PRAMANA
— journal of
physics

Vol. 43, No. 1,
July 1994
pp. 67–72

Enhanced bistability with the line-narrowed modes in a nonlinear modulated index Fabry–Perot cavity

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MS received 25 March 1994

Abstract. We exploit the recently reported line narrowed modes [1] for observation of bistability in a nonlinear Fabry–Perot cavity with intracavity index modulation. We show that the frequency response of the structure exhibits larger bistability loops for the line narrowed modes.

Keywords. Bistability; multi stability.

PACS No. 42·65

In the past two decades there has been a lot of interest in distributed feedback structures in the context of lasing [2, 3] and other applications [[4], Chapters 7 and 10]. Nonlinear distributed feedback structures have been exploited to show the existence of optical bistability [5] and localized solutions in them [6–11]. Chaotic response of such structures has also been reported [12–14]. Very recently it was demonstrated that one can narrow down the resonances if the distributed feedback medium is enclosed in a Fabry–Perot (FP) cavity [1]. The interplay of the distributed feedback and the localized feedback due to the FP mirrors can diversely affect the various resonances leading to line narrowing of some selective modes. The line narrowing was shown to be quite useful in the context of cavity quantum electrodynamics applications, in particular, for vacuum field Rabi splitting [1]. The line narrowing effect can also be exploited for optical bistability since it is well established that the narrower the resonances the easier it is to deform them to obtain bistable/multistable output. In this paper we consider a FP cavity with intracavity index modulation and with Kerr type nonlinearity. In the framework of a coupled wave theory [[4] pp 189–199] we obtain the equations for the forward and backward propagating waves. These equations are integrated to calculate the frequency response of the structure in the domain which encompasses the line narrowed resonance along with others. We show that the nonlinearity induced bending of the line narrowed resonance dominates over those of other resonances leading to a larger bistability loop. We also study the power dependence of the transmission coefficient when the frequencies are chosen to be detuned by the same amount from the centre of the mode resonances. We demonstrate that the line narrowed modes offer well defined multistable behaviour.

We consider a FP cavity of length L with intracavity index modulation. The intracavity medium is assumed to possess Kerr type nonlinearity. Thus, in the approximation that depth of modulation n_1 is much smaller than background refractive index n_0 the dielectric function for the intracavity medium can be written as

$$\varepsilon(z,E) = n_0^2 + 2n_0 n_1 \cos(Kz+\theta) + 4\pi\chi_n |E|^2, \quad (1)$$

where K is the grating constant ($=2\pi/\Lambda, \Lambda$ is the grating period). Phase θ can be adjusted to have sine or cosine modulation and χ_n is the nonlinearity parameter. The smallness of the modulation amplitude enables one to develop a coupled mode theory, whereby the field inside the cavity can be expressed as a sum of counter propagating terms as follows:

$$E(z) = E_f + E_b = \mathscr{E}_f(z)\mathrm{e}^{ikz} + \mathscr{E}_b(z)\mathrm{e}^{-ikz}, \quad (2)$$

with $k = \dfrac{\omega}{c} n_0$.

Henceforth, it is assumed that the grating vector K is close to twice the momentum of the field in order to have efficient coupling between the forward and backward propagating waves. On substituting (2) in the wave equation and making slowly varying envelope approximation the coupled amplitude equations can be obtained and can be written as follows:

$$\frac{\mathrm{d}\mathscr{E}_f}{\mathrm{d}z} = i\beta\mathscr{E}_b \mathrm{e}^{-i\delta z + i\theta} + i\alpha\mathscr{E}_f(|\mathscr{E}_f|^2 + 2|\mathscr{E}_b|^2), \quad (3)$$

$$\frac{\mathrm{d}\mathscr{E}_b}{\mathrm{d}z} = -i\beta\mathscr{E}_b \mathrm{e}^{i\delta z - i\theta} - i\alpha\mathscr{E}_b(|\mathscr{E}_b|^2 + 2|\mathscr{E}_f|^2). \quad (4)$$

In eqs (3), (4)

$$\delta = 2\frac{\omega}{c} n_0 - K, \quad \beta = \frac{\omega n_1}{c}, \quad \alpha = 2\pi\omega\chi_n/cn_0.$$

