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Plasma-maser interaction of Langmuir wave with kinetic Alfvén wave turbulence

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Abstract. A theoretical study is made on the generation mechanism of Langmuir mode wave in the presence of kinetic Alfvén wave turbulence in a magnetized plasma on the basis of plasma-maser interaction. It is shown that a test high frequency Langmuir mode wave is unstable in the presence of low frequency kinetic Alfvén wave turbulence. The growth of the Langmuir wave occurs due to direct and polarization coupling terms. Because of the universal existence of the kinetic Alfvén waves in large scale plasmas, the results have potential importance in space and astrophysical radiation processes.

Keywords. Weak turbulence theory; wave particle interaction; resonant and nonresonant waves; growth rate.

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1. Introduction

According to the recent weak turbulence theory [1], the lowest order mode–mode coupling processes are composed of three parts. The first two processes are the three waves resonance and the nonlinear scattering resonance; the conditions for which are $\mathbf{K}-\mathbf{k}=\mathbf{K}'$, $\Omega-\omega=\Omega'$ and $\Omega-\omega-(K_\parallel-k_\parallel)v_\parallel+a\Omega_e=0$ respectively. These processes are extensively studied by many authors [2]. The third process which we consider here is the plasma-maser. The plasma-maser effect occurs when nonresonant as well as resonant plasma oscillations are present. The resonant waves are those for which the Cherenkov resonance condition $\omega-k_\parallel v_\parallel=0$ is satisfied, while the nonresonant waves are those for which both the Cherenkov and scattering conditions are not satisfied: $\Omega-K_\parallel v_\parallel\neq 0$ and $\Omega-\omega-(K_\parallel-k_\parallel)v_\parallel+a\Omega_e\neq 0$. It corresponds to vortex correction, and is effective without electron population inversion which differs it markedly from other maser effects (e.g., inverse nonlinear Landau interaction). The most important characteristics of plasma-maser is the energy up-conversion from the resonant low frequency mode to the non-resonant high frequency mode in open plasma system where energy and particle sources are available [3] (in the form of continuous beam injection, external magnetic field etc.). For closed system without magnetic field, the plasma-maser invariably cancels out with the reverse absorption process due to quasilinear process.

In recent years, there has been an increasing interest in theoretical, experimental and observational studies in plasma-maser [4, 5, 6]. This new mode coupling process has been applied to auroral kilometric radiation, type III radio emission, chorus related electrostatic bursts, Jovian kilometric radiation and Saturnian kilometric radiation. The nature of plasma-maser based on quantum electrodynamical methods is investigated [7]. The nonlinear effect of the resonant and non-resonant modes simultaneously on the evolution of electron distribution function, known as the inverse plasma-maser, has also been studied [8]. Recently, the basic physics involved in the process, i.e., the energy and momentum conservation relations among the waves and the electrons is also investigated [4, 9]. In [9], it has been shown that the energy and momentum conservation relations between particle kinetic energy and wave energy is satisfied for the plasma-maser process, while the Manley–Rowe relation for plasma waves is violated and as a result an efficient energy up-conversion from the low frequency resonant mode to high frequency nonresonant mode becomes possible. The violation of Manley–Rowe relation for open system is already mentioned by Krlin [10].

Almost all the previous studies in plasma-maser, except [6], electrostatic turbulences are considered. But, due to its small scale length (on the order of Debye length), electrostatic wave is not dominant turbulent fluctuations in cosmic plasmas. On the other hand, magnetohydrodynamic waves have a large scale size because the parallel wavelength is comparable to the system size. MHD turbulence occurs in laboratory settings such as fusion confinement systems (e.g., reverse-field pinch) and astrophysical systems (e.g., solar corona, earth's magnetosheath etc.). Here, we show that nonlinear interaction of kinetic Alfvén wave turbulence with a test high frequency electrostatic (Langmuir) wave can lead to amplification of the high frequency wave through the plasma-maser interaction.

We study the interaction between stationary kinetic Alfven wave and electrons in an open system. Electrons that have velocities close to the phase velocity of kinetic Alfvén wave experience mixed mode perturbations between kinetic Alfvén wave and electrostatic waves. As a result, the growth of the test electrostatic wave occur through the plasma-maser process.

The organization of the paper is as follows. In § 2, the formulation and basic assumptions of the plasma-maser process are given and the nonlinear dielectric constant of the Langmuir mode in presence of the kinetic Alfvén wave turbulence is derived. The nonlinear dielectric constant is composed of two parts: direct and polarization mode coupling terms. The growth rate of the Langmuir mode through the form of the Landau resonance is obtained for direct and polarization terms respectively in § 3. Finally, § 4 contains a discussion on the study and its potential importance and applicability.

2. Formulations

We derive the dispersion relation of the Langmuir mode in the presence of stationary kinetic Alfvén wave turbulence propagating in the $x-z$ plane with wave vector $\mathbf{k}(=k_\perp, 0, k_\parallel)$. The wave fields are $\mathbf{E}_l(=E_{l\perp}, 0, E_{l\parallel})$ and $\mathbf{B}_l(=0, B_{ly}, 0)$. The kinetic Alfvén wave turbulence is assumed to be driven by electron beam. The electron distribution function is

$$f_{oe}(v) = \left(\frac{m}{2\pi T_e}\right)^{3/2} \exp\left(-\frac{mv_\perp^2}{2T_e}\right) \exp\left(-\frac{m(v_\parallel - v_0)^2}{2T_e}\right), \tag{1}$$

where the symbols have their usual meaning. We assume that the electron beam is injected continuously from system outside to keep the distribution function stationary.

