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High pressure measurement with simple piston gauge in static condition

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Abstract. This paper describes the characterization of the newly developed piston gauge pressure standard at the National Physical Laboratory (NPL), with particular reference to its fall rate, engagement length of the piston and the deceleration rate for the measurement of hydraulic hydrostatic pressure up to 60 MPa. The low pressure effective area of the gauge derived from its dimensional measurements when compared with the value obtained by its direct calibration against NPL transfer pressure standard agrees within 0·025%. The pressure gauge is quite stable, reproducible and has a sensitivity of 3 ppm. Though the theoretically calculated value of the pressure coefficient is low as compared to the experimentally observed one in its absolute terms, the pressure dependent effective area agrees within ± 0·025% over whole of the pressure range which is well within the uncertainty statement of the two independent techniques used.

Keywords. Piston gauge; pressure measurement; pressure standard.

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1. Introduction

With the advancement of scientific and technological research and rapid industrialisation in the developing countries a need is felt to have a reliable, reproducible secondary pressure standard with long-term stability. The pressure can be measured by simple Bourdon gauges or the liquid column manometer. The use of the former is limited due to the large uncertainty in the pressure measured whereas the use of the latter is not common due to requirement of large column height. However, the pressure balance/piston gauges are widely used as primary pressure standard [1–3] over a wide range of pressure, from a few kilo pascals to several hundred mega pascal, with a relatively low uncertainty of the order of a few ppm in the measured pressure.

In piston gauges a cylindrical piston rotates in a closely fitted circular cylinder. The pressure at the base of the piston is defined as the ratio of the total downward force on the piston to the effective area (A_e) of the gauge when floating at its operating level. The accuracy of measuring pressure using these gauges depends on the accuracy of both force and pressure dependent effective area measurement. The pressure dependent effective area is most often attributed to the elastic distortion of the piston-cylinder assembly. At low pressures (barometric pressure region) the changes in the effective area caused by the elastic deformation of piston-cylinder assembly are very small and are usually negligible [3]. The effective area of simple geometry large diameter piston gauges can be calculated analytically by the dimensional measurement and thermo-mechanical properties of the piston-cylinder assembly [1, 4, 5]. At higher pressure besides the dimensional uncertainty there is an additional uncertainty in determining the effective area of a piston gauge due to the relatively large elastic distortion of the piston-cylinder assembly. As the high pressure pistons and cylinders, in general have relatively small diameters, a given dimensional uncertainty results in larger uncertainty in both the low pressure effective area and the elastic distortion coefficient of the gauge. The estimated accuracy of the pressure measured by the gauge is mainly limited by how precisely the pressure distortion coefficient of the piston gauge can be determined at least for gauges of the simple design in which the pressurising fluid acts only on the inside (and sometimes end) surface of the cylinder. In gauges of the re-entrant design the pressurising fluid acts over some or all of the outer surface of the cylinder as well. In such cases the ability to measure or predict the overall distortion coefficient of the piston and cylinder becomes quite complicated [2].

Different ways of dealing with the pressure coefficient and hence the pressure dependent effective area have led to various piston gauge designs, various materials from which the piston cylinders are made and various pressure ranges for which they are to be used. More recently, we [6] have shown an agreement of 5 ppm in the effective area values of the piston gauge of 28 MPa full scale pressure, obtained from the standard gauge used either in the primary controlled clearance, or simple mode. The use of the controlled clearance piston gauge is quite complicated, time consuming and it is an expensive piece of equipment compared to the simple piston gauge. Simple piston gauge is easy to use and one needs only a few operating parameters to calculate pressure which minimises errors. It is a much preferred method to measure pressure up to about 500 MPa.

With this aim in view a piston gauge to be used as transfer pressure standard/master standard in industrial organisations up to 60 MPa in hydraulic hydrostatic pressure region was designed and fabricated at NPL. The systematic study of its metrological characterisation, in particular, the parameters like the best piston position, fall rates, and rotation speed etc. have been made and the results of these studies are reported here. Though the results presented here relate to one of the four such gauges developed at NPL, they are representative of all the four gauges.

2. Theory

The pressure generated at the reference level of a piston gauge is given by Heydemann and Welch [1]

$$P = \left[\sum_{i=1}^{n} M_i(1 - \rho_{\mathrm{air}}/\rho_i)g + \gamma c \right] \Big/ (A_e(\rho)) \tag{1}$$

where M_i is mass of the ith weight, ρ_{air} is the density of air in the vicinity of the piston, ρ_i is the density of the ith weight, g is the local acceleration due to gravity, γ is the surface tension of the operating fluid, c is the circumference of the piston and A_e is the effective area of the gauge.

