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Generation of ultrasonic waves in water by an elliptical Gaussian laser beam

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Abstract. The paper presents an analysis of ultrasonic wave generation in water by intensity modulated elliptical Gaussian laser beams. It is found that generated acoustic radiation is highly directional both in polar and azimuthal directions. An increase in the assymmetry of the transverse intensity distribution from the Gaussian dependence enhances the source directivity considerably. An obvious conclusion is that elliptical Gaussian beams are better choice in applications where it is desired to communicate in an approximately prescribed small solid angle.

Keywords. Thermal generation of sound; thermoacoustic source; far-field directivity pattern; directional acoustic sources.

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1. Introduction

The generation of sound in water by an intensity modulated laser beam has been a fascinating subject since last three decades as it opens up a way to communicate with submarines under water from air. Laser generated acoustic signals are currently being used in optoacoustic spectroscopy [1], in optoacoustic nondestructive testing [2], thermoacoustic medical diagnostics [3], scanning laser acoustic microscopy [4] etc.

There are several mechanisms through which optical energy is converted into the acoustic energy, but the thermal mechanism offers a great advantage over the others because it is easily controllable. This is the reason why it is being used in all the potential applications of laser acoustics. The physics of the process can be easily understood. When an intensity modulated laser beam passes through water it produces an exponentially shaded sound source along its direction of propagation. Since the laser beam is intensity modulated the heating of the medium results in a periodic sequence of condensations and rarefactions which propagate through the medium. The frequency of the acoustic wave is the same as the modulating frequency of the laser beam. Such sources of acoustic perturbation are known as thermoacoustic sources (TS).

Characteristics of TS generated by the absorption of intensity modulated CW laser beams by a water body have been studied by Westervelt and Larson [5], Lyamshev and Sedov [6], Berthelot and Busch-Vishniac [7]. Experimental verification of thermoacoustic radiation has been carried out by various workers [8, 9]. Excitation of acoustic pulses by pulsed laser and moving pulsed laser sources has also been investigated [10–15]. Readers are referred to Lyamshev and Sedov [6] for a review of some of the earlier work in this area.

A common feature of these investigations is the study of the effect of Gaussian intensity distribution in the laser beam on the directional characteristics of the emitted acoustic field. Though such a laser beam produces a highly directional acoustic array its directivity pattern is symmetric around the acoustic array. This symmetry of directivity pattern is a result of the symmetry of the intensity distribution of the laser beam around its axis. Naturally, laser beams with assymmetric intensity distribution will produce acoustic fields which will depend both on polar and azimuthal angles. Since a preferentially directed wave will have its energy confined within a relatively small solid angle, the range is enhanced in certain directions and as a result such sources would be favourable in comparison to symmetric sources for communication in a particular direction. Study of acoustic radiation by an assymmetric array is therefore pertinent.

In this note we have investigated directivity pattern of TS produced by intensity modulated elliptical Gaussian laser beams. Such beams are easy to obtain as for example by sending the circular Gaussian beam through a cylindrical lens [17] and are more suitable for long distance communication as comparatively they spread much less [18]. An expression for acoustic pressure in the far-field approximation is given in § 2. Section 3 provides main results of our study and discusses the benefits achievable by the use of elliptical Gaussian laser beams.

2. Analysis

Consider an intensity modulated laser beam propagating in the positive z-direction and incident normally on the free air-water surface at $z = 0$. We assume that the laser beam is of moderate power so that change of aggregate state of the medium does not take place in the laser absorption zone and hence, the sound generation mechanism is purely thermal in nature. Absorption of laser produces a thermoacoustic array as shown in figure 1. The sound pressure field p satisfies inhomogeneous Helmholtz equation [5]

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{c_p}\frac{\partial q}{\partial t} \tag{1}$$

where c is the velocity of sound in water, β is the coefficient of thermal expansion and c_p the specific heat at constant presure. q is the amount of heat added in water per unit volume per unit time. The required boundary condition for p may be written as

$$p = 0 \text{ at } z = 0 \tag{2}$$

Heat deposition q in the array region may be written as

$$q(x, y, z, t) = A\alpha I(x, y)\exp(-\alpha z)(1 + \mathrm{m}\cos\omega t) \tag{3}$$

where A is the transmittivity of laser light in water, α the attenuation coefficient, ω the modulation frequency and m is the modulation index $(0 < m < 1)$. For elliptical Gaussian laser beam I may be written as

$$I = I_0\exp[-(x^2/a^2 + y^2/b^2)] \tag{4}$$

where I_0 is the intensity on the axis and a and b $(a > b)$ are constants; $a = b$ represents a laser beam with Gaussian intensity distribution. In view of (3), (1) may be rewritten as

$$\nabla^2 p + k^2 p = \frac{i\omega m A\alpha\beta}{c_p} I(x, y) e^{-\alpha z} \tag{5}$$

where $k = \omega/c$. The time factor $\exp(-i\omega t)$ is implicit in (5) and only the real part of p is the physical solution.