Thus, in spite of the quasilinear interaction between kinetic Alfvén wave and electrons, the electron distribution function remains stationary. This is the main reason why the Manley–Rowe relation is violated for open plasma system.

The basic equations are the Vlasov-Poisson equations

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}-\frac{e}{m}\left(\mathbf{E}(\mathbf{r},t)+\frac{\mathbf{v}\times\mathbf{B}(\mathbf{r},t)}{c}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right]f_e(\mathbf{r},\mathbf{v},t)=0, \tag{2}$$

$$\nabla\cdot\mathbf{E}(\mathbf{r},t)=-4\pi e\int f_e(\mathbf{r},\mathbf{v},t)\,\mathrm{d}\mathbf{v}, \tag{3}$$

where the notations are standard. According to the linear response theory of turbulent plasma, the unperturbed electron distribution function and fields are,

$$F_{oe}=f_{oe}+\varepsilon f_{1e}+\varepsilon^2 f_{2e},$$
$$\mathbf{E}_{oe}=\varepsilon\mathbf{E}_l,\quad \mathbf{B}_{oe}=\mathbf{B}_0+\varepsilon\mathbf{B}_l, \tag{4}$$

where ε ($\ll 1$) is a small parameter associated with the turbulence fields and $\mathbf{B}_0(=0, 0, B_0)$ is the external magnetic field.

We assume that the system is open. Further, the fluctuation level of the kinetic Alfvén wave is assumed to be a given quantity and the interaction between the low frequency Alfvén wave turbulence and the high frequency Langmuir mode is considered through resonant electrons. With the assumption that the density gradient be negligibly small, the linear response of the electron distribution function (f_{1e}) due to the kinetic Alfvén wave reduces to,

$$f_{1e}(\mathbf{k},\omega)=\frac{ie}{m}\frac{1}{\omega-k_\parallel v_\parallel+i0}\mathrm{E}_{l\parallel}(\mathbf{k},\omega)\frac{\partial}{\partial v_\parallel}f_{oe}, \tag{5}$$

where $i0$ is the small imaginary part.

We now perturb the quasi-steady state by a high frequency test Langmuir mode wave field $\mu\delta\mathbf{E}_h$ with a propagation vector $\mathbf{K}(=0,0,K)$ and a frequency Ω, here $\mu\ll\varepsilon$. Thus the total perturbed electric and magnetic fields and the electron distribution functions are,

$$\delta\mathbf{E}=\mu\delta\mathbf{E}_h+\mu\varepsilon\delta\mathbf{E}_{lh},$$
$$\delta\mathbf{B}=0+\mu\varepsilon\delta\mathbf{B}_{lh}, \tag{6}$$
$$\delta f=\mu\delta f_h+\mu\varepsilon\delta f_{lh}+\mu\varepsilon^2\Delta f.$$

Linearizing the Vlasov equation (2) to the orders μ, $\mu\varepsilon$ $\mu\varepsilon^2$ we obtain,

$$P\delta f_h-\frac{e}{m}\delta\mathbf{E}_h\cdot\frac{\partial}{\partial\mathbf{v}}f_{oe}=0 \tag{7}$$

$$P\delta f_{lh}-\frac{e}{m}\left(\mathbf{E}_l+\frac{\mathbf{v}\times\mathbf{B}_l}{c}\right)\cdot\frac{\partial}{\partial\mathbf{v}}\delta f_h-\frac{e}{m}\delta\mathbf{E}_h\cdot\frac{\partial}{\partial\mathbf{v}}f_{1e}$$
$$-\frac{e}{m}\left(\delta\mathbf{E}_{lh}+\frac{\mathbf{V}\times\delta\mathbf{E}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\mathbf{v}}f_{oe}=0 \tag{8}$$

$$P\Delta f - \frac{e}{m}\left\langle\left(\mathbf{E}_l + \frac{\mathbf{v}\times\mathbf{B}_l}{c}\right)\cdot\frac{\partial}{\partial\mathbf{v}}\delta f_{lh}\right\rangle - \frac{e}{m}\left\langle\left(\delta\mathbf{E}_{lh} + \frac{\mathbf{v}\times\delta\mathbf{B}_{lh}}{\mathbf{c}}\right)\cdot\frac{\partial}{\partial\mathbf{v}}f_{1e}\right\rangle = 0 \tag{9}$$

where $\langle\cdots\rangle$ means average over the kinetic Alfvén waves and P is the operator,

$$P \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}} - \frac{e}{m}\mathbf{v}\times\mathbf{B}_0\cdot\frac{\partial}{\partial\mathbf{v}}$$

In (4), (6) and (9), we omitted the second-order field quantities, which can be justified under random phase approximations.

