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Photon polarisation asymmetries in $(p, p'\gamma)$ experiments

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Abstract. The Stokes parameters characterising the polarization of the photon in the γ-decay of $^{12}C^*(1^+)$ are examined in correlation with the scattered proton in $^{12}C(p, p')\,^{12}C^*(1^+)$. This opens up the possibility of efficiently determining empirically the six inelastic scattering amplitudes utilizing only 13 measurements. We identify several alternative sets of such measurements to encourage experimental efforts in this direction.

Keywords. Inelastic scattering; amplitude determination; proton polarization; Stokes parameters; correlation observables.

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1. Introduction

It is well-known that electron scattering experiments have made important contributions to the study of the electromagnetic structure of the nuclei. It was expected that the study of scattering of hadrons or of reactions involving hadrons on nuclei at intermediate energies would provide useful and complementary information on the nuclear structure. It was initially believed that the hadron interaction with a nucleon bound in the nucleus is not much different from that with a free nucleon at these energies. Alternatively, it was hoped that any deviations in the hadron-nucleon interaction could also be investigated assuming apriori knowledge of nuclear structure derived from electron scattering and other considerations.

After moderate initial successes with cross-section data, a real challenge was posed to these beliefs by the advent of data on the spin observables. It is interesting to note that a similar development took place at high energies, where spin polarization data spelt disaster to Regge Models. In this context, therefore, much work is being done experimentally to determine not only the differential cross-section, but also all the attendant spin observables on hadron-nucleus scattering—both elastic and inelastic.

Of particular interest are the exclusive inelastic nucleon-nucleus scattering studies leading to well-defined excited states of the target nucleus like $^{12}C(p, p')^{12}C^*(1^+)$ on which several experimental studies have been reported recently. Even if we include data obtained by employing polarized protons and measuring the proton polarization, a complete knowledge of the inelastic scattering amplitudes is not forthcoming empirically since the process involves six non-zero amplitudes which are complex. In view of this, suggestions have been made [1, 2] to supplement these measurements with measurements of correlated $(\mathbf{p}, \mathbf{p}'\gamma)$ observables like polarized proton asymmetry, proton polarization and proton spin transfers apart from the coincidence cross-section. Although the measurements already suggested [2] are comprehensive enough to serve the purpose and although measurements of photon polarizations at these energies are probably difficult, it is at least of theoretical interest to investigate $(p, p'\gamma)$ reactions in a more complete manner including those observables associated with photon polarizations. It is encouraging to note in this context that Fasano *et al*, [3] have recently discussed photon polarization-based observables in meson photo-production involving photons at energies which are even higher by an order of magnitude. The importance of a photon polarization sensitive measurement in $^{12}C(\mathbf{p}, \mathbf{p}'\gamma)^{12}C$ reaction for the determination of the real part of the density matrix element ρ_{10} was pointed out by Piekarewicz [4]. A special merit in taking photon polarization based observables into consideration in $^{12}C(p, p'\gamma)^{12}C$ is that they facilitate direct determination of the magnitudes of the six independent amplitudes, thus effecting reduction in the associated statistical errors. The relative phases can then be determined with greater ease. Alternatively, the photon polarization based observables may also be utilized to economise the number of observables that need to be measured to effect unambiguous determination of the inelastic amplitudes.

In § 2, six independent amplitudes associated with the inelastic scattering process are defined and the Fano statistical tensors t^k_q characterising the $^{12}C^*(1^+)$ are given explicitly in terms of these amplitudes. In § 3, we outline the conventions adopted in defining the photon observables and discuss the γ-decay of polarized $^{12}C^*(1^+)$. We also define in full all the photon-proton correlation observables. In § 4, we describe the inversion problem and show how with a proper choice of observables, one can directly determine the magnitudes as well as effect a reduction in the number of observables.