In writing (3)–(4) we ignored the quickly oscillating spatial components like $\mathrm{e}^{\pm i3kz}$. For the set of equations (3), (4) a closed form solution could not be obtained and we resort to numerical methods to solve them. In order to obtain transmission characteristics of the structure eqs (3), (4) are to be supplemented by the FP boundary conditions. Assuming that both the mirrors of the FP cavity are identical with amplitude reflection and transmission coefficients r and t, respectively, with $|r|^2 + |t|^2 = 1$ (mirrors are lossless) the boundary conditions can be written as follows

$$\mathscr{E}_f(0) = t\mathscr{E}_i + r\mathscr{E}_b(0), \quad (5)$$

$$\mathscr{E}_b(L)\mathrm{e}^{-ikL} - r\mathscr{E}_f(L)\mathrm{e}^{ikL} = 0, \quad (6)$$

$$\mathscr{E}_t = t\mathscr{E}_f(L), \quad (7)$$

$$\mathscr{E}_r = t\mathscr{E}_b(0) - r\mathscr{E}_i. \quad (8)$$

We now switch to dimensionless quantities as follows:

$$K \to KL = \tilde{K}, \quad \beta \to \beta L = \tilde{\beta}, \quad \delta \to \delta L = \tilde{\delta}$$

$$\mathscr{E}_{f,b} \to \sqrt{\alpha L}\mathscr{E}_{f,b} = \tilde{\mathscr{E}}_{f,b}, \quad z \to z/L = \tilde{z}.$$

Restricting to cosine modulation ($\theta = 0$), eqs (3) and (4) can be written in terms of the dimensionless quantities as follows

$$\frac{d\tilde{\mathscr{E}}_f}{d\tilde{z}} = i\tilde{\beta}\mathscr{E}_b e^{-i\tilde{\delta}\tilde{z}} + i\tilde{\mathscr{E}}_f(|\tilde{\mathscr{E}}_f|^2 + 2|\tilde{\mathscr{E}}_b|^2), \quad (9)$$

$$\frac{d\tilde{\mathscr{E}}_b}{d\tilde{z}} = -i\tilde{\beta}\mathscr{E}_b e^{-i\tilde{\delta}\tilde{z}} - i\tilde{\mathscr{E}}_b(|\tilde{\mathscr{E}}_b|^2 + 2|\tilde{\mathscr{E}}_f|^2). \quad (10)$$

Henceforth, we treat $\tilde{\mathscr{E}}_t (= \sqrt{\alpha L}\mathscr{E}_t)$ as a parameter in terms of which both $\tilde{\mathscr{E}}_f$ and $\tilde{\mathscr{E}}_b$ can be specified at $\tilde{z} = 1$ ($z = L$) as follows

$$\tilde{\mathscr{E}}_f(1) = \tilde{\mathscr{E}}_t/t, \quad (11)$$

$$\tilde{\mathscr{E}}_b(1) = r\tilde{\mathscr{E}}_f(1)e^{i(\tilde{K}+\tilde{\delta})}. \quad (12)$$

Equations (9) and (10) can be solved with initial conditions (11) and (12) and the solution can be obtained at $\tilde{z} = 0$. In terms of $\tilde{\mathscr{E}}_f(0)$ and $\tilde{\mathscr{E}}_b(0)$ the scaled incident and reflected amplitudes can be evaluated by using the following equations:

$$\tilde{\mathscr{E}}_i = \frac{\tilde{\mathscr{E}}_f(0) - r\tilde{\mathscr{E}}_b(0)}{t}, \quad (13)$$

$$\tilde{\mathscr{E}}_r = t\tilde{\mathscr{E}}_b(0) - r\tilde{\mathscr{E}}_i. \quad (14)$$

For a given $\tilde{\mathscr{E}}_t$ once the incident and reflected amplitudes, $\tilde{\mathscr{E}}_i$ and $\tilde{\mathscr{E}}_r$ are known, it is straightforward to calculate the intensity transmission and reflection coefficients. Note that the above procedure can lead to a simple calculation of the power dependence of the transmission/reflection coefficients. However, for a given input power and given frequency of the incident radiation calculation of the transmission coefficient is not straightforward. One has to solve a nonlinear equation to obtain the value of $\tilde{\mathscr{E}}_t$ which corresponds to the given $\tilde{\mathscr{E}}_i$.