The effective area A_e of a controlled clearance piston gauge is typically expressed as [7].

$$A_e = \pi r_p^2(1 + b_p P)[1 + d(P_z - P_j)][1 + (\alpha_p + \alpha_c)(T - T_r)] \tag{2}$$

where r_p is the piston radius for zero applied pressure, b_p is the pressure distortion coefficient of the piston, P is the pressure at the reference level of the piston, alpha p, c are the thermal expansion coefficients of the piston (p) and cylinder (c). T is the absolute temperature of the piston and the cylinder, T_r is the absolute reference temperature, d is the jacket pressure coefficient, P_z is the zero clearance jacket pressure, P_j is the operating jacket pressure when dealing with the controlled clearance primary pressure standard. These parameters can either be measured or determined from the knowledge of the mechanical properties of the material which the piston and cylinder are made of. In practice r_p, t, d, P_z and P_j are usually measured, b_p is calculated, alpha p, c are usually obtained from the manufacturer without the need of any other more accurate pressure standard.

If the same controlled clearance piston gauge is used in the simple mode then the applied jacket pressure P_j in eq. (2) becomes zero. It is also possible to derive an independent expression for the effective area of the simple piston gauge

$$A_e = \pi r_p^2(1 + H/r_p)(1 + b_{p/c}P) \times [1 + (\alpha_p + \alpha_c)(T - T_r)] \tag{3}$$

where H is the gap within the piston and cylinder for low applied pressure. In practice it is very difficult to measure H dimensionally. The simplest way is to determine the area of the piston and cylinder to an accuracy of 1 ppm by dimensional metrology, whereas b_p/c, the pressure distortion coefficient of the piston-cylinder combination, can either be determined on the basis of elastic theory or experimentally by cross-floating the test gauge against another piston gauge the pressure dependent effective area of which is known to a fairly high accuracy. The theoretical determination of b_p/c based on elastic theory requires the measurement of outer and inner diameters of the cylinder with a fairly high accuracy which is difficult to obtain as discussed above. Moreover, the value of b_p/c calculated theoretically from elastic theory is based on a few approximations and these along with the larger uncertainty in the diameter measurement add a large amount of uncertainty in the estimation of b_p/c value theoretically. It would not be out of place to mention that generally the small diameter piston and cylinder are used to measure pressure beyond 10 MPa. In the case of small diameter piston-cylinder, experimental methods are generally used to obtain the pressure distortion coefficient.

However, in these studies the value of b_p/c has been determined by both these methods. The effective area of a piston gauge under test can be expressed [5] as

$$A_e = \{A_0(1 + b_p)[1 + (\alpha_p + \alpha_c)(T - T_r)]\}_T = \sum_{i=1}^{n} \frac{\left[M_i\left(1 - \frac{\rho_{air}}{\rho_{mi}}\right)_g\right]_T}{P_s + \Delta P} \tag{4}$$

P_s is the pressure at the reference level of the standard gauge, Δp is the head correction $(\rho_f - \rho_{air})\,gh$, where h is the height difference between the reference levels of the two gauges and ρf is the density of the pressure transmitting fluid, Δp can be positive or negative depending on whether the reference level of the standard is lower or higher than that of the test gauge.

3. Experiment

The schematic representation of the NPL-designed piston gauge pressure standard used in these studies, designated as NPL-60, is equipped with a piston-cylinder device of a simple geometry and capable of measuring/generating full scale pressure up to 60 MPa, (see figure 1). The pressure measurement range can be extended upto 100 MPa by using different piston-cylinder assembly. The piston is rotated manually to relieve friction. The piston of die steel and the cylinder of special steel are hardened to Rockwell C 60. The nominal area of the piston is 1·64 E-5 meter square. Two NPL transfer and secondary pressure standards i.e. NPL-100 and NPL-140 used during these studies have the total estimated uncertainty of 74 ppm at 3 sigma level to the full scale pressure of 100 and 140 MPa, respectively. The characteristics of these standards are well documented during the international intercomparison exercise carried out by the high pressure working group of BIPM in hydraulic hydrostatic pressure region up to 100 MPa [8]. The metrological characteristics of these two standards are given in table 1 alongwith the parameters associated with the new NPL-60 standard as derived during these studies.