2. Nuclear polarization in the transverse frame

The six independent amplitudes characterising the inelastic scattering $^{12}C(p, p')$ $^{12}C^*(1^+)$ can be defined in several ways such that the requirements of rotation and parity invariance are satisfied. If $\mathbf{P}_i$ and $\mathbf{P}_f$ denote the proton momenta in the centre of mass system, Piekarewicz *et al*, [1] define a Cartesian co-ordinate system through

$$\mathbf{N} = \mathbf{P}_i \times \mathbf{P}_f, \quad \mathbf{K} = \mathbf{P}_i + \mathbf{P}_f, \quad \mathbf{Q} = \mathbf{N} \times \mathbf{K} \tag{2.1}$$

and express the amplitude for the nuclear transition $0^+ \to 1^+$ through

$$\begin{aligned} T = {} & A_{NO}(\hat{\boldsymbol{\Sigma}}\cdot\hat{N}) + A_{NN}(\hat{\boldsymbol{\Sigma}}\cdot\hat{N})(\boldsymbol{\sigma}\cdot\hat{N}) + A_{KK}(\hat{\boldsymbol{\Sigma}}\cdot\hat{K})(\boldsymbol{\sigma}\cdot\hat{K}) + A_{KQ}(\hat{\boldsymbol{\Sigma}}\cdot\hat{K})(\boldsymbol{\sigma}\cdot\hat{Q}) \\ & + A_{QK}(\hat{\boldsymbol{\Sigma}}\cdot\hat{Q})(\boldsymbol{\sigma}\cdot\hat{K}) + A_{QQ}(\hat{\boldsymbol{\Sigma}}\cdot\hat{Q})(\boldsymbol{\sigma}\cdot\hat{Q}) \end{aligned} \tag{2.2}$$

where $\hat{\boldsymbol{\Sigma}}_M = |1^+ M\rangle\langle 0^+|$ is called the polarization operator of the nucleus and $\boldsymbol{\sigma}$, the Pauli spin matrices. With the Z-axis chosen along $\mathbf{Q}$, X and Y respectively along $\mathbf{N}$ and $\mathbf{K}$, the T-matrix is given explicitly by,

$$T^P = \begin{vmatrix} (-1/\sqrt{2})(A_{NO} - iA_{KQ}) & (-1/\sqrt{2})(A_{NN} - A_{KK}) \\ (-1/\sqrt{2})(A_{NN} + A_{KK}) & (-1/\sqrt{2})(A_{NO} + iA_{KQ}) \\ A_{QQ} & -iA_{QK} \\ iA_{QK} & -A_{QQ} \\ (1/\sqrt{2})(A_{NO} + iA_{KQ}) & (1/\sqrt{2})(A_{NN} + A_{KK}) \\ (1/\sqrt{2})(A_{NN} - A_{KK}) & (1/\sqrt{2})(A_{NO} - iA_{KQ}) \end{vmatrix} \tag{2.3}$$

where the rows are labelled by the final spin states $|1,1/2\rangle$, $|1,-1/2\rangle$, $|0,1/2\rangle$, $|0,-1/2\rangle$, $|-1,1/2\rangle$, $|-1,-1/2\rangle$ and columns by the initial spin states $|0,1/2\rangle$, $|0,-1/2\rangle$ of the spin zero nucleus and the incident proton.

Helicity formalism is also used in this particular context by Lyndon *et al* [5]. Since invariance under parity for the helicity amplitudes requires

$$T^H_{-\mu',-\mu'_p;0,-\mu_p} = (-1)^{1+\mu_p-\mu'_p-\mu'} \quad T^H_{\mu',\mu'_p;0,\mu_p} \tag{2.4}$$

where μ', μ'_p and μ_p denote the helicities of $^{12}C^*(1^+)$ nucleus, scattered proton and the incident proton respectively, the T-matrix assumes the form,

$$T^H = \begin{vmatrix} H_1 & -H_6 \\ H_2 & H_5 \\ H_3 & H_4 \\ H_4 & -H_3 \\ H_5 & -H_2 \\ H_6 & H_1 \end{vmatrix} \tag{2.5}$$

where the rows and columns are labelled as in (2.3) but in terms of the helicity states.