The solution of (5) may be written as

$$p = -\frac{i\omega m A\alpha\beta}{c_p} \int G(\mathbf{R}|\mathbf{R}') I(x', y') \exp(-\alpha z') \mathrm{d}v' \tag{6}$$

where $G(\mathbf{R}|\mathbf{R}')$ is the Green's function of the homogeneous wave equation (eq. (5)). The Green's function which satisfies the boundary condition at $z = 0$ may be written as

$$G = \frac{1}{4\pi}\left[\frac{e^{ik}|\mathbf{R} - \mathbf{r}_1|}{|\mathbf{R} - \mathbf{r}_1|} - \frac{e^{ik}|\mathbf{R} - \mathbf{r}_2|}{|\mathbf{R} - \mathbf{r}_2|}\right]. \tag{7}$$

Since we are interested in the far field acoustic pattern the following approximations may be appropriate (see figure 1)

$$|\mathbf{R} - \mathbf{r}_1| \approx R_1(z') - r(x', y')\cos\alpha_1$$

$$|\mathbf{R} - \mathbf{r}_2| \approx R_2(-z') - r(x', y')\cos\alpha_2$$

where

$$R_1(z') = \sqrt{(x^2 + y^2 + (z - z')^2)}, \quad R_2(-z') = \sqrt{(x^2 + y^2 + (z + z')^2)}$$

$$r(x', y') = \sqrt{(x'^2 + y'^2)},$$

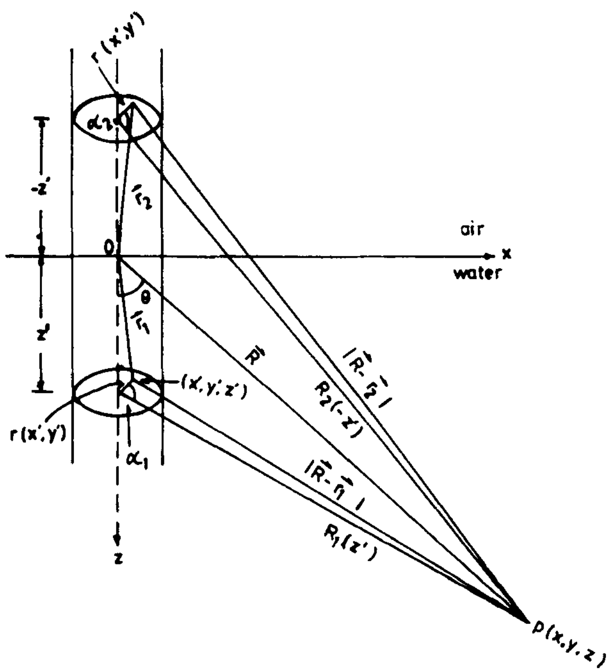

Figure 1. Schematics of thermoacoustic array.

α_1 and α_2 are respectively the angles which $\mathbf{R}_1(z')$ and $\mathbf{R}_2(-z')$ make with $\mathbf{r}(x',y')$. With these substitutions, (6) yields

$$p = -\frac{Am\beta}{2\pi c_p}\frac{e^{ikR}}{R}F(\theta,\Phi) \tag{8}$$

where the directivity pattern $F(\theta,\Phi)$ is given by

$$F(\theta,\Phi) = \frac{\pi\omega\alpha abI_0 k\cos\theta}{\alpha^2+k^2\cos^2\theta}\exp\left[-\frac{k^2a^2\sin^2\theta}{4}\left(\cos^2\Phi+\frac{b^2}{a^2}\sin^2\Phi\right)\right] \tag{9}$$