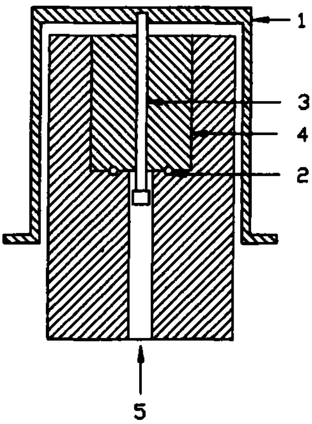

Figure 1. Schematic cross-sectional view of the piston and cylinder assembly for the pressure standard (NPL-60).
(1) Weight loading table; (2) "O" ring: (3) piston (4) cylinder; (5) pressure input.

Table 1. Description of the piston gauges used.

Parameter	NPL-140	NPL-100	NPL-60 (Test)
Range (MPa)	0·24 – 240	0·2 – 100	5 – 60
Material of piston and cylinder	cemented tungsten carbide	tungsten carbide	Steel
Effective area at zero applied pressure and at ref. temperature ($E\text{-}5\,m^2$)	1·680244	9·80487	1·646127
Coefficient of thermal expansion of piston and cylinder ($E\text{-}6\,c^{-1}$)	9·1	9·0	23·0
Distortion coefficient ($E\text{-}12\,Pa^{-1}$)	2·76	0·3	4·3
Estimated total uncertainty of the effective area (3 sigma) ppm.	73	74	110

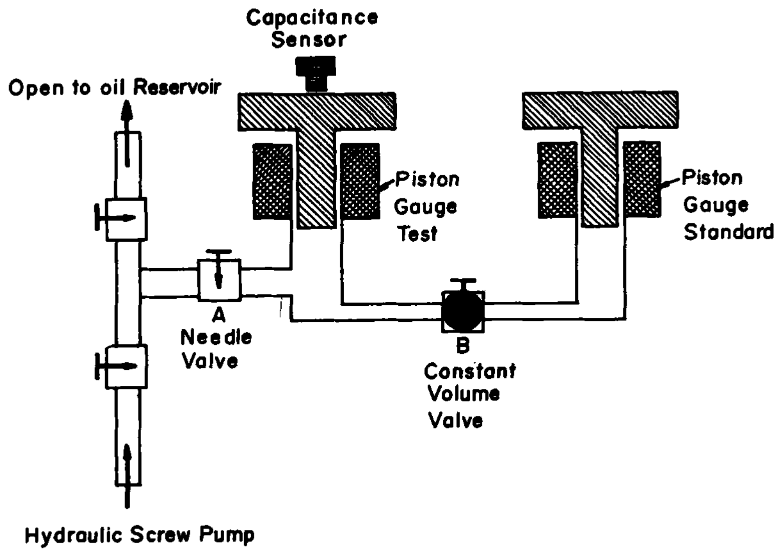

Figure 2. Experimental crossfloat arrangement between the test gauge and the standard piston gauge.

The piston gauges used during these studies were kept on heavy nonmagnetic stainless steel base in order to minimise vibrations and magnetic effects. The effective area of the NPL-60 piston gauge, hereafter designated as test gauge, was determined by direct calibration using the well-known crossfloat technique [1, 2, 7]. A schematic experimental arrangement for the crossfloat of the piston gauges is shown in figure 2. It comprises a manually operated needle valve and the hydraulic screw pump to generate the desired pressure and a constant volume valve B to isolate the piston gauges from one another. The bare minimum length of all the connecting pressure lines is used to have a better crossfloat sensitivity. Before the crossfloat both piston gauges were levelled to ensure the verticality of the axis and the system was checked for leaks to the full scale pressure of 60 MPa. The piston gauges were loaded with the weights calculated to generate the desired pressure and then pressurised to float at the reference level. They were then isolated from rest of the pressurising system by closing valve A and subsequently from each other by closing valve B.

The fall rate of the test gauge, the area of which was being evaluated was measured using a capacitance type linear voltage displacement transducer to avoid any extraneous force on the piston. The output of the transducer was directly recorded on a X-*T* chart recorder moving at a constant speed of 6 cm/min which monitored both the position and the fall rate of the piston. The change in piston position is known with a resolution better than 5×10^{-3} mm and the time interval is determined better than 0·2 s.