We find however that it is more advantageous to choose a special transverse frame, (Z-axis along $\mathbf{N}$) which can be identified with the ($\mathbf{L},\mathbf{S},\mathbf{N}$), ($\mathbf{L}',\mathbf{S}',\mathbf{N}'=\mathbf{N}$) co-ordinate system employed by McClelland *et al* [6]. The advantage inherent in this choice has also been noted by Moravcsik *et al* [7]. The speciality of this transverse frame is that the X-axis for the initial $p-{}^{12}C$ system is chosen along $\mathbf{P}_i$ whereas it (X') is chosen along the direction $\mathbf{P}_f$ for the $p'-{}^{12}C^*(1^+)$ system. The respective Y-axes are so chosen to form a right handed co-ordinate system i.e.,

$$\hat{Y} = \frac{\mathbf{N}\times\mathbf{P}_i}{|\mathbf{N}\times\mathbf{P}_i|} \quad \text{and} \quad \hat{Y}' = \frac{\mathbf{N}\times\mathbf{P}_f}{|\mathbf{N}\times\mathbf{P}_f|} \tag{2.6}$$

The requirement of parity conservation in this frame reads as,

$$T^t_{M'm';Om} = (-1)^{M'+m'-m}\, T^t_{M'm';Om} \tag{2.7}$$

so that, the T-matrix assumes the attractive form [1].

$$T^t = \begin{vmatrix} O & D \\ A & O \\ B & O \\ O & E \\ O & F \\ C & O \end{vmatrix} \tag{2.8}$$

where it may be observed that half the number of elements are zero. Though labelled in the same way, it may be noted that the basis states used for $\boldsymbol{T}$ in (2.3), (2.5) and (2.8) are different; but they could be related to each other through appropriate rotations:

$$T^t_{M'm';Om} = \sum_{\substack{M'',m''\\ m'''}} D^{1^*}_{M''M'}(0,\pi/2,\pi/2+\theta_{\text{cm}}/2)\, D^{1/2^*}_{m''m'}(0,\pi/2,\pi/2+\theta_{\text{cm}}/2)$$

$$T^P_{M''m'';Om'''}\, D^{1/2}_{m'''m}(0,\pi/2,\pi/2-\theta_{\text{cm}}/2) \tag{2.9}$$

and

$$T^{t}_{M'm';Om} = \sum_{\mu',\mu'_p,\mu_p} D^{1*}_{\mu' M'}(\pi/2,\pi/2,0) D^{1/2*}_{\mu'_p m'}(\pi/2,\pi/2,\pi) T^{H}_{\mu'\mu'_p;O\mu_p} D^{1/2}_{\mu_p m}(\pi/2,\pi/2,\pi) \tag{2.10}$$

where θ_{cm} is the centre of mass scattering angle.

The density matrix $\boldsymbol{\rho}$ characterising the final spin state of the $p' - {}^{12}\text{C}^*(1^+)$ system may now be defined in the special transverse frame through its elements

$$\rho_{M'm';Mm} = \sum_{m'',m'''} T^{t}_{M'm';Om''} \rho^{p}_{m''m'''} T^{t*}_{Mm;Om'''} \tag{2.11}$$

where $\boldsymbol{\rho}^p$ denotes the density matrix characterising the state of polarization of the incident proton beam. Following Ravishankar and Ramachandran [8,9] and observing that $T^{t}_{M'm';Om}$ transforms under rotations in the nuclear spin space as a spherical tensor A of rank 1 i.e.,

$$T^{t}_{M'm';Om} = (-1)^{M'} A_{-M'}(m',m) \tag{2.12}$$

and defining A^+ following Schwinger [10] through $A^+_M = (-1)^M$ times the hermitian conjugate of A_{-M}, the state of polarization of ${}^{12}\text{C}^*(1^+)$ may be characterised by the Fano statistical tensors t^k_q [11] given by

$$t^k_q = (-1)^{k+1}\sqrt{3}\sum_{M,M'} A_M \rho^p A^+_{M'} C(11k; MM'q). \tag{2.13}$$

