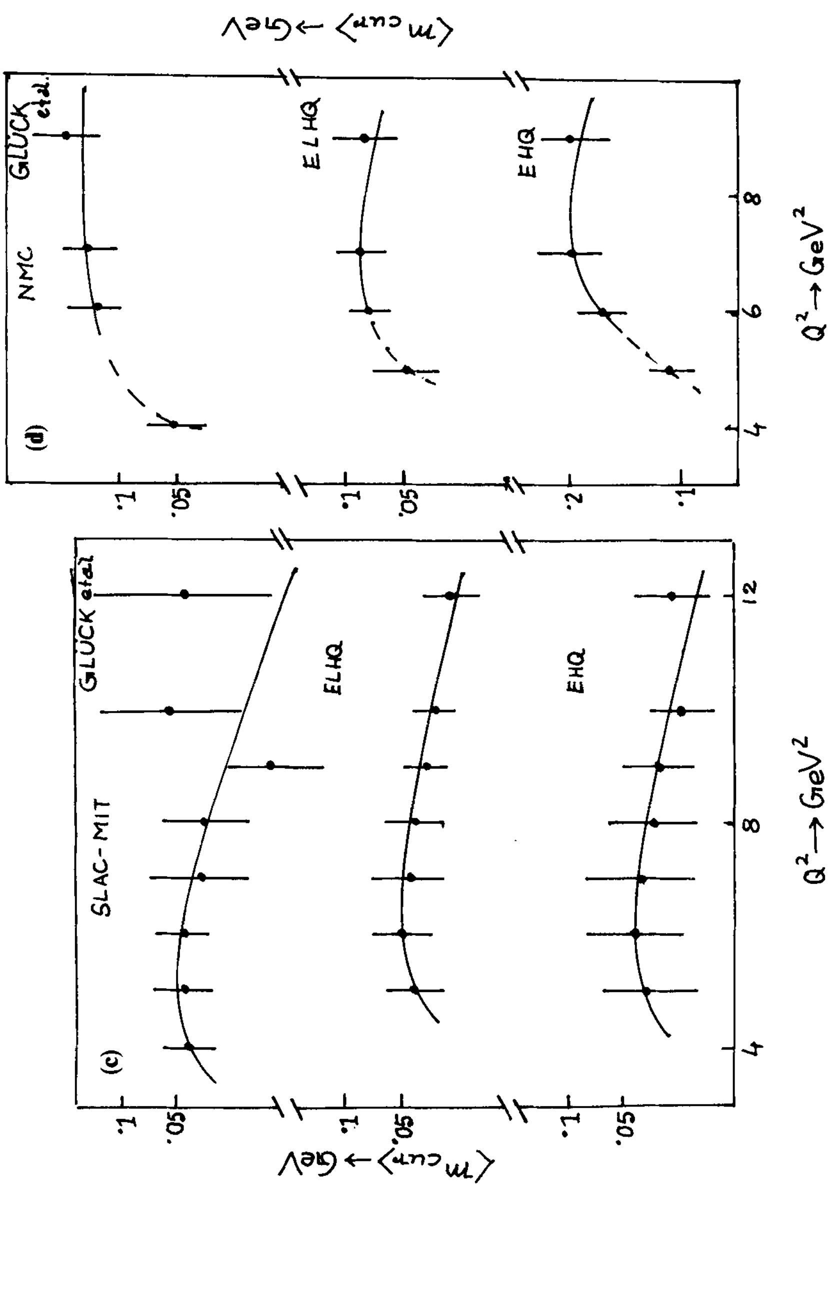


Figure 2(a–d). x averaged current quark mass vs Q^2 using the inputs of Glück *et al* [24], ELHQ [25] and EHQ [26]. Data taken from SLAC-MIT [3] and NMC [28] respectively. The dashed section of the curves represents m_{current} extrapolated down to $Q^2 \sim Q_0^2$.

These curves and the table demonstrate the following features of the quark mass.