All the measurements were made in an environment which provided stable temperature conditions of 23 ± 1 c. The temperature of the NPL-140 pressure standard was measured within 0·1 c by a mercury-in-glass thermometer placed near the pressure column. The temperature of the NPL-100 pressure standard was measured with a PRT attached just near the piston and its output was read with an auto ranging digital multimeter having a resolution of 2 mΩ corresponding to a temperature resolution of 0·005 c. The temperature of the test gauge was measured with a PRT attached to the pressure column and the output of the PRT is read on the digital display having a resolution corresponding to 0·01 c.

By adjusting fractional weights on the test gauge which was generating comparatively lower pressure, crossfloat equilibrium was achieved as determined when the test gauge had the same fall rate irrespective of whether the valve B was closed or opened. The pressure was then raised and the same procedure was repeated, up to the full scale pressure of 60 MPa. A period of 30 min between the two successive observations was found adequate to allow the system to return to equilibrium and about 10 min was required to repeat observations at each pressure point.

As it was not possible to bring the reference level of individual piston gauges to the same operating level during crossfloat, a pressure head correction term was applied to compensate for the difference in the operating levels. The reference level of NPL-60 was lower by 0·050 m against NPL-100 whereas it was 0·105 m against NPL-140 standard.

Throughout these studies, the piston of the NPL-60 was manually rotated clockwise so as to maintain the piston rotational speed of 25 ± 10 rpm. The NPL-100 and NPL-140 pressure standards were rotated anti-clockwise with a rotational speed of 25 ± 5 rpm by a pulley system coupled to a dc motor.

4. Results and discussion

The performance of the gauge is evaluated by measuring its deceleration rate and sink rate at different applied pressures. It is desirable to have small sink rate to get high accuracy during the crossfloat equilibrium of the two gauges. Moreover, it helps to trace out the leakage in the system, if any, and also the deterioration of the piston and cylinder surfaces over a period of time. During these studies the manually rotated piston has sufficiently large deceleration rate (figure 3) owing to large diameter masses used. Typically, the sink rate of 4·6 E-4 m/min at 5 MPa is observed during

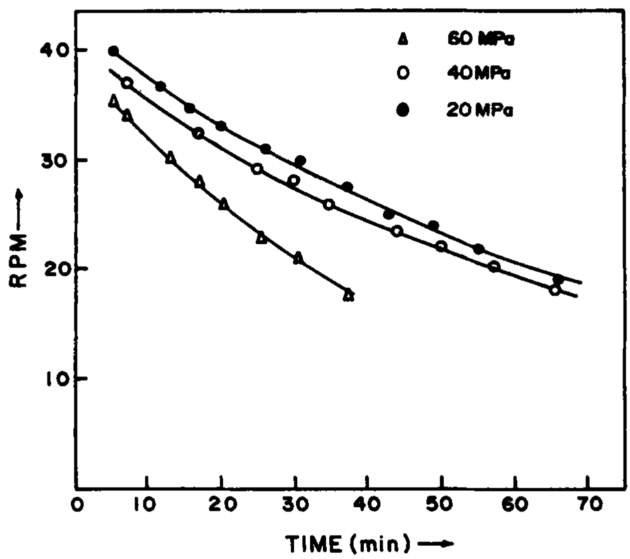

Figure 3. Deceleration rate of the piston with time operating at different applied pressures.

these studies which increases to 9·2 E-4 m/min at its full scale pressure value of 60 MPa which is sufficiently high for pressure measurements. The sink rates of the gauge were also measured at different engagement length of the piston and at different applied pressure. In all the cases the sink rates were well within the measurement uncertainty i.e. 2·5%.

Dimensional metrology has been used to characterise the test gauge (NPL-60). For calculating its effective area the model of Meyers and Jessup [9] has been used which assumes the piston cylinder as merely straight and free from taper, and there is no dependence of viscous drag on fluid viscosity. In practice, since the piston cylinder never posseses perfect geometry one could follow the method developed by Dadson *et al* [10] to derive the effective area of the gauge by simply averaging the diameter measurements. In these studies, the effective area was calculated by simply averaging the dimensional measurements taken at different points in a plane perpendicular to the piston-cylinder axis and in different plane along the axis. All the measurements were made on CMM machine (Model UPMC 850 CARAT, Carl Zeiss) having an uncertainty of 1 μm in the range used here. The diameter was measured at approximately five locations along the length of piston and cylinder. The measurements were repeated after the piston and cylinder had been rotated by 90 degrees. The straightness measurements of the piston as shown in figure 4 were taken in auto mode whereas the same could not be done for the inner diameter of the cylinder due to the limitations of the probe size. These measurements were made along the entire length of piston and cylinder at four different positions along the circumference and were repeated at least twice. The straightness and roundness measurements of both the pieces were made throughout its length of 40 mm and were within two micron. The diameter measurements of the piston and the cylinder were made and averaged. These values were then used as discussed above to calculate the effective area and the elastic distortion coefficient [1, 7] of the piston-cylinder assembly. Combining the elastic distortion 3·47 E-12/Pa, with the low pressure effective area 1·646863 E-5 sq meter as calculated, and taking into account the uncertainty due to roundness and straightness, the pressure dependent effective area of NPL-60 has an uncertainty of 200 ppm at its full scale pressure value of 60 MPa.