Taking Fourier transform in space and time for the various quantities, we obtain from (3) the non-linear dielectric constant of the Langmuir mode in the presence of the kinetic Alfvén wave turbulence as (see Appendix),

$$\varepsilon_h(\mathbf{K},\Omega) = \varepsilon_0(\mathbf{K},\Omega) + \varepsilon_d(\mathbf{K},\Omega) + \varepsilon_p(\mathbf{K},\Omega), \tag{10}$$

where $\varepsilon_0(\mathbf{K},\Omega)$ is the linear part given by

$$\varepsilon_0(\mathbf{K},\Omega) = 1 + \frac{\omega_{pe}^2}{K}\int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - Kv_\parallel}\frac{\partial}{\partial v_\parallel}f_{oe}, \tag{11}$$

$\varepsilon_d(\mathbf{K},\Omega)$ is the direct mode coupling term given by

$$\begin{aligned}\varepsilon_d(\mathbf{K},\Omega) = {} & -\frac{\omega_{pe}^2}{K}\left(\frac{e}{m}\right)^2\sum_{\mathbf{k}}|E_{l\parallel}(\mathbf{k})|^2\sum_{a=-\infty}^{\infty}\int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - Kv_\parallel} \\ & \times\left[\frac{\partial}{\partial v_\parallel} + \frac{a\Omega_e}{v_\perp\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]\frac{J_a^2(k_\perp v_\perp/\Omega_e)}{\Omega - \omega - (K - k_\parallel)v_\parallel + a\Omega_e} \\ & \times\frac{\partial}{\partial v_\parallel}\frac{1}{k_\parallel v_\parallel - \omega + i0}\frac{\partial}{\partial v_\parallel}f_{oe} - \frac{\omega_{pe}^2}{K}\left(\frac{e}{m}\right)^2\sum_{\mathbf{k}}[E_{l\perp}(\mathbf{k})\mathbf{E}_{l\parallel}(-\mathbf{k})] \\ & \times\sum_{a=-\infty}^{+\infty}\int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - Kv_\parallel}\left[\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega}\frac{\partial}{\partial v_\parallel}\right] \\ & \times\frac{J_a^2(k_\perp v_\perp/\Omega_e)}{\Omega - \omega - (K - k_\parallel)v_\parallel + a\Omega_e}\frac{a\Omega_e}{k_\perp v_\perp}\frac{\partial}{\partial v_\parallel}\frac{1}{k_\parallel v_\parallel - \omega + i0}\frac{\partial}{\partial v_\parallel}f_{oe}\end{aligned} \tag{12}$$

and $\varepsilon_p(\mathbf{K},\Omega)$ is the polarization mode coupling term given by

$$\begin{aligned}\varepsilon_p(\mathbf{K},\Omega) = {} & \frac{\omega_{pe}^2}{K}\left(\frac{e}{m}\right)^2\sum_{\mathbf{k}}\frac{\omega_{pe}^2(\Omega - \omega)}{R(\mathbf{K} - \mathbf{k})[(\Omega - \omega)^2 - c^2k_\perp^2]}[|E_{l\perp}(\mathbf{k})|^2(A\times B) \\ & + |E_{l\parallel}(\mathbf{k})|^2[(C + D)\times(F + G)] + [E_{l\perp}(\mathbf{k})E_{l\parallel}(-\mathbf{k})][A\times(F + G)] \\ & + [E_{l\parallel}(\mathbf{k})E_{l\perp}(-\mathbf{k})][(C + D)\times B]]\end{aligned} \tag{13}$$

where,

$$A = \int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - Kv_\parallel}\left[\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\frac{\partial}{\partial v_\perp} - \frac{k_\parallel v_\perp}{\omega}\frac{\partial}{\partial v_\parallel}\right]$$

$$\times \frac{J_a^2}{\Omega - \omega - (K - k_\parallel)v_\parallel + a\Omega_e} \frac{a\Omega_e}{k_\perp v_\perp} \left[\frac{\partial}{\partial v_\parallel} - \frac{a\Omega_e}{v_\perp(\Omega - \omega)} \times \left(v_\parallel \frac{\partial}{\partial v_\perp} - v_\perp \frac{\partial}{\partial v_\parallel} \right) \right] f_{oe}$$

$$B = \int 2\pi v_\perp v_\parallel \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{J_a^2}{\Omega - \omega - (K - k_\parallel)v_\parallel + a\Omega_e} \frac{a\Omega_e}{k_\perp v_\perp} \times \left[\left(1 - \frac{k_\parallel v_\parallel}{\omega} \right) \frac{\partial}{\partial v_\perp} - \frac{k_\parallel v_\perp}{\omega} \frac{\partial}{\partial v_\parallel} \right] \frac{1}{\Omega - K v_\parallel} \frac{\partial}{\partial v_\parallel} f_{oe}$$

$$C = \int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - K v_\parallel} \left[\frac{\partial}{\partial v_\parallel} + \frac{a\Omega_e}{v_\perp \omega} \left(v_\parallel \frac{\partial}{\partial v_\perp} - v_\perp \frac{\partial}{\partial v_\parallel} \right) \right] \times \frac{J_a^2(k_\perp v_\perp/\Omega_e)}{\Omega - \omega - (K - k_\parallel)v_\parallel + a\Omega_e} \left[\frac{\partial}{\partial v_\parallel} - \frac{a\Omega_e}{v_\perp(\Omega - \omega)} \left(v_\parallel \frac{\partial}{\partial v_\perp} - v_\perp \frac{\partial}{\partial v_\parallel} \right) \right] f_{oe}$$