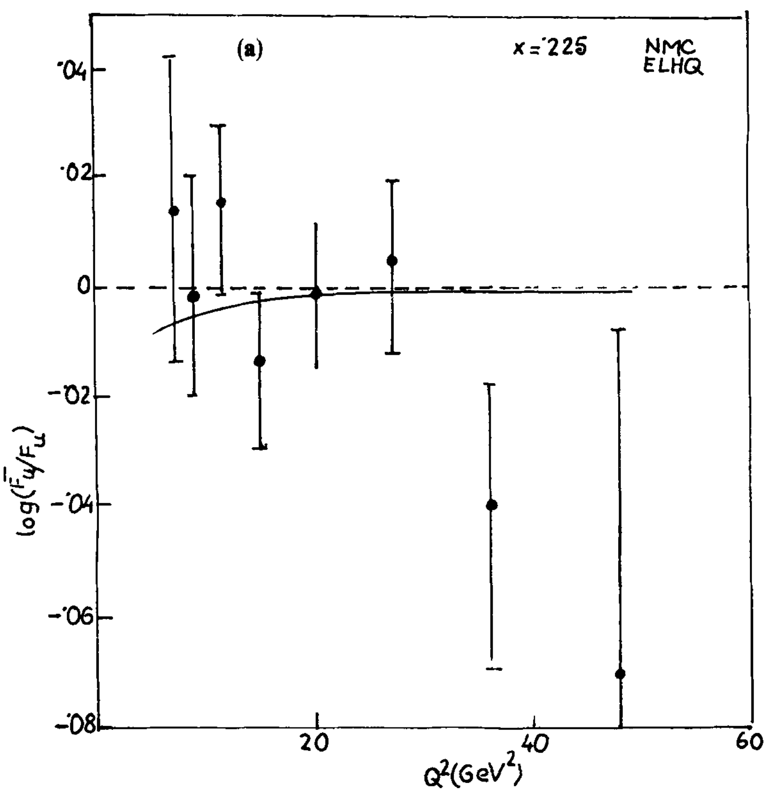
(i) The dynamical quark mass depends on the inputs as well as on data. But such variation does not change its order of magnitude. We find $m_u \sim 0{\cdot}126 - 0{\cdot}656$ GeV while $m_d \sim 0{\cdot}373 - 0{\cdot}664$ GeV.
(ii) Inclusion of HT enhances the dynamical mass slightly but it falls to its constituent limit as Q^2 increases.
(iii) Figure 2 (a–d) shows that away from $Q^2 \sim Q_0^2$, the current mass falls. But if they are extrapolated down to $Q^2 \sim Q_0^2$ (shown by dashed section of the curves), they acquire maxima at around 6–7 GeV^2.

This peculiar feature of current quark mass is due to the following reasons:

(a) The dynamical and current quark masses are additive even in the limit $Q^2 \to Q_0^2$
(b) Both $m_{\mathrm{dyn}}(Q^2)$ and $m_{\mathrm{cur}}(Q^2)$ are positive definite
(c) $m_{\mathrm{current}}(Q^2)$ should vanish both at $Q^2 \approx Q_0^2$ and $Q^2 \to \infty$.

Any deviation of the behaviour of current quark mass from figure 2(a–d) represents the breakdown of the naive additivity relation (Equation 2.37).

Finally, in figure 3(a, b), we show representative estimates of log $\bar{F}_u/F^u$ using eq. (2.13) with NMC data and ELHQ inputs. Theoretically one expects a decrease of m_q as Q^2 increases (represented by solid lines) while asymptotically it should vanish [represented by dashed line]. The representative results of figure 3(a, b) are compatible with this expectation.