Φ is the angle between xz plane and the plane containing the field point and the z axis. It may be pointed out that setting $a=b$ (reducing to Gaussian beam) reduces the expression for $F(\theta,\Phi)$ to that obtained by Lyamshev [16] as it should. The directivity pattern normalized with the directivity pattern of the acoustic field produced by a Gaussian laser beam of equal power may be cast as

$$F_N(\theta,\Phi) = \exp\left[\frac{k^2a^2\sin^2\theta}{4}\left(b/a-\cos^2\Phi\frac{b^2}{a^2}-\sin^2\Phi\right)\right] \tag{10}$$

3. Discussion

The function $F(\theta,\Phi)$ of (9) represents the angular dependence of the pressure field at a large distance R from the point of incidence of an elliptic Gaussian laser beam incident normally on the water surface. As against this $F_N(\theta,\Phi)$ of (10) expresses the ratio of the fields obtained by two sources, one generated by an elliptic Gaussian beam and the other by a circular Gaussian beam at a point in the direction (θ,Φ) from the point of incidence of the laser beams at a large but equal distance R. The Φ dependence is naturally identical in the two functions. One easily obtains that $\partial F_N/\partial\Phi = \partial F/\partial\Phi = 0$ for the $\Phi = m\pi/2$ where m is an integer. It established that $F(\theta,\Phi)$ (as also $F_N(\theta,\Phi)$) acquires its maximum along $\Phi=\pi/2$ and $3\pi/2$, and minimum along $\Phi=0$ and π. In addition, towards $\Phi=\pi/2$ and $3\pi/2$, $F_N(\theta,\Phi)$ is always greater than one. Thus, one finds that the sound field of the source generated by the elliptic Gaussian beam has an enhanced intensity towards $\Phi=\pi/2$ and $3\pi/2$ directions.

The dependence on θ can also be analysed from (9). For this let us choose $\Phi=\pi/2$ or $3\pi/2$ the directions in which most of the sound is propagating. In that case $\partial F/\partial\theta=0$ demands

$$k^2\cos^2\theta = \left(\frac{1}{b^2}-\frac{\alpha^2}{2}\right) \pm \left[\left(\frac{1}{b^2}-\frac{\alpha^2}{2}\right)^2 - \frac{2\alpha^2}{b^2}\right]^{1/2}.$$

For $k^2\cos^2\theta$ to be real and positive we must have $\alpha^2b^2 < 2/3$. For lasers with small attenuation coefficients α and a reasonable size of the beam diameter this condition is invariably satisfied. For example for an Nd-glass laser $\alpha \approx 0{\cdot}17\,\text{cm}^{-1}$. If the radius of the beam is even 2 cm (b is always < 2 cm), we shall have $\alpha^2b^2 \approx 0{\cdot}1 < 2/3$. For such a system one can easily obtain for the maximum of $F(\theta,\Phi)$ an approximate relation $k^2\cos^2\theta \approx \alpha^2$, which for a sound wave of 60 kHz yields $\theta \approx 86{\cdot}1^\circ$.

Figure 2 shows angular plot of $F_N(\theta,\Phi)$ versus Φ for different values of a and b, the frequency of the generated acoustic wave is $f = 60$ kHz ($f=\omega/2\pi$) and the field point is located at a polar angle $\theta = 75^\circ$. For $f = 60$ kHz and $\theta = 75^\circ$, values of

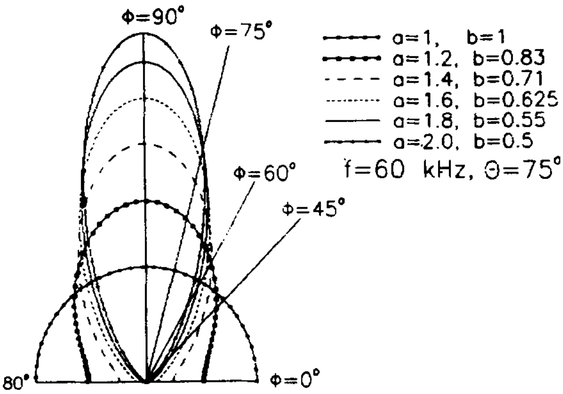

Figure 2. Angular plot of $F_N(\theta, \Phi)$ as a function of Φ.