$$D = \int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - K v_\parallel} \frac{\partial}{\partial v_\parallel} \frac{1}{\omega - k_\parallel v_\parallel + i0} \frac{\partial}{\partial v_\parallel} f_{oe}$$

$$F = \int 2\pi v_\perp v_\parallel \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{J_a^2(k_\perp v_\perp/\Omega_e)}{\Omega - \omega - (K - k_\parallel)v_\parallel + a\Omega_e} \left[\frac{\partial}{\partial v_\parallel} + \frac{a\Omega_e}{v_\perp \omega} \times \left(v_\parallel \frac{\partial}{\partial v_\perp} - v_\perp \frac{\partial}{\partial v_\parallel} \right) \right] \frac{1}{\Omega - K v_\parallel} \frac{\partial}{\partial v_\parallel} f_{oe}$$

$$G = \int 2\pi v_\perp v_\parallel \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{J_a^2(k_\perp v_\perp/\Omega_e)}{\Omega - \omega - (K - k_\parallel)v_\parallel + a\Omega_e} \frac{\partial}{\partial v_\parallel} \frac{1}{k_\parallel v_\parallel - \omega + i0} \frac{\partial}{\partial v_\parallel} f_{oe} \tag{14}$$

$$R(\mathbf{K} - \mathbf{k}) = 1 + \frac{\omega_{pe}^2(\Omega - \omega)}{(\Omega - \omega)^2 - c^2 k_\perp^2} \int 2\pi v_\perp v_\parallel \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{J_a^2(k_\perp v_\perp/\Omega_e)}{\Omega - \omega - (K - k_\parallel)v_\parallel + a\Omega_e} \times \left[\frac{\partial}{\partial v_\parallel} + \frac{a\Omega_e}{k_\perp v_\perp} \left(v_\parallel \frac{\partial}{\partial v_\perp} - v_\perp \frac{\partial}{\partial v_\parallel} \right) \right] f_{oe}. \tag{15}$$

In evaluating (10), according to the linear response theory of a turbulent plasma, we keep linear terms with respect to μ up to $\mu\varepsilon^2$. The terms of order ε^2 and higher order give nonlinear frequency shift for kinetic Alfvén wave which can be neglected because $\Omega \gg \omega$.

3. Plasma-maser interaction

We consider the plasma-maser interaction between electron and kinetic Alfvén turbulence. The condition for the plasma-maser is $\omega = k_\parallel v_\parallel, \Omega > K v_\parallel$. We first estimate the linear part of the dielectric constant of Langmuir mode (11). This can be expressed as,

$$\varepsilon_0(\mathbf{K}, \Omega) \simeq 1 - \left(\frac{\omega_{pe}}{\Omega} \right)^2. \tag{16}$$

With $\varepsilon_0(\mathbf{K},\Omega)=0$, we get the linear dispersion relation for Langmuir mode. The growth rate of Langmuir mode is given by

$$\gamma(\mathbf{K},\Omega) = -\left.\frac{\mathrm{Im}\,\varepsilon_h(\mathbf{K},\Omega)}{\dfrac{\partial}{\partial\Omega}\varepsilon_0(\mathbf{K},\Omega)}\right|_{\Omega=\Omega_r}, \tag{17}$$

where Im shows that imaginary part of the dielectric constant and Ω_r is the real frequency of the Langmuir mode. We obtain from (16),

$$\frac{\partial}{\partial\Omega}\varepsilon_0(\mathbf{K},\Omega) \simeq 2\frac{\omega_{pe}^2}{\Omega^3} \simeq \frac{2}{\omega_{pe}}. \tag{18}$$

3.1 *Growth rate due to the direct coupling term*

As is well-known, the plasma-maser effect is particularly important for strongly magnetised plasma, $\Omega_e > \omega_{pe}$. Here we estimate the imaginary part of the direct mode coupling term, (12). In estimating (12) [and (13) as well] we keep contributions only for $a=0$, because for kinetic Alfvén wave, the finite Larmor radius effect for electrons is neglected [11], i.e., $k_\perp \rho_e \ll 1$. Furthermore, for strongly magnetized plasma considered here ($\Omega_e > \omega_{pe}$), the dominant contribution comes from $a=0$. The terms which are neglected in estimating (12) and (13) can be shown to be smaller by a factor of the order of $(\omega_{pe}/\Omega_e)^2$. Thus (12) reduces to,

$$\varepsilon_d(\mathbf{K},\Omega) = -\frac{\omega_{pe}^2}{K}\left(\frac{e}{m}\right)^2 \sum_{\mathbf{k}} |E_{l\parallel}(\mathbf{k})|^2 \int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - Kv_\parallel} \times \frac{\partial}{\partial v_\parallel}\frac{J_0^2}{\Omega-\omega-(K-k_\parallel)v_\parallel}\frac{\partial}{\partial v_\parallel}\frac{1}{k_\parallel v_\parallel - \omega + i0}\frac{\partial}{\partial v_\parallel} f_{oe}. \tag{19}$$