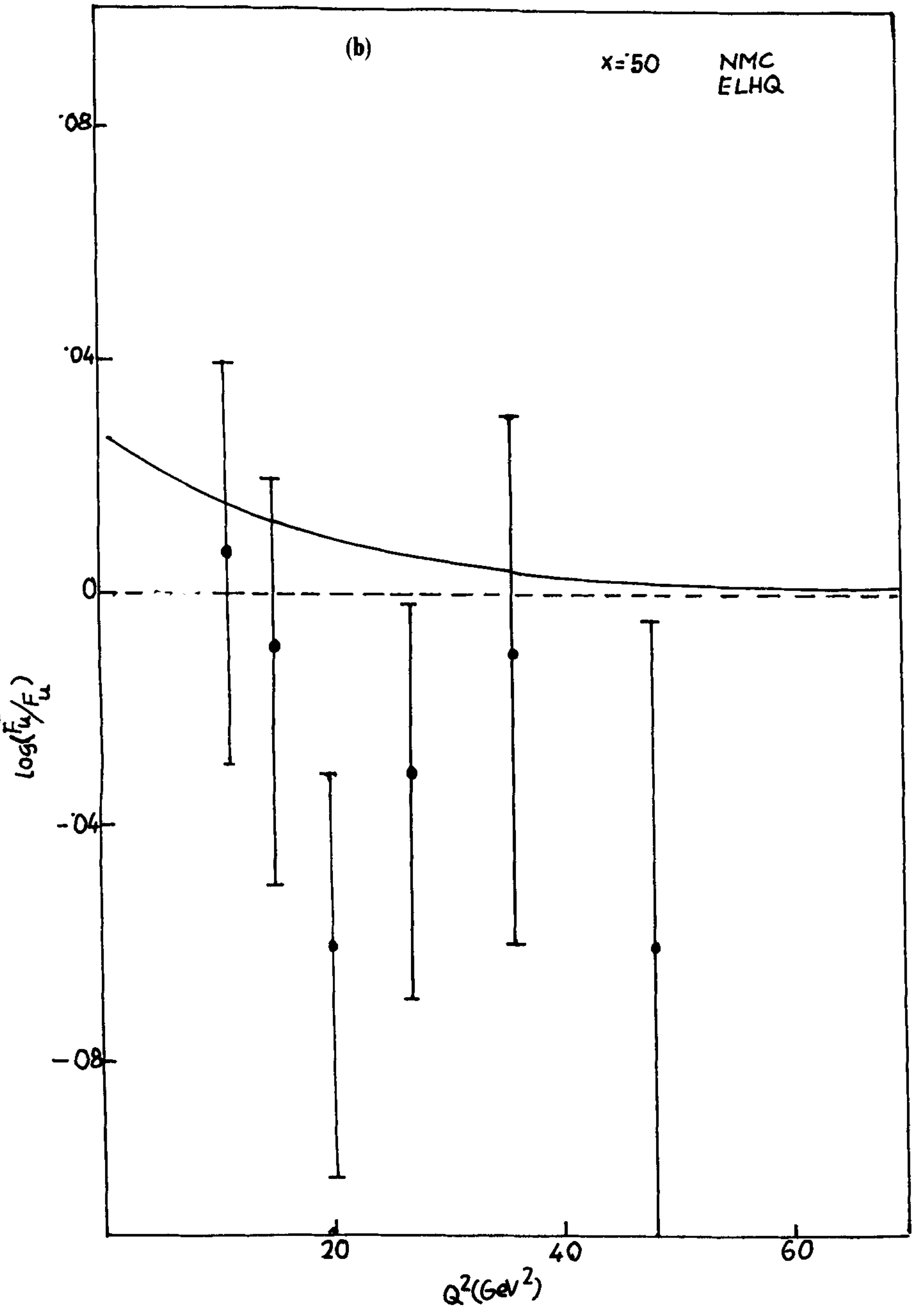


Figure 3(a–b). $F^{\bar{u}}(x, Q^2)/F^{u}(x, Q^2)$ vs Q^2 for $x = 0{\cdot}225$ and $0{\cdot}5$ respectively using NMC data and ELHQ input.

4. Conclusions

We have thus shown that the dynamical and the current quark masses for each flavour can be extracted from structure functions by improving the formalism reported earlier [1]. For up and down quark, the results are reasonable but for s quark the results are more sensitive to x values and inputs used.

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Appendix A

We record below the parameters of eq. (2.24) occurring in the inputs of Glück *et al* [24] ELHQ [25] and EHQ [26]

Glück et al [24] *distribution* $Q_0^2 = 4\,\mathrm{GeV}^2$

$$n^u = 1, \quad a_1^u = 1{\cdot}230, \quad p_1^u = 0{\cdot}421, \quad \alpha_1^u = 2, \quad l_1^u = 3{\cdot}37,$$

$$n^d = 1, \quad a_1^d = 0{\cdot}541, \quad p_1^d = 0{\cdot}364, \quad \alpha_1^d = 2, \quad l_1^d = 5{\cdot}09,$$

$$n^s = n^s = 1, \quad a_1^s = 0{\cdot}25, \quad p_1^s = 0, \quad \alpha_1^s = 1, \quad l_1^s = 7,$$