Alternatively, the effective area of the test gauge was determined by calibrating it against NPL standard gauges NPL-100 and NPL-140 using a well-known crossfloat

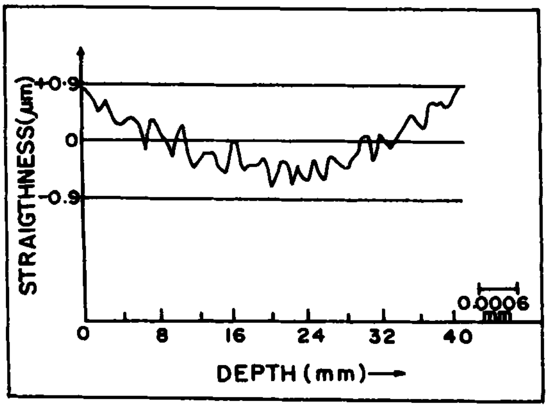

Figure 4. Dimensional measurement of straightness of the piston along its length of 40 mm. (Magnification 16300).

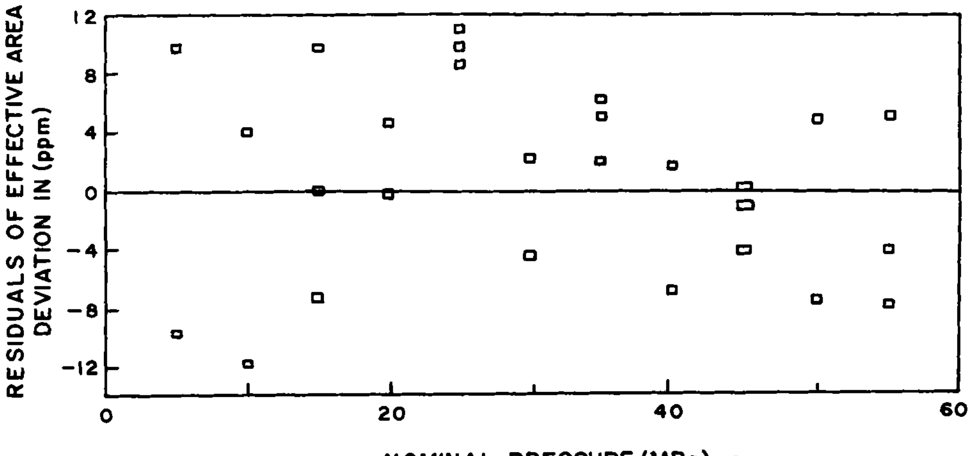

Figure 5. Deviation of measured values of the effective area of NPL-60 from the best linear fit $A_e = 1{\cdot}647127$ E-5 (1 + 4·3 E-12 P) when calibrated by NPL-100.

technique. A computer programme [11] that determines the effective area and the pressure coefficient of the test gauge based upon those of the standard was used to analyse the data. This program is based upon equations (1–4) above and also provides the standard deviation of the residuals of the area and the standard deviation of the coefficients.

In all the calibrations of the test gauge against the NPL-100 pressure standards, three test cycles were carried out. In one cycle the pressure was increased to 55 MPa and then decreased from 55 to 5 MPa and in the other two cycles the measurement proceeded from the highest pressure to the lowest and back to the highest. In one test cycle 11 observations were taken leading to a total of 33 observations for complete calibration.