From (19) we evaluate the imaginary part as,

$$\mathrm{Im}\,\varepsilon_d(\mathbf{K},\Omega) = -b\sqrt{\pi}\sum_{\mathbf{k}} \frac{|E_{l\parallel}(\mathbf{k})|^2}{4\pi N T_e}\frac{3K}{|k_\parallel|}\frac{v_A^* - v_0}{v_e}\exp\left[-\left(\frac{v_A^*-v_0}{v_e}\right)^2\right]. \tag{20}$$

Here,

$$b = \int 2\pi v_\perp J_0^2 f_{oe}(v_\perp)\mathrm{d}v_\perp \tag{21}$$

Inserting (20) in (17), we obtain,

$$\frac{\gamma_d}{\omega_{pe}} = \frac{3}{2}b\sqrt{\pi}\sum_{\mathbf{k}} \frac{|E_{l\parallel}(\mathbf{k})|^2}{4\pi N T_e}\frac{K}{|k_\parallel|}\frac{v_A^* - v_0}{v_e}\exp\left[-\left(\frac{v_A^*-v_0}{v_0}\right)^2\right]. \tag{22}$$

The normalized turbulence energy of Alfvén wave (W_T) is given by

$$W_T = \sum_{\mathbf{k}} \frac{|B_{ly}|^2}{16\pi N T_e}, \tag{23}$$

where B_{ly} is the magnetic field of the kinetic Alfvén wave. Inserting the relation between B_{ly} and $E_{l\parallel}$, we obtain,

$$W_T = \sum_{\mathbf{k}} \frac{|E_{l\parallel}(\mathbf{k})|^2}{16\pi N T_e} \left(\frac{k_\perp}{k_\parallel}\right)^2 \left(\frac{c}{v_A^*}\right)^2 (Q+1)^2. \tag{24}$$

Here Q is related to the amplitude ratio of the electric field components of the kinetic Alfvén wave:

$$\frac{E_{l\parallel}(\mathbf{k})}{E_{l\perp}(\mathbf{k})} = -\frac{k_\parallel}{k_\perp}\frac{T_e}{T_i}[1 - I_0(\beta_i)\exp(-\beta_i)] \equiv \frac{k_\parallel}{k_\perp} Q^{-1}. \tag{25}$$

Here, T_e, T_i and I_0 are electron, ion temperatures and modified Bessel function respectively; $\beta_i = (k_\perp \rho_i)^2/2$ and ρ_i is ion gyroradius.

Thus in terms of normalized turbulence energy of the kinetic Alfvén wave, (22) reduces to,

$$\frac{\gamma_d}{\omega_{pe}} = 6b\sqrt{\pi}\sum_{\mathbf{k}} W_T \left(\frac{k_\parallel}{k_\perp}\right)^2 \left(\frac{v_A^*}{c}\right)^2 (Q+1)^{-2} \frac{K}{|k_\parallel|} \times \frac{v_A^* - v_0}{v_e} \exp\left[-\left(\frac{v_A^* - v_0}{v_e}\right)^2\right] \tag{26}$$

Thus, growth occurs $\gamma_d > 0$ for $\Omega/K < 0$ under the condition $v_0 > v_A^*$, in presence of kinetic Alfvén wave turbulence.

3.2 *Growth rate due to polarization coupling term*

Next, we estimate the growth rate from the polarization mode coupling term (13). This contribution to the growth rate vanishes only for unmagnetized system. For an open system, such as a magnetized plasma, the polarization mode coupling gives the dominant contribution in plasma-maser instability. After a lengthy, but straightforward calculations, we find that the dominant imaginary part contribution of (13) comes from $|E_{l\parallel}(\mathbf{k})|^2 \mathrm{Im}[(C+D)\times(F+G)]$. With $a = 0$, C, D, F and G reduces to [from (14)]

$$C = \int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - Kv_\parallel}\frac{\partial}{\partial v_\parallel}\frac{J_0^2}{\Omega - \omega - (K - k_\parallel)v_\parallel}\frac{\partial}{\partial v_\parallel} f_{oe}$$

$$D = \int 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{1}{\Omega - Kv_\parallel}\frac{\partial}{\partial v_\parallel}\frac{1}{\omega - k_\parallel v_\parallel + i0}\frac{\partial}{\partial v_\parallel} f_{oe}$$

$$F = \int 2\pi v_\perp v_\parallel \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{J_0^2}{\Omega - \omega - (K - k_\parallel)v_\parallel}\frac{\partial}{\partial v_\parallel}\frac{1}{\Omega - Kv_\parallel}\frac{\partial}{\partial v_\parallel} f_{oe}$$

$$G = \int 2\pi v_\perp v_\parallel \mathrm{d}v_\perp \mathrm{d}v_\parallel \frac{J_0^2}{\Omega - \omega - (K - k_\parallel)v_\parallel}\frac{\partial}{\partial v_\parallel}\frac{1}{k_\parallel v_\parallel - \omega + i0}\frac{\partial}{\partial v_\parallel} f_{oe}. \tag{27}$$

It is easy to show that,

$$\mathrm{Im}(C+D)\times(F+G)] = \frac{3(\Omega - \omega)}{\Omega^5}(1 - b^2)\frac{m\sqrt{\pi}}{T_e}\frac{K^2}{|k_\parallel|}\frac{v_A^* - v_0}{v_e} \times \exp\left[-\left(\frac{v_A^* - v_0}{. v_e}\right)^2\right]. \tag{28}$$