Writing explicitly

$$A_1 = -\begin{vmatrix} 0 & F \\ C & 0 \end{vmatrix} \quad A_0 = \begin{vmatrix} B & 0 \\ 0 & E \end{vmatrix} \quad A_{-1} = -\begin{vmatrix} 0 & D \\ A & 0 \end{vmatrix} \tag{2.14a}$$

$$A^+_1 = \begin{vmatrix} 0 & A^* \\ D^* & 0 \end{vmatrix} \quad A^+_0 = \begin{vmatrix} B^* & 0 \\ 0 & E^* \end{vmatrix} \quad A^+_{-1} = \begin{vmatrix} 0 & C^* \\ F^* & 0 \end{vmatrix} \tag{2.14b}$$

$$\rho^p = (1/2)(\mathbf{1} + \boldsymbol{\sigma}\cdot\mathbf{P}) \tag{2.15}$$

where $\mathbf{P}$ denotes the initial proton (beam) polarization. The scattered proton polarization $\mathbf{P}'$ can be obtained from its density matrix

$$\rho^{p'} = (\text{Tr}\,\rho^{p'}/2)(\mathbf{1} + \boldsymbol{\sigma}\cdot\mathbf{P}') \tag{2.16}$$

whose elements are given by

$$\rho^{p'}_{m'm} = \sum_M \rho_{Mm';Mm}. \tag{2.17}$$

3. Photon polarization asymmetries

The basis states of polarization [12–18] characterising the photon emitted by ${}^{12}\text{C}^*(1^+)$ can conveniently be chosen to be right circular and left circular,

$$\hat{\varepsilon}_{+1} = (-1/\sqrt{2})(\hat{\varepsilon}_x + i\hat{\varepsilon}_y) \tag{3.1a}$$

$$\hat{\varepsilon}_{-1} = (1/\sqrt{2})(\hat{\varepsilon}_x - i\hat{\varepsilon}_y) \tag{3.1b}$$

so as to satisfy standard conventions [19] for the rotation matrices D^j so that the density matrix $\boldsymbol{\rho}^\gamma$ takes the form,

$$\boldsymbol{\rho}^\gamma = 1/2\begin{vmatrix} S_0 + S_z & S_x - iS_y \\ S_x + iS_y & S_0 - S_z \end{vmatrix} \tag{3.2}$$

where $S_0 = \mathrm{Tr}\,\boldsymbol{\rho}^\gamma$ denotes the intensity and $S_x = -s_1$, $S_y = s_2$ and $S_z = s_3$ in terms of the Stokes parameters defined by Born and Wolf [14].

The electromagnetic transition ${}^{12}\mathrm{C}^*(1^+) \to {}^{12}\mathrm{C}(0^+) + \gamma$ is governed by the transition matrix element (in the radiation gauge)

$$\langle 0^+; \mathbf{k}\mu | T^\gamma | 1^+ M\rangle = \langle 0^+ | a_{\mathbf{k}\mu} \mathbf{j}\cdot\mathbf{A} | 1^+ M\rangle \tag{3.3}$$

where

$$\mathbf{A} = (2\pi)^{-3/2}\int \mathrm{d}^3k'(2\mathrm{k}')^{-1/2}\sum_{\lambda=+1,-1}\hat{\varepsilon}_\lambda[a_{\mathbf{k}'\lambda}e^{ik'\cdot x} + a^+_{\mathbf{k}'\lambda}e^{-ik'\cdot x}] \tag{3.4}$$

in terms of the photon creation and annihilation operators $a^+_{\mathbf{k}'\lambda}$ and $a_{\mathbf{k}'\lambda}$ respectively. Carrying out a simple quantum field theoretic calculation and expressing

$$\hat{\varepsilon}_\mu e^{-i\mathbf{k}\cdot\mathbf{r}} = (2\pi)^{1/2}\sum_{L=1}^{\infty}\sum_{M=-L}^{L}(i)^L(2L+1)^{1/2}[-\mu A^{(m)}_{LM} + iA^{(e)}_{LM}]$$
$$D^L_{M\mu}(\phi_\gamma, \theta_\gamma, 0) \tag{3.5}$$