Figure 5 shows a plot of residuals in the effective area of NPL-60 as a function of nominal applied pressure, when NPL-60 is crossfloated against NPL-100. This figure gives the deviation of the measured values of the effective area, in ppm, for the individual measured pressures, from the best low order fit equation $A_e = A_0(1 + b_p)$ where $A_0 = 1{\cdot}647127$ E-5 and $b = 4{\cdot}3$ E-12 and P is in Pa. The residuals of the effective area of the NPL-60 pressure standard are randomly distributed within $\pm$ 12 ppm. It may be concluded that the effective area of NPL-60 gauge has an uncertainty of 110 ppm (3 sigma) over the full scale pressure of 60 MPa. These uncertainties represent the combined systematic uncertainty of NPL-100 (74 ppm 3 sigma) with the random uncertainty of the individual calibrations.

Due to the relatively large difference in the values of b of NPL-60 obtained experimentally and by calculation from elastic theory, it is desirable to check the consistency of the effective area of NPL-60 obtained by these two independent methods. The NPL-60 was again calibrated against other NPL secondary standard which in turn has the tracability to NPL primary standard and has also been used in BIPM international intercomparison exercise up to 100 MPa [8]. This would show the existence of a relative systematic error if any, which would in turn increase the confidence in the estimated uncertainties of the test gauge.

NPL-60 was compared with NPL-140 secondary transfer pressure standard at 11 different standard pressures for a total of 33 observations. The deviations in the

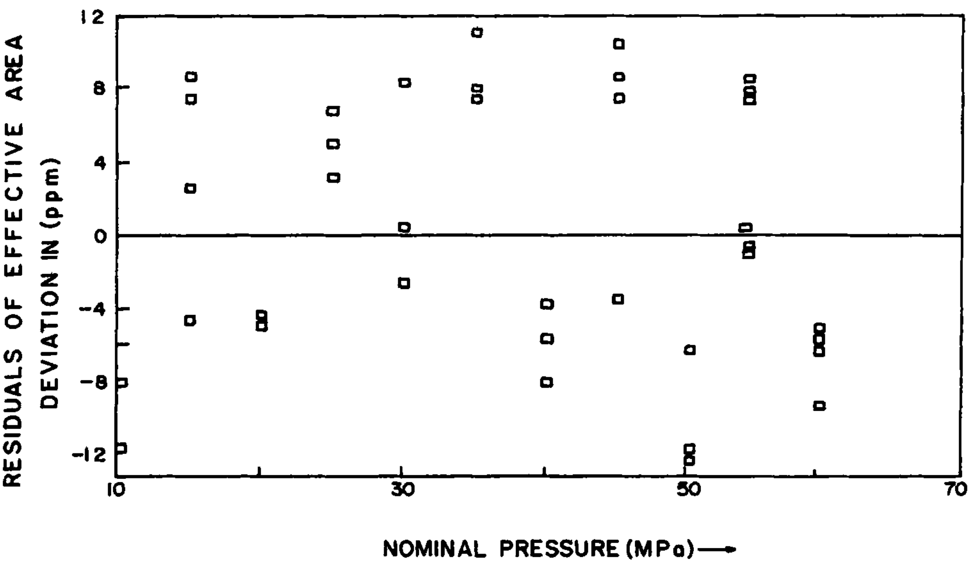

Figure 6. Deviation of measured values of the effective area of NPL-60 from the best linear fit $A_e = 1{\cdot}647188$ E-5 (1 + 4·4 E-12 P) when calibrated by NPL-140.

effective area, in ppm, for the individual measured pressures, from the linear best fit equation, $A_e = 1{\cdot}647188$ E-5 (1 + 4·43 E-12) where A_e is in m^2 and P is in pascal, are shown in figure 6. Similar scatter within ± 12 ppm in the residuals (observed minus calculated) of the effective area of NPL-60 is observed when it was calibrated against NPL-100 (figure 5) except in the low pressure region. The values of low pressure effective area (A_e) of NPL-60 as obtained by crossfloating against NPL-100 differ by 37 ppm from the values obtained against its calibration by NPL-140. Though the 37 ppm difference is outside the 3 sigma standard deviation of A_0 coefficient but considering the uncertainty associated with individual standards (table 1), this is well within the uncertainty statement of the standards. Further, b value of NPL-60 obtained from NPL-100 as compared to that obtained during the crossfloat against NPL-140 differ by 0·13 E-12/Pa, in absolute term, considering the 3 sigma standard deviation of these measured values i.e., 0·23 E-12/Pa and 0·24 E-12/Pa, respectively, the difference is not unreasonable. Additionally, this observed difference in b causes a relative difference in the effective area value by less than 1 ppm at a measured pressure of 5 MPa which increases to 7 ppm at its full scale pressure of 60 MPa (figure 7). The overall difference in A_e value of NPL-60 obtained by two independent calibrations against NPL-100 and NPL-140 is 37 ppm at 5 MPa increased to 43 ppm at 60 MPa is mainly dominated due to the basic difference of 37 ppm observed in its low pressure effective area value obtained by two independent calibrations. These results are hence within the measurement uncertainties associated with the individual transfer standard (table 1).