Next, we estimate $R(\mathbf{K}-\mathbf{k})$. For $\Omega > \omega$, (15) gives to the lowest order,

$$\frac{1}{R(\mathbf{K}-\mathbf{k})[(\Omega-\omega)^2 - c^2 k_\perp^2]} \approx \frac{1}{c^2 k_\perp^2}. \tag{29}$$

From (17), (28) and (29) we obtain the growth rate due to the polarization term as,

$$\frac{\gamma_p}{\omega_{pe}} = 12\sqrt{\pi}(1-b^2)\sum_{\mathbf{k}} W_T (Q+1)^{-2} \frac{K}{|k_\parallel|}\left(\frac{k_\parallel}{k_\perp}\right)^2 \left(\frac{k_e}{k_\perp}\right)^2 \left(\frac{v_e}{c}\right)^2 \times \left(\frac{v_A^*}{c}\right)^2 \left(\frac{v_A^* - v_0}{v_e}\right) \exp\left[-\left(\frac{v_A^* - v_0}{v_e}\right)^2\right] \tag{30}$$

for strongly magnetized plasma. Here W_T is the normalized turbulence energy of the kinetic Alfvén wave [eq. (24)], and k_e is the electron Debye wave number. Since $b \ll 1$, for $k_\perp \rho_e \ll 1$, $1-b^2$ is always a positive quantity. Accordingly, a Langmuir wave with negative phase velocity ($\Omega/K < 0$) always grows in the presence of kinetic Alfvén wave turbulence with $v_0 > v_A^*$. Equations (26) and (30) are the main results of our investigation.

It is straightforward to compare the magnitudes of the two growth rates, equations (26) and (30):

$$R = \frac{\gamma_p}{\gamma_d} = 2\frac{1-b^2}{b}\left(\frac{k_e}{k_\perp}\right)^2 \left(\frac{v_e}{c}\right)^2.$$

For typical plasma parameters $k_\perp \rho_e = 0{\cdot}5$, $k_e/k_\perp = 10^3$, $v_e/c = 0{\cdot}1$, we find that $R \gg 1$, and therefore, we conclude that the polarization term gives the main destabilizing effect in the plasma-maser interaction. As an illustration of our process, we estimate the dominant contribution to the growth rate. Taking typical plasma parameters and MHD data [6], $k_\perp/k_\parallel = 10$, $v_A^*/c = 0{\cdot}01$, $K/k_\parallel = 10^2$ and $Q = 1{\cdot}5$, we obtain $\gamma_p/\omega_{pe} \approx 10^{-1} W_T$.

4. Discussion

The stability of stationary kinetic Alfvén wave turbulence driven by a weak electron beam in an open plasma system against a test high frequency nonresonant wave is studied. It is shown that the growth of the high frequency wave occurs through the plasma-maser interaction between beam electrons and the kinetic Alfvén wave. The growth originates from two processes, viz., direct and polarization mode coupling. In contrast to the ion acoustic turbulence case [1], where $\gamma/\omega_{pe} \approx (m/M)^{1/2}(kK/k_e^2)$ $W_T \ll W_T$, the growth rate with MHD wave turbulence is large, because here $\gamma/\omega_{pe} \approx 10^{-1} W_T$. Accordingly, the plasma-maser interaction of Langmuir wave with kinetic Alfvén wave turbulence considered in this paper has a large growth rate in contrast to earlier results based on electrostatic turbulence.

The plasma-maser is one of the three mode–mode coupling processes in plasmas, the other two being the three-wave resonant interaction and the nonlinear Landau interaction. These belong to the lowest order mode-coupling processes, and potentially give same order contribution to the growth rate. Following the prediction of the plasma-maser effect, the reverse absorption process was sought for some time.

Recently, it has been pointed out that plasma-maser emission, which comes from the imaginary part of the nonlinear dielectric constant, $\mathrm{Im}\,\varepsilon_h(\mathbf{K},\Omega)$, is accompanied by absorption due to the quasilinear effect, $\partial^2\varepsilon_0(\mathbf{K},\Omega,f_{oe}(t))/2\partial\Omega\partial t$. For closed system there is exact cancellation between the above two effects [3]. In contrast, plasma-maser is effective in open system where energy and momentum sources and sinks are available. Furthermore, for a magnetized plasma with an external magnetic field, cancellation between emission and absorption processes does not occur, since the presence of the magnetic field is equivalent to externally driven rotation. From this point of view the magnetized plasma is an open system.

Here we comment on the direct and polarization term contributions to the growth rate in the plasma-maser process in unmagnetized and magnetized plasmas. It was expected that plasma-maser process depends on the nature of polarization of the interacting waves in addition to the external environment of the system, and the polarization term was shown to contribute to the process even for unmagnetized plasma [12]. But the mistake was clarified in a recent communication [13]. This paper shows that for unmagnetized plasma ($b = 1$), polarization contribution to the growth rate vanishes [see (30)]. In other words, irrespective of the fact that whether the interacting waves are longitudinal or transverse, only the direct coupling term contributes to the growth rate for unmagnetized plasma. Accordingly, the energy (momentum) source of the process lies in the external environment only. Recently, Nambu *et al* [4] examined the transition from magnetized to unmagnetized plasma and showed that for pure nonresonant test wave $(\mathbf{K},\Omega)$ the polarization coupling term due to plasma-maser vanishes for unmagnetized plasma.