Table 1. $F_N(\theta, \Phi)$ for different values of b/a and Φ; $f = 60\,\mathrm{kHz}$, $\theta = 75°$ and $ab = 1$; Φ in degrees.

			b/a			
Φ	1·0	0·69	0·51	0·39	0·30	0·25
0	1·00	0·52	0·24	0·10	0·04	0·01
15	1·00	0·56	0·28	0·12	0·05	0·02
30	1·00	0·69	0·41	0·22	0·11	0·05
45	1·00	0·91	0·71	0·50	0·32	0·19
60	1·00	1·19	1·20	1·10	0·94	0·76
75	1·00	1·46	1·78	1·98	2·07	2·08
90	1·00	1·57	2·06	2·45	2·77	3·01
105	1·00	1·46	1·78	1·98	2·08	2·09
120	1·00	1·19	1·21	1·11	0·94	·0·76
135	1·00	0·91	0·71	0·50	0·32	0·19
150	1·00	0·69	0·42	0·22	0·11	0·05
165	1·00	0·56	0·28	0·13	0·05	0·02
180	1·00	0·52	0·24	0·10	0·04	0·01
195	1·00	0·56	0·28	0·12	0·05	0·02
210	1·00	0·69	0·41	0·22	0·11	0·05
225	1·00	0·90	0·70	0·49	0·32	0·19
240	1·00	1·19	1·20	1·10	0·93	0·75
255	1·00	1·45	1·78	1·97	2·06	2·07
270	1·00	1·57	2·06	2·45	2·77	3·01
285	1·00	1·46	1·79	1·99	2·08	2·10
300	1·00	1·19	1·21	1·11	0·95	0·77
315	1·00	0·91	0·71	0·50	0·32	0·19
330	1·00	0·69	0·42	0·23	0·11	0·05
345	1·00	0·56	0·28	0·13	0·05	0·02
360	1·00	0·52	0·24	0·10	0·04	0·01

$F_N(\theta, \Phi)$ for different values of Φ and b/a are given in table 1. Figure 3 shows angular plot of $F_N(\theta, \Phi)$ which modulation frequency f for $\theta = 45°$, $a = 1{\cdot}4\,\mathrm{cm}$ and $b/a = 0{\cdot}507$. Figure 4 shows angular plot of $F(\theta, \Phi)$ with θ for $f = 60\,\mathrm{kHz}$, $\Phi = \pi/2$, $\alpha = 0{\cdot}17\,\mathrm{cm}^{-1}$, $a = 2\,\mathrm{cm}$ and $b = 0{\cdot}5\,\mathrm{cm}$. For $\Phi = \pi/2$, $\alpha = 0{\cdot}17\,\mathrm{cm}^{-1}$ and $f = 60\,\mathrm{kHz}$, values of $F(\theta, \Phi)/B$ (where $B = I_0 \pi c$) for different values of θ and b/a are given in table 2. It is

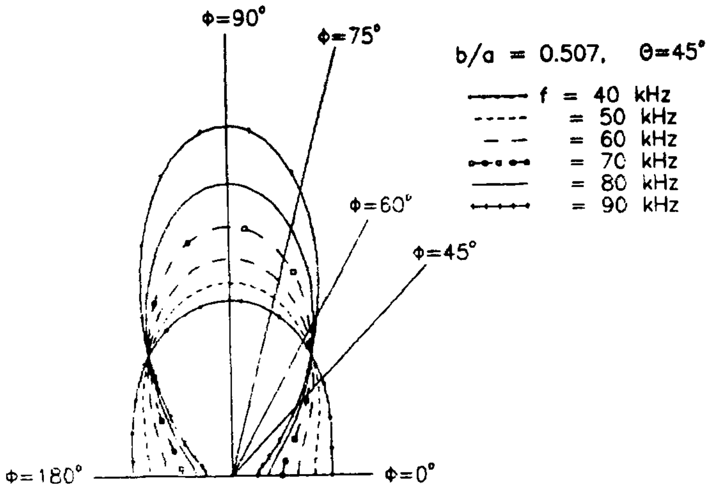

Figure 3. Angular plot of $F_N(\theta, \Phi)$ as a function of f.