ELHQ [25] *distribution* $Q_0^2 = 5\,\mathrm{GeV}^2$

$$n^u = 1, \quad a_1^u = 1{\cdot}78, \quad p_1^u = 0{\cdot}5, \quad \alpha_1^u = 1{\cdot}51, \quad l_1^u = 3{\cdot}5,$$

$$n^d = 1, \quad a_1^d = 0{\cdot}67, \quad p_1^d = 0{\cdot}4, \quad \alpha_1^d = 1{\cdot}51, \quad l_1^d = 4{\cdot}5,$$

$$n^{u_s} = n^{d_s} = 1, \quad a_1^{u_s} = a_1^{d_s} = 0{\cdot}182, \quad p_1^{u_s} = p_1^{d_s} = 0, \quad \alpha_1^{u_s} = \alpha_1^{d_s} = 1, \quad l_1^{u_s} = l_1^{d_s} = 8{\cdot}54,$$

$$n^{s_s} = 1, \quad a_1^s = 0{\cdot}081, \quad p_1^s = 0, \quad \alpha_1^s = 1, \quad l_1^s = 8{\cdot}54$$

EHQ [26] *distribution* $Q_0^2 = 5\,\mathrm{GeV}^2$

$$n^u = 1, \quad a_1^u = 2{\cdot}406, \quad p_1^u = 0{\cdot}60, \quad \alpha_1^u = 1, \quad l_1^u = 3{\cdot}1,$$

$$n^d = 1, \quad a_1^d = 2{\cdot}144, \quad p_1^d = 0{\cdot}70, \quad \alpha_1^d = 1{\cdot}2, \quad l_1^d = 4{\cdot}8,$$

$$n^{\bar{s}} = 1, \quad a_1^{\bar{s}} = 0{\cdot}086, \quad p_1^{\bar{s}} = 0, \quad \alpha_1^{\bar{s}} = 1, \quad l_1^{\bar{s}} = 8{\cdot}5,$$

$$n^{\bar{u}} = 1, \quad a_1^{\bar{u}} = 0{\cdot}20, \quad p_1^{\bar{u}} = 0, \quad \alpha_1^{\bar{u}} = 1, \quad l_1^{\bar{u}} = 11{\cdot}2,$$

$$n^{\bar{d}} = 1, \quad a_1^{\bar{d}} = 0{\cdot}20, \quad p_1^{\bar{d}} = 0, \quad \alpha_1^{\bar{d}} = 1, \quad l_1^{\bar{d}} = 5{\cdot}8.$$

Appendix B

Extraction of F^u, F^d in the experimental data.

We show the extraction of F^u and F^d from SLAC MIT data [3] with ELHQ input at $x = 0{\cdot}25$, $Q_0^2 = 5\,\mathrm{GeV}^2$. Similar analysis can be done for other representative points for different inputs as well:

From equations (2.21), (2.22) and (2.23) of the text and using ELHQ inputs (Appendix A) one obtains at $x = 0{\cdot}25$

$$F^u \to 0{\cdot}236; \; F^d \to 0{\cdot}027; \; F^s \to 0{\cdot}002.$$

It leads to the theoretical value of structure function at $x = 0{\cdot}25$ and $Q_0^2 = 5\,\mathrm{GeV}^2$ to be

$$F(x = 0{\cdot}25, \quad Q^2 = 5\,\mathrm{GeV}^2) \approx 0{\cdot}292.$$

It gives

$$\left.\frac{F^u}{F}\right|_{x=0\cdot25} \approx 0\cdot902.$$

The experimental value of structure function from SLAC-MIT data at the corresponding values of x and Q^2 is

$$\bar{F} = 0\cdot2959 \pm 0\cdot0112.$$

This yields

$$F^u \simeq 0\cdot902 \times \bar{F} \approx 0\cdot27 \pm 0\cdot01.$$

Similarly for d quark at the same x and Q^2 yield

$$\left.\frac{F^d}{F}\right|_{x=0\cdot25} \approx 0\cdot09 \quad \text{so that } \bar{F}^d \simeq 0\cdot09 \times \bar{F}$$

$$\simeq 0\cdot026 \pm 0\cdot001$$

and so on.

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