The kinetic Alfvén wave was introduced by Hasegawa and Chen in relation to plasma heating [14]. It can be excited by a MHD surface wave through a resonant mode conversion process and is shown to cause electron acceleration which may be applicable to aurora formation [15]. The kinetic Alfvén wave can also be generated by the drift wave instability in a plasma with β larger than the electron to ion mass ratio. Further, the kinetic Alfvén wave has a scale length of MHD waves in the direction parallel to the magnetic field and a scale length of ion gyroradius in the perpendicular direction. Hence the scale size of a plasma volume affected by the kinetic Alfvén wave is much larger than the scale sizes of plasma volumes affected by either electrostatic turbulence or cyclotron wave turbulence. Due to these reasons, the existence of such wave is considered to be an universal property of large scale plasmas [16].

In this work, we have considered the plasma-maser process including a test electrostatic wave for an open plasma system with a magnetic field. It is shown that kinetic Alfvén wave turbulence can radiate unstable Langmuir mode. The growth rate of the unstable Langmuir mode is given by (26) [direct coupling term] and (30) [polarization coupling term]. Our results and mechanism have potential importance in explaining electrostatic turbulences occurring in the magnetosheath, the generation of plasma wave in the earth's bowshock and numerous other astrophysical radiation processes. This point may be appreciated from the following observation. It is well known that for nonlinear scattering resonance with $a = 0$ $(\Omega - \omega - (K - k_\parallel)v_\parallel = 0)$, $\Omega \sim \omega_{pe} \gg \omega$ and $K \ll k_\parallel$, one finds $k_\parallel \lambda_{De} \approx 1$ and $\beta \approx m_e/m_i$. On the other hand, for plasma-maser, one gets $k_\parallel \sim \omega/v_A \ll \Omega_i/v_A = \omega_{pi}/c$. Since $\omega_{pi}/c \ll 1/\lambda_{De}$, the parallel wavelength which satisfies the plasma-maser is much larger than that of the nonlinear scattering resonance. This is the reason why plasma-maser including the kinetic Alfvén wave is important for cosmic plasmas. The primary condition necessary for this mechanism is the presence of a turbulence field from which energy can be transferred to the radiation field via resonant electrons.

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Appendix

Here we show a brief derivation of (10). The unperturbed orbits for electrons are,

$$\begin{aligned} x' &= x - \frac{v_\perp}{\Omega_e}\sin(\phi - \Omega_e\tau) + \frac{v_\perp}{\Omega_e}\sin\phi \\ y' &= y + \frac{v_\perp}{\Omega_e}\cos(\phi - \Omega_e\tau) - \frac{v_\perp}{\Omega_e}\cos\phi \\ z' &= z + v_\parallel \tau. \end{aligned} \tag{A1}$$

The Fourier component of the high frequency electron number density modification from (7) is,

$$\delta f_h(\mathbf{K},\Omega) = \frac{ie}{m}\frac{1}{\Omega - Kv_\parallel}\,\delta E_h(\mathbf{K},\Omega)\frac{\partial}{\partial v_\parallel}f_{oe} \tag{A2}$$

In a similar manner, we get from (8) and (9),

$$\begin{aligned} \delta f_{lh}(\mathbf{K}-\mathbf{k},\Omega-\omega) &= \frac{ie}{m}\sum_{a,b}\frac{J_b(k_\perp v_\perp/\Omega_e)\exp[i(a-b)\phi]}{(\Omega-\omega)-(K-k_\parallel)v_\parallel + a\Omega_e} \\ &\times\left\{E_{l\perp}(\mathbf{k}')\frac{(J_{a+1}+J_{a-1})}{2}\left[\frac{\partial}{\partial v_\perp} - \frac{k'_\parallel}{\omega'}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]\delta f_h(\mathbf{K}-\mathbf{k}-\mathbf{k}')\right. \\ &+ E_{l\parallel}(\mathbf{k}')\left[J_a\frac{\partial}{\partial v_\parallel} + \frac{(J_{a+1}+J_{a-1})}{2}\frac{k'_\perp}{\omega'}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]\delta f_h(\mathbf{K}-\mathbf{k}-\mathbf{k}') \\ &+ \delta E_{lh}(\mathbf{K}-\mathbf{k})\left[J_a\frac{\partial}{\partial v_\parallel} - \frac{(J_{a+1}+J_{a-1})}{2}\frac{k_\perp}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]f_{oe} \\ &\left.+ J_a\delta E_h(\mathbf{K}-\mathbf{k}-\mathbf{k}')\frac{\partial}{\partial v_\parallel}f_{1e}(\mathbf{k}')\right\} \end{aligned} \tag{A3}$$