To have a check on the consistency in the value of the effective area of NPL-60, the crossfloat data have also been evaluated with reference to pressure generated by the two gauges (NPL-60 and NPL-140). Using (4) the pressure measured simultaneously by these two gauges at a specified reference level were calculated from 33 direct crossfloat intercomparison discussed earlier. The fractional pressure difference is

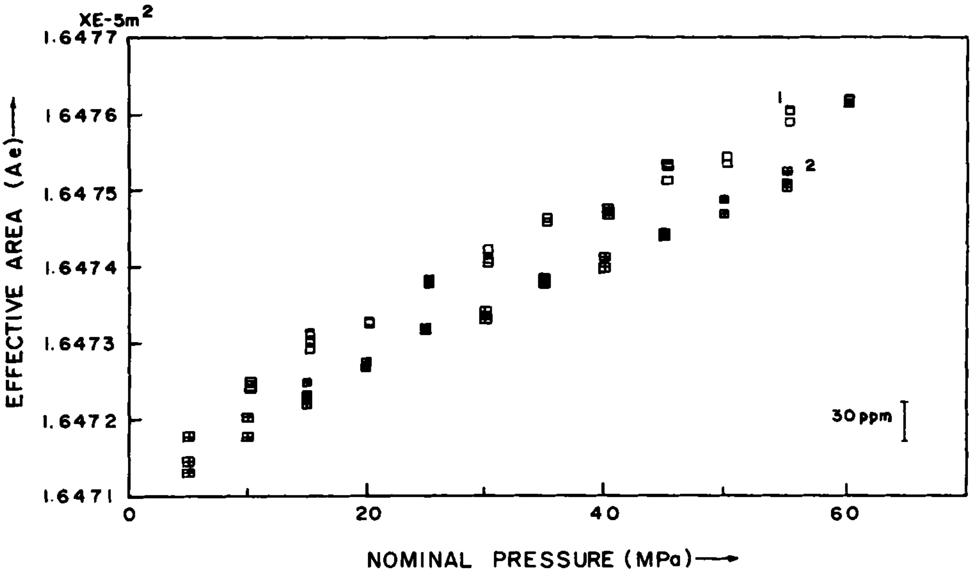

Figure 7. Effective area A_e of NPL-60 obtained by direct crossfloating against NPL-140 (1) and by crossfloating against NPL-100(2).

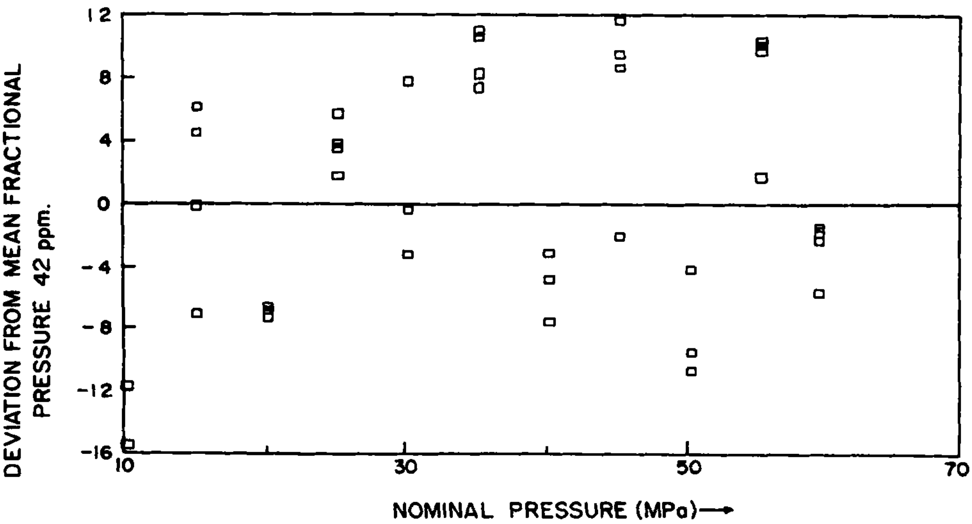

Figure 8. Fractional pressure differences from the mean value of 42 ppm between pressure measurements with the NPL-60 and NPL-140 pressure standard as a function of nominal applied pressure.