In what follows we present the results of numerical calculations. The following system parameters were chosen for calculation: $KL/2 = 439\pi$, $r^2 = 0{\cdot}7$, $n_0 = 2{\cdot}3$, $n_1 = 10^{-2}$. We first present the results pertaining to the frequency response of the system for given input fields. In figure 1 we have shown the transmission coefficient for two different values of $|\tilde{\mathscr{E}}_i|^2$ namely $|\tilde{\mathscr{E}}_i|^2 = 1{\cdot}0 \times 10^{-5}$ (dashed curve) and 0·1 (solid curve), as functions of dimensionless $\tilde{\delta}$. Various peaks for the case of low input intensity are labelled by the corresponding mode indices (see Dutta Gupta *et al* [1]). Since the input intensity for this case is vanishingly small the response agrees quite well with response of a linear structure (with $\alpha = 0$). As was shown in ref. 15, the mode labelled by $+1$ is the line narrowed mode and the resonance corresponding to it is the narrowest one. We now concentrate on the case when the incident intensity is increased to 0·1. The nonlinearity comes into play and bends all the resonances leading to a shift of their peaks. The broader the resonance (e.g., -2, -1 modes) the less is the effect on them. The bending is maximum for the $+1$ mode which has the highest quality factor compared to all others. Thus, the line narrowed mode exhibits the largest bistability loop demonstrating its potentials for various applications. In other words, for moderate incident powers when other modes cannot exhibit bistability the line narrowed mode can show well defined and well separated multivalued output.

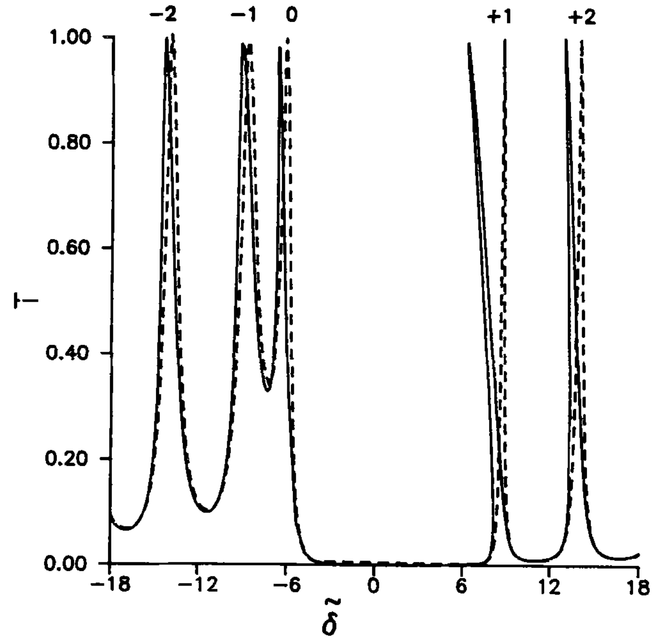

Figure 1. Transmission coefficient T as a function of dimensionless detuning $\tilde{\delta}$ for a cosine modulated FP cavity. The dashed curve corresponds to low input intensity ($|\tilde{\mathscr{E}}_i|^2 = 1{\cdot}0 \times 10^{-5}$) and is almost identical with the linear response. Various resonances are labelled by their corresponding mode numbers [1]. The solid curve shows the response for a large input intensity ($|\tilde{\mathscr{E}}_i|^2 = 0{\cdot}1$). Other parameters are as follows: $r^2 = 0{\cdot}7$, $KL/2 = 439\pi$, $n_0 = 2{\cdot}3$, $n_1 = 0{\cdot}01$.

We now turn to the bistability in the input-output characteristics of the system. In figure 2 we have shown the transmission coefficient as a function of incident intensity $|\tilde{\mathscr{E}}_i|^2$ for fixed frequencies (fixed $\tilde{\delta}$). Results for $\tilde{\delta} = -9{\cdot}12$ ($= 5{\cdot}69$) are shown in figure 2a (2b). The chosen values of $\tilde{\delta}$ correspond to an equal amount of offset in the same direction from the 0 and the $+1$ mode resonances of the linear structure. It is clear from a comparison of figures 2a and 2b that the high-to-low switching threshold is an order of magnitude lower in the case of $+1$ mode. However, one has to drive the system stronger for the $+1$ mode in order to go to the upper transmission branch. Besides, one has the possibility of multistability with $+1$ and $+2$ modes, whereas, multistability is not exhibited in figure 2a.

In conclusion, we studied an index modulated Fabry–Perot cavity with Kerr type nonlinearity. We obtained the frequency response of the structure for given input intensities. In the frequency response the line narrowed modes are shown to lead to larger bistability loops. Moreover, we studied the power dependence of the transmission coefficient at given frequencies and demonstrated multistability with line narrowed modes.

The author would like to thank R W Boyd for the hospitality at Institute of Optics, University of Rochester, where the work was carried out under the NSF Project no. INT910085.