in terms of the standard multipole solutions [19], we have

$$\langle 0^+; \mathbf{k}\mu | T^\gamma | 1^+ M\rangle = Q(-1)^{1-M}\mu D^1_{-M\mu}(\phi_\gamma, \theta_\gamma, 0) \tag{3.6}$$

where Q collectively denotes all the factors including the reduced matrix element for the nuclear transition and $(\theta_\gamma, \phi_\gamma)$ denote the polar angles of $\mathbf{k}$ with respect to the $p' - {}^{12}\mathrm{C}^*(1^+)$ co-ordinate system defined earlier. The γ-decay of the 15·11 MeV ($J^\pi = 1^+; T = 1$) state to the ($J^\pi = 0^+; T = 0$) ground state is a M1 transition with strength 0·531 ± 0·011 W.u [20]. The branching ratio for this transition is 97%, the width of the level being 38·5 ± 0·8 eV. The life time is short enough that the photon can unambiguously be identified with the associated inelastic scattering event and the coincidence cross-section and analysing powers have already been measured experimentally [5, 26].

The density matrix $\boldsymbol{\rho}^\gamma$ for the photon is now defined in terms of its elements,

$$\boldsymbol{\rho}^\gamma_{\mu\mu'} = \sum_{M,M'}\langle 0^+; \mathbf{k}\mu | T^\gamma | 1^+ M\rangle \rho_{MM'}\langle 0^+; \mathbf{k}\mu' | T^\gamma | 1^+ M'\rangle^* \tag{3.7}$$

where the elements $\rho_{MM'}$ of the nuclear density matrix are given in terms of the t^k_qs defined earlier by (2.13) through

$$\boldsymbol{\rho}_{MM'} = 3^{-1/2}\sum_{k=0}^{2}\sum_{q=-k}^{k}(-1)^M(-1)^{k+1}C(11k; -MM'q)t^k_q. \tag{3.8}$$

Substituting (3.8) in (3.7) and making use of the standard properties of the D^j – matrices, we get

$$\boldsymbol{\rho}^\gamma_{\mu\mu'} = (|Q|^2/3^{1/2})\sum_k\sum_q(-1)^k t^k_q C(11k; \mu - \mu' q')D^k_{qq'}(\phi_\gamma, \theta_\gamma, 0) \tag{3.9}$$

Information regarding all the photon-proton correlation observables are now contained in the matrix elements of

$$S_0(\theta_\gamma,\phi_\gamma;\mathbf{P}) = (2|Q|^2/3^{1/2}) \sum_{k=0,2} \sum_{q=-k}^{k} [4\pi/(2k+1)]^{1/2}$$

$$C(11k;1-10)(t^k_q\, Y^*_{kq}(\theta_\gamma,\phi_\gamma)) = (1/2)[S_{00} + \sum_{j=x,y,z} \sigma_j S_{0j}] \tag{3.10}$$

$$S_x(\theta_\gamma,\phi_\gamma;\mathbf{P}) = (|Q|^2/3^{1/2}) \sum_{q=-2}^{+2} t^2_q(D^2_{q2}(\phi_\gamma,\theta_\gamma,0) + D^2_{q-2}(\phi_\gamma,\theta_\gamma,0))$$

$$= (1/2)[S_{x0} + \sum_{j=x,y,z} \sigma_j S_{xj}] \tag{3.11}$$

$$S_y(\theta_\gamma,\phi_\gamma;\mathbf{P}) = (i|Q|^2/3^{1/2}) \sum_{q=-2}^{+2} t^2_q(D^2_{q2}(\phi_\gamma,\theta_\gamma,0) - D^2_{q-2}(\phi_\gamma,\theta_\gamma,0))$$

$$= (1/2)[S_{y0} + \sum_{j=x,y,z} \sigma_j S_{yj}] \tag{3.12}$$

$$S_z(\theta_\gamma,\phi_\gamma;\mathbf{P}) = (-2|Q|^2/3)(2\pi)^{1/2} \sum_{q=-1}^{+1} t^1_q\, Y^*_{1q}(\theta_\gamma,\phi_\gamma)$$

$$= (1/2)[S_{z0} + \sum_{j=x,y,z} \sigma_j S_{zj}] \tag{3.13}$$

which can readily be expressed in terms of A, B, C, D, E and F on making use of (2.13) and (2.14).