$$\begin{aligned} \Delta f(\mathbf{K},\Omega) &= \left(\frac{ie}{m}\right)^2\sum_{a,b}\frac{1}{\Omega-Kv_\parallel}\left\langle\left\{E_{l\perp}(\mathbf{k})\frac{(J_{b+1}+J_{b-1})}{2}\right.\right. \\ &\times\left[\frac{\partial}{\partial v_\perp} - \frac{k_\parallel}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right] + E_{l\parallel}(\mathbf{k})\left[J_b\frac{\partial}{\partial v_\parallel}\right. \\ &\left.\left.+\frac{(J_{b+1}+J_{b-1})}{2}\frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]\right\}\frac{\exp[i(a-b)\phi]}{(\Omega-\omega)-(K-k_\parallel)v_\parallel + a\Omega_e} \end{aligned}$$

$$\times\left\{E_{l\perp}(\mathbf{k}')\frac{(J_{a+1}+J_{a-1})}{2}\left[\frac{\partial}{\partial v_\perp}-\frac{k'_\perp}{\omega'}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]\delta f_h(\mathbf{K}-\mathbf{k}-\mathbf{k}')\right.$$
$$+E_{l\parallel}(\mathbf{k}')\left[J_a\frac{\partial}{\partial v_\parallel}-\frac{(J_{a+1}+J_{a-1})}{2}\frac{k'_\perp}{\omega'}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]\delta f_h(\mathbf{K}-\mathbf{k}-\mathbf{k}')$$
$$+\delta E_{lh}(\mathbf{K}-\mathbf{k})\left[J_a\frac{\partial}{\partial v_\parallel}-\frac{(J_{a+1}+J_{a-1})}{2}\frac{k_\perp}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]f_{oe}$$
$$\left.+J_a\delta E_h(\mathbf{K}-\mathbf{k}-\mathbf{k}')\frac{\partial}{\partial v_\parallel}f_{1e}(\mathbf{k}')\right\}+\delta E_{lh}(\mathbf{K}-\mathbf{k})\left[\frac{\partial}{\partial v_\parallel}-\cos\phi\frac{k_\perp}{\Omega-\omega}\right.$$
$$\left.\left.\times\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]f_{1e}(\mathbf{k})\right\rangle \qquad \text{(A4)}$$

where $f_{1e}(\mathbf{k})$ is given by (5).

The Maxwell equation for mixed mode field can be written as,

$$\nabla\times\delta\mathbf{B}_{lh}(\mathbf{K}-\mathbf{k})=\frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{E}_{lh}(\mathbf{K}-\mathbf{k})-\frac{4\pi e}{c}\int\mathbf{v}\delta f_{lh}(\mathbf{K}-\mathbf{k})\mathrm{d}\mathbf{v}. \qquad \text{(A5)}$$

Accordingly, the Fourier component of the mixed mode electric field is given by,

$$\delta E_{lh}(\mathbf{K}-\mathbf{k})=\frac{4\pi ie(\Omega-\omega)}{(\Omega-\omega)^2-c^2k_\perp^2}\int v_\parallel\delta f_{lh}(\mathbf{K}-\mathbf{k})\mathrm{d}\mathbf{v} \qquad \text{(A6)}$$

Substituting (A3) into (A6), we get,

$$\delta E_{lh}(\mathbf{K}-\mathbf{k})=$$
$$-\frac{4\pi e^2(\Omega-\omega)}{mR(\mathbf{K}-\mathbf{k})[(\Omega-\omega)^2-c^2k_\perp^2]}\sum_{a,b}\int v_\parallel\frac{J_a(k_\perp v_\perp/\Omega_e)\exp[i(a-b)\phi]}{(\Omega-\omega)-(K-k_\parallel)v_\parallel+a\Omega_e}$$
$$\times\left\{E_{l\perp}(\mathbf{k}')\frac{(J_{a+1}+J_{a-1})}{2}\left[\frac{\partial}{\partial v_\perp}-\frac{k'_\parallel}{\omega'}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]\delta f_h(\mathbf{K}-\mathbf{k}-\mathbf{k}')\right.$$
$$+E_{l\parallel}(\mathbf{k}')\left[J_a\frac{\partial}{\partial v_\parallel}+\frac{(J_{a+1}+J_{a-1})}{2}\frac{k'_\perp}{\omega'}\left(v_\parallel\frac{\partial}{\partial v_\perp}-v_\perp\frac{\partial}{\partial v_\parallel}\right)\right]\delta f_h(\mathbf{K}-\mathbf{k}-\mathbf{k}')$$
$$\left.+J_a\delta E_h(\mathbf{K}-\mathbf{k}-\mathbf{k}')\frac{\partial}{\partial v_\parallel}f_{1e}(\mathbf{k}')\right\}\mathrm{d}\mathbf{v} \qquad \text{(A7)}$$

where $R(\mathbf{K}-\mathbf{k})$ is given by (15).

Finally, from (3), the Fourier component of the electric field of the Langmuir mode can be written as,

$$\delta E_h(\mathbf{K},\Omega)=-\frac{4\pi e}{K}\int[\delta f_h(\mathbf{K},\Omega)+\Delta f(\mathbf{K},\Omega)]\mathrm{d}\mathbf{v} \qquad \text{(A8)}$$

Substituting (A2), (A3), (A4) and (A7) into the rhs of (A8) and assuming random phase approximation for the kinetic Alfvén wave, and performing the integration with respect to ϕ (only $a=b$ terms do survive), we obtain (10).

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