defined as

$$\frac{\Delta P}{P} = \frac{P_{P60} - P_{P140}}{P_{P140}}$$

where $P60$ is the applied pressure as measured by NPL-60 and $P140$ by NPL-140 transfer pressure standard. Figure 8 shows the deviation from the mean value of

42 ppm of the fractional pressure difference $\Delta P/P$ as a function of the nominal applied pressure. The distribution of the fractional pressure difference along its mean value is random and the overall behavior of the fractional pressure difference as expected is almost the same as that of the residuals of the effective area of the NPL-60 obtained during its calibration either against NPL-100 or NPL-140. So it is considered here that there is no significant discrepancy in the estimation of the pressure distortion of NPL-60 as determined by two different ways i.e. calibration against NPL standard and from elastic theory as discussed earlier. Though the systematic difference of 42 ppm exists throughout the pressure range: it is relatively small compared to the estimated uncertainty in the effective area of the standards used (table 1). The uncertainty in the measurement of pressure using a piston gauge arises from two main sources: (1) inherent uncertainties associated with the gauge itself and (2) other uncertainties associated with the local experimental conditions. The former is mainly attributed to the determination of the effective area of the piston-cylinder assembly and uncertainties in the mass of the load and/or piston. However the latter arises from the experimental procedures including (i) uncertainty associated with the measurement of temperature, (ii) correction due to any difference of reference levels and (iii) the resolution of the balancing criteria when the two systems are in equilibrium.

During the crossfloat of NPL-60 either against NPL-100 or NPL-140 fractional masses were so adjusted as not to contribute more than 1·2 ppm uncertainty at the minimum pressure of 5 MPa, which reduces to less than 1 ppm at the full scale pressure of 60 MPa. As the reference levels were measured to an estimated accuracy of 5 E–4 m and temperature was read with an estimated accuracy of $\pm 0{\cdot}1\,^{\circ}\mathrm{C}$, the contribution to the total estimated uncertainties in effective area due to temperature and difference in reference level is not significant compared to the total uncertainties associated with the standards.

5. Conclusion

1. Both methods of expressing the results of the measurements namely the comparison of simultaneous pressure measurement to a common reference level and the comparison of the effective area of NPL-60 as obtained by NPL-100 and NPL-140 standards show an agreement which is significantly better than that of the estimated total uncertainty of the two standards.

2. The NPL-60 standard can be used as a pressure standard with confidence where the accuracy requirement in pressure measurement is not more than 0·03%.

3. The larger uncertainty in the effective area of NPL-60 determined on the basis of the dimensional measurements is due to the fact that diameter measurements could not be performed with better accuracy than that used here.

4. These uncertainty values can further be improved by reducing the uncertainty in the effective area of NPL-100 and/or NPL-140 pressure standards or by making the dimensional measurements of NPL-60 with better accuracy.

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References

[1] P L M Heydemann and B E Welch, *Experimental thermodynamics* edited by B LeNeindre and B Vodar (Guildford: Butterworth 1975) vol. II 147
[2] J K N Sharma and K K Jain, *Pramana – J. Phys.* **27**, 417 (1986)
[3] J K N Sharma, K K Jain and A K Bandyopadhyay, *Rev. Sci. Instrum.* **59**, 2063 (1988)
[4] R S Dadson, S L Lewis and G N Peggs, *The pressure balance: theory and practice* (National Physical Laboratory, Teddington, England HMSO) (1982)
[5] J K N Sharma, K K Jain, C D Ehrlich, J C Houck and D B Ward, *J. Res. Natl. Bur. Stand.* (1993) (to be published)
[6] K Jain, C Ehrlich and J Houck, *Rev. Sci. Instrum.* **63**, 3127 (1992)
[7] K Jain, C Ehrlich, J Houck and J K N Sharma, *Meas. Sci. Technol.* **4**, 249 (1993)
[8] J C Legras, G F Molinar, K Schmalhofer, J K N Sharma, M Thrane and D B Walker, *BIPM Internal Report* 90/10 (1990)
[9] C H Meyers and R S Jessup, *J. Res. Natl. Bur. Stand.* **6**, 1061 (1931)
[10] R S Dadson, R G P Greig and A Horner, *Metrologia* **1**, 55 (1965)
[11] J C Houck, G F Molinar and R Menghenzani, *J. Res. Natl. Bur. Stand.* **88**, 253 (1983)