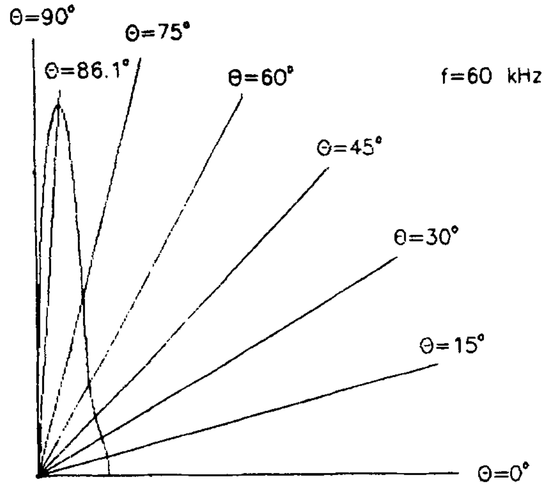

Figure 4. Angular plot of $F(\theta, \Phi)/B$ as a function of θ. $\alpha = 0{\cdot}17\,\mathrm{cm}^{-1}$, $a = 2\,\mathrm{cm}$, $b = 0{\cdot}5\,\mathrm{cm}$, $\Phi = 90^\circ$ and $f = 60\,\mathrm{kHz}$. $B = I_0 \pi c$ and $F(\theta, \Phi)/B$ in cgs units.

obvious from these figures and tables that the sound field has a preferential direction in the $\theta \approx 86{\cdot}1^\circ$, $\Phi = \pi/2$ and $3\pi/2$, and the solid angle carrying most of the sound field narrows as the sound frequency and the ellipticity of the laser beam are increased.

Thus it is clear from the above discussion that using an elliptic Gaussian beam one can get an acoustic field which depends both on polar (θ) as well as azimuthal (Φ) directions. Particularly, towards $\Phi = \pi/2$ and $3\pi/2$ acoustic field enhances whereas towards $\Phi = 0$ and π it diminishes. Since, acoustic field also depends on $1/R$, acoustic field generated by an elliptic Gaussian laser beam can travel larger distance towards $\Phi = \pi/2$ and $3\pi/2$, and less distance towards $\Phi = 0$ and π in comparison to acoustic field generated by a Gaussian laser beam of equal power. Thus using elliptic Gaussian laser beam one can get acoustic field above noise level to larger distances towards $\Phi = \pi/2$ and $3\pi/2$. Once acoustic field above noise level is available, one can easily electrically amplify it, but no amplifier can detect/amplify acoustic signals unless it

Table 2. $F(\theta,\Phi)/B$ for different values of b/a and θ. $f=60\,\text{kHz}$, $\Phi=90°$, $ab=1$ and $\alpha=0{\cdot}17\,\text{cm}^{-1}$; $F(\theta,\Phi)/B$ in cgs units and θ in degrees.

	b/a					
θ	1	0·69	0·51	0·39	0·3	0·25
0	0·17	0·17	0·17	0·17	0·17	0·17
5	0·17	0·17	0·17	0·17	0·17	0·17
10	0·16	0·17	0·17	0·17	0·17	0·17
15	0·16	0·16	0·17	0·17	0·17	0·17
20	0·15	0·16	0·16	0·17	0·17	0·17
25	0·14	0·15	0·16	0·17	0·17	0·17
30	0·13	0·15	0·16	0·17	0·17	0·18
35	0·12	0·14	0·16	0·17	0·18	0·18
40	0·11	0·14	0·16	0·17	0·18	0·19
45	0·11	0·14	0·16	0·18	0·19	0·20
50	0·10	0·14	0·16	0·18	0·20	0·21
55	0·10	0·14	0·17	0·19	0·21	0·22
60	0·10	0·15	0·18	0·21	0·23	0·25
65	0·11	0·16	0·20	0·24	0·26	0·28
70	0·12	0·18	0·24	0·28	0·31	0·34
75	0·14	0·22	0·29	0·35	0·39	0·43
80	0·18	0·29	0·39	0·47	0·53	0·58
84	0·24	0·39	0·52	0·62	0·71	0·78
85	0·25	0·41	0·55	0·66	0·75	0·82
86	0·26	0·42	0·56	0·68	0·77	0·85
87	0·25	0·41	0·54	0·66	0·75	0·82
88	0·21	0·34	0·45	0·55	0·62	0·68
89	0·12	0·20	0·26	0·32	0·36	0·39
90	0·00	0·00	0·00	0·00	0·00	0·00

exists above noise level. The elliptic Gaussian beam provides a simple means of availability of acoustic field over noise level to larger distances in comparison to a Gaussian laser beam of equal power. Hence, it is a positive advantage as far as underwater sound communication via laser source is concerned.

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