4. The inversion problem

To determine empirically the amplitudes A, B, C, D, E and F we need not have to make use of all the observables defined in the previous section. Although there are several ways [2] in which one can choose the requisite observables, all of them require some involved manipulations in order to determine the amplitudes unambiguously. One outstanding advantage in including some of the photon polarization observables is that it enables us to determine straightaway the magnitudes of the six amplitudes through the simple process of solving six linear equations. With this end in view, we first consider the inelastic differential cross-section,

$$(\mathrm{d}\sigma/\mathrm{d}\Omega_p)_{\mathrm{unpol}} = \mathrm{Tr}[\rho^p(\mathbf{P}=0)] = (1/2)(|A|^2+|B|^2+|C|^2+|D|^2+|E|^2+|F|^2) \tag{4.1}$$

for $^{12}C(p,p')^{12}C^*(1^+)$ using initially unpolarized protons and

$$(\mathrm{d}\sigma/\mathrm{d}\Omega_p)_{\mathrm{pol}} = \mathrm{Tr}[\rho^p(\mathbf{P}\neq 0)] = (\mathrm{d}\sigma/\mathrm{d}\Omega_p)_{\mathrm{unpol}} + (1/2)P_z(|A|^2+|B|^2+ + |C|^2-|D|^2-|E|^2-|F|^2) \tag{4.2}$$

with initially polarized protons. We may next choose to take into consideration the measurement of the transverse (or Z) component of the scattered proton polarization given by

$$(\mathrm{d}\sigma/\mathrm{d}\Omega_p)_{\mathrm{unpol}} P'_z(\mathbf{P}=0) = (1/2)(-|A|^2+|B|^2-|C|^2+|D|^2-|E|^2+|F|^2) \tag{4.3}$$

when the initial beam is unpolarised and

$$(\mathrm{d}\sigma/\mathrm{d}\Omega_p)_{\mathrm{pol}} P'_z(\mathbf{P} \neq 0) = (\mathrm{d}\sigma/\mathrm{d}\Omega_p)_{\mathrm{unpol}} P'_z(\mathbf{P} = 0) + (1/2)P_z(-|A|^2 + |B|^2 - |C|^2 - |D|^2 + |E|^2 - |F|^2) \tag{4.4}$$

with initially polarized proton beam. The above four observables have already been measured experimentally [6, 21–28]. To generate two more linear equations involving the squares of the amplitudes, we suggest here the measurement of the Stokes parameter S_z of forward produced photons. We have

$$S_{z0}(0,0;\mathbf{P}=0) = -(1/2)|Q|^2(|D|^2 - |F|^2 + |A|^2 - |C|^2) \tag{4.5}$$

with initially unpolarized protons and

$$S_{z0}(0,0;\mathbf{P}\neq 0) = S_{z0}(0,0;\mathbf{P}=0) + (1/2)|Q|^2 P_z(|D|^2 - |F|^2 - |A|^2 + |C|^2) \tag{4.6}$$

using initially polarized proton beam. It is transparent that equations (4.1) to (4.6) are readily invertible and hence $|A|^2$, $|B|^2$, $|C|^2$, $|D|^2$, $|E|^2$ and $|F|^2$ can be determined. It may be noted that this can be done without introducing large statistical uncertainties in the inversion process. Since the overall phase is anyway not determinable, we are now left with the problem of determining the five relative phases. It is convenient to choose B to be real and positive and the relative phases of A, C, D, E, F may be denoted respectively by δ_A, δ_C, δ_D, δ_E and δ_F. On making use of the observables