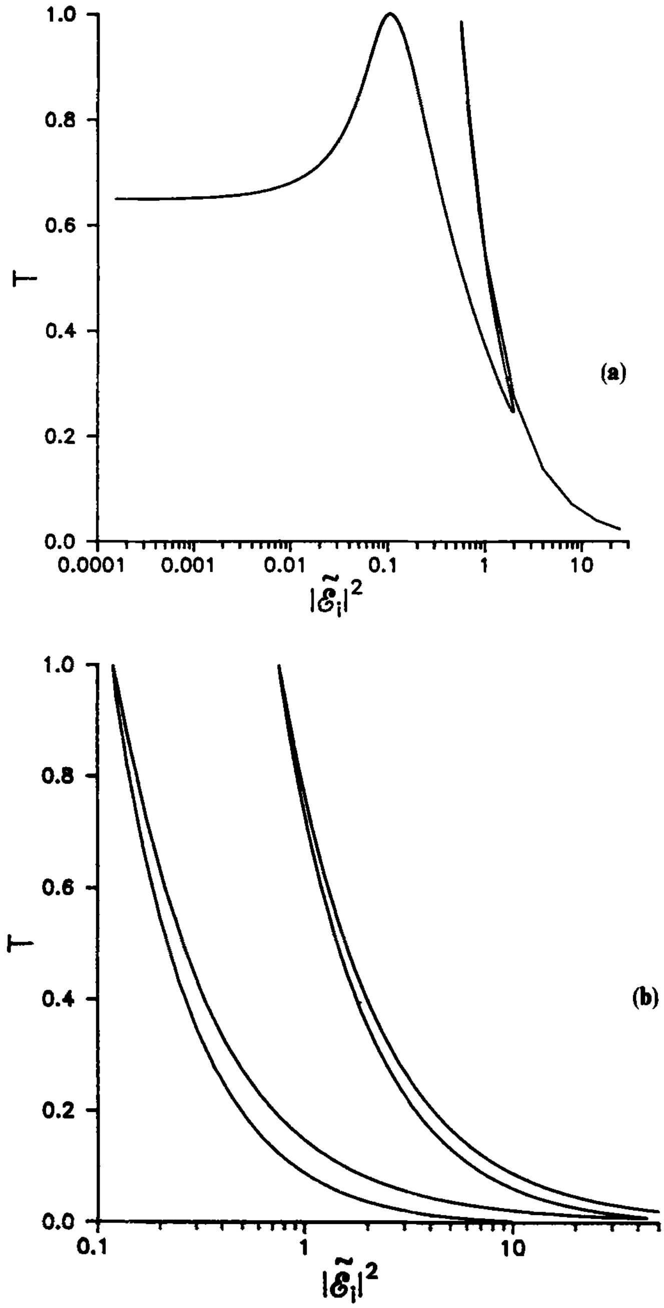

Figure 2. Transmission coefficient T as a function of incident intensity $|\tilde{\mathscr{E}}_i|^2$ for (**a**) $\tilde{\delta} = -9{\cdot}12$ and for (**b**) $\tilde{\delta} = 5{\cdot}69$. Other parameters are as in figure 1. The cases (a) and (b) correspond to an equal amount of shift from the linear 0 and + 1 mode resonances, respectively.

References

[1] S Dutta Gupta and G S Agarwal, *Opt. Commun.* **103**, 122 (1993)
[2] H Kogelnik and C V Shank, *J. Appl. Phys.* **43**, 2327 (1972)
[3] M Sargent III, W H Swatner and J D Thomas, *IEEE J. Quant. Electron.* **QE-16**, 465 (1980)
[4] P Yeh, *Optical waves in layered media* (Wiley, New York, 1988)
[5] H G Winful, J H Marburger and E Garmine, *Appl. Phys. Lett.* **35**, 379 (1979)
[6] W Chen and D L Mills, *Phys. Rev. Lett.* **58**, 160 (1987)
[7] J Coste and J Peyreud, *Phys. Rev.* **B39**, 13086 (1989)
[8] J Coste and J Peyreud, *Phys. Rev.* **B39**, 13096 (1989a)
[9] C M de Sterke and J E Sipe, *Phys. Rev.* **A38**, 5149 (1988)
[10] C M de Sterke and J E Sipe, *Phys. Rev.* **B39**, 5163 (1989)
[11] C M de Sterke and J E Sipe, *Phys. Rev.* **A42**, 550 (1990)
[12] M Grabowski and P Hawrylak, *Phys. Rev.* **B41**, 5783 (1990)
[13] P Hawrylak, M Grabowski and P Wilson, *Phys. Rev.* **B40**, 6398 (1989)
[14] C M de Sterke and J E Sipe, *Phys. Rev.* **A42**, 2858 (1990a)