$$S_{zx}(\pi/2,0;\mathbf{P}=0) = -(|Q|^2/\sqrt{2})\mathrm{Re}(FE^* + BA^* + CB^* + ED^*) \tag{4.7}$$

$$S_{zx}(\pi/2,0;\mathbf{P}\neq 0) = S_{zx}(\pi/2,0;\mathbf{P}=0) + (|Q|^2/\sqrt{2})P_z\mathrm{Re}(FE^* - BA^* - CB^* + ED^*) \tag{4.8}$$

$$S_{zy}(\pi/2,\pi/2;\mathbf{P}=0) = (|Q|^2/\sqrt{2})\mathrm{Re}(FE^* + BA^* - CB^* - ED^*) \tag{4.9}$$

and

$$S_{zy}(\pi/2,\pi/2;\mathbf{P}\neq 0) = S_{zy}(\pi/2,\pi/2;\mathbf{P}=0) + (|Q|^2/\sqrt{2})P_z\mathrm{Re}(FE^* - BA^* + CB^* - ED^*) \tag{4.10}$$

we can determine the real parts of FE^*, BA^*, CB^*, and ED^* which contain cosines of $(\delta_F - \delta_E)$, δ_A, δ_C and $(\delta_E - \delta_D)$. To avoid discrete ambiguities, we need one more observable [29] which involves a combination of these relative phases and which is linearly independent of equations (4.7), (4.8), (4.9) and (4.10). For this purpose we choose to measure

$$S_{zx}(\pi/2,\pi/2;\mathbf{P}=0) = -(|Q|^2/\sqrt{2})\mathrm{Im}(FE^* + BA^* + CB^* + ED^*) \tag{4.11}$$

We then have δ_A, δ_C, $\delta_E - \delta_D$ and $\delta_E - \delta_F$ unambiguously. To determine δ_D, δ_E and δ_F individually, we can make use of the observables

$$S_{zx}(0,0;P_x,0,0) = -|Q|^2 P_x \mathrm{Re}(AD^* - CF^*) \tag{4.12}$$

and

$$S_{zx}(0,0;0,P_y,0) = -|Q|^2 P_y \mathrm{Im}(AD^* - CF^*) \tag{4.13}$$

which together readily yield δ_D, δ_E and δ_F unambiguously since the magnitudes of the amplitudes are already known.

It is worth emphasizing that the number of observables chosen here is only 13 as compared to sets suggested earlier [2] each of which contained a total of 16 observables. It may moreover be noted that the inversion procedure suggested here to determine the magnitudes could reasonably be expected to lead to greater precision, provided the Stokes parameters $S_{z0}(0,0;\mathbf{P}=0)$ and $S_{z0}(0,0;\mathbf{P}\neq 0)$ are sufficiently large. To obtain an estimate of these observables we may use the existing non-relativistic [30] or relativistic impulse approximation theories [31]. Using nuclear structure inputs of Cohen and Kurath [32] and Lee and Kurath [33] computer codes DW81 [30] and its relativistic counterpart DREX [31] have already been developed. The estimates for the amplitudes based on these codes have been made available to us by Piekarewicz [34]. We find that the ratio S_{z0}/S_{00} at forward angles of the photon with $\mathbf{P}=0$ vary between $\pm 40\%$ whereas S_{z0}/S_{00} with $P_z = 0{\cdot}4$ varies between $\pm 30\%$ [35].

The set of observables (4.1) to (4.13) is by no means unique. Choosing only the Stokes parameter which is connected with the circular polarization asymmetry of the photon polarisation (which is likely to be the easiest to determine experimentally at these energies), we can identify several alternative sets (as many as 16 each of which contain only 13 observables) which can lead to unambiguous determination of inelastic scattering amplitudes. A large number of other sets of possible measurements could also be identified if we take into consideration the other two Stokes parameters. It may also be emphasized finally that a completely empirical programme of determining unambiguously the $^{12}C(p,p')^{12}C^*(1^+)$ scattering amplitudes cannot be envisaged without including at least some observables associated with $^{12}C(p,p'\gamma)^{12}C$.

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