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Inflationary cosmology

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Abstract. We review the basic features of the inflationary scenarios associated with phase transitions in the very early universe.

1. Introduction

In this review on inflationary cosmology [1], we try to bring out its salient features without entering into details of specific models. We restrict ourselves to scenarios which are associated with phase transitions. With a brief survey of the standard model of cosmology [2], we go on to explain the problems it faces when extrapolated back to early times. We then sketch Guth's proposal, the so-called 'old' inflationary scenario. It is followed by an outline ot the 'new' inflationary scenario, which was proposed to rectify the defects of the old one. We state some conditions to be satisfied by any viable model for this scenario. We also indicate how this scenario may explain the small scale density inhomogeneity of the present-day universe in terms of the quantum fluctuations in the inflationary epoch.

2. Standard Model of cosmology

The universe is assumed to be homogeneous and isotropic, whence follows the Robertson-Walker form of the metric ,

$$ds^2 = dt^2 - a^2(t)\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\}. \tag{1}$$

By convention r and $a(t)$ are rescaled so that k has discrete values, +1, 0 and -1 corresponding to closed, flat and open universe respectively. For $k = 0$, the metric assumes the simple form,

$$ds^2 = dt^2 - a^2(t)\mathrm{d}\mathbf{x}^2. \tag{2}$$

Here r denotes the 'coordinate distance', which remains unchanged under coosmological expansion and gives the physical distance when multiplied by the scale factor $a(t)$. See Fig.1.

In this metric the energy-momentum tensor of a perfect fluid is

$$T_{\mu\nu} = pg_{\mu\nu} + (p+\rho)u_\mu u_\nu, \tag{3}$$

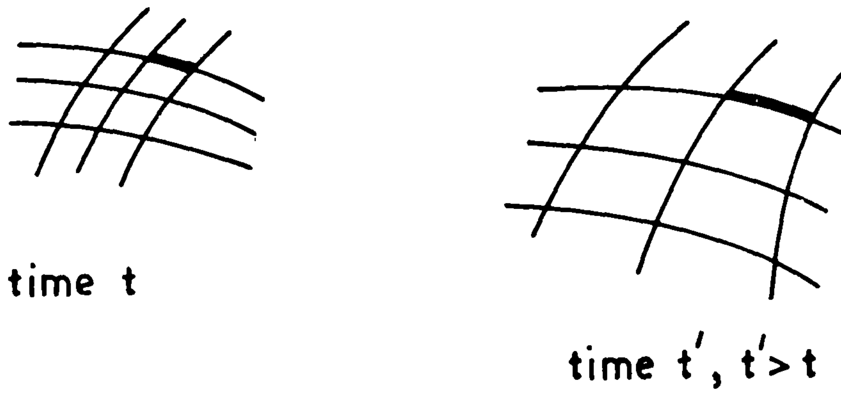

Figure 1. Increase of physical distance under cosmological expansion. The thick segments are corresponding physical distances at times t and $t' > t$.

where u^{μ} is the velocity 4-vector of the fluid. In the frame where the fluid is at rest, $u^{\mu} = (1,0,0,0)$ and p and ρ are the pressure and the energy density of the fluid respectively. Then Einstein's field equation gives the time evolution of the scale factor $a(t)$:

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{4}$$

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a, \tag{5}$$

where G is Newton's Gravitation constant, $G = M_p^{-2}$ and $M_p = 1.22 \times 10^{19}$GeV is the Planck mass. Also, covariant conservation of energy momentum yields

$$\frac{d}{dt}(a^3\rho) = -p\frac{da^3}{dt}. \tag{6}$$

Of the last three equations only two are independent. Together with the equation of state, they govern the dynamics of the standard cosmology.

The universe expands adiabatically to a good accuracy, so that the entropy per comoving volume $S = sa^3$, where s is the entropy density, is constant. The very early universe is dominated by the thermal radiation of effectively massless particles ($m \ll T$). We may here ignore the curvature term and Eq.(4) may be solved to give

$$T^2 t = cM_p, \tag{7}$$

where the constant c depends on the effective degrees of freedom N_{eff} and is given by $c = \frac{1}{4}(45/\pi^3 N_{eff})^{1/2}$.

The Hubble 'constant' is $H(t) = \dot{a}/a$ and the critical density $\rho_c(t)$ corresponds to the actual density for a $k = 0$ universe, which from (4) is

$$\rho_c(t) = \frac{3H^2}{8\pi G}. \tag{8}$$

Defining $\Omega(t) = \rho(t)\rho_c^{-1}(t)$, (4) becomes

$$\Omega - 1 = \frac{k}{Ha^2}. \tag{9}$$

Presentday observation limits Ω_0 to

$$0.1 < \Omega_0 < 1.$$

(The present value of any quantity will be denoted by a subscript o).

3. Problems of the Standard Model

The Standard Model has many impressive successes. But when it is pushed sufficiently backward in time, several problems arise.

3.1. Flatness/Oldness problem [3].

The only time scale in the Einstein equation is the Planck time, $t_p = (M_p)^{-1} = 5.4 \times 10^{-44}$ sec. The age of the universe is $\sim 10^{10}$ years $\sim 10^{61} \times t_p$. Thus although the universe is very old, the value of $\Omega(t_0)$ is close to unity.

Note that for a $k = 0$ universe, $\Omega = 1$ always. But this is a very special case. Recall that the metric is defined for all k values on the entire real line,which are scaled to +1, 0 and -1. So $k = 0$ is a set of measure zero on the real line. Thus the most likely value of k is nonzero. We shall ignore the $k = 0$ case in the following.

The unnaturalness of $\Omega_0 \sim 1$ will be evident if we calculate its value in the very early universe. From the conservation of of entropy $a(t)T(t) = a_0 T_0$, with $a_0 > 10^{28}$ cm, $T_0 = 2.7^0 K$ and the solution (7) of Einstein's equation, we get

$$|\Omega(t) - 1| \,|\simeq 10^{-21}/(T\ GeV)^{-2} \tag{10}$$

in the early universe. Thus at $T \sim 1$ MeV ($t \sim 1$ sec.,big bang nucleosynthesis), 10^{14} GeV ($t \sim 10^{-34}$sec, GUT phase transition) and 10^{19} GeV ($t \sim 10^{-44}$ sec, Planck time), the values of $|\Omega - 1|$ was less than 10^{-15}, 10^{-49} and 10^{-59} respectively. The puzzle is then why Ω was so close to 1 at these early times.

3.2. Horizon problem [4].

The universe is very homogeneous on a large scale as evidenced by the uniformity of Cosmic Microwave Background Radiation (CMBR), whose anisotropy is $< 10^{-4}$. It is difficult to understand this striking uniformity in the Standard Model, where the horizon distance is short.

Consider a setup with two microwave antennae pointing in the opposite directions (Fig.2). Each is receiving now the radiation that is believed to have been emitted or last scattered at the time of hydrogen recombination $t_R \sim 10^5$ years, when the temperature was $T_R \sim 4000^o K$. Accordingly, such measurements provide information on the density inhomogeneity prevailing at that time. The coordinate distance between the source of CMBR and us is

$$R = \int_{t_R}^{t_0} \frac{dt'}{a(t')}, \tag{11}$$

while the coordinate horizon distance at time t_R is

$$l_H = \int_0^{t_R} \frac{dt'}{a(t')}. \tag{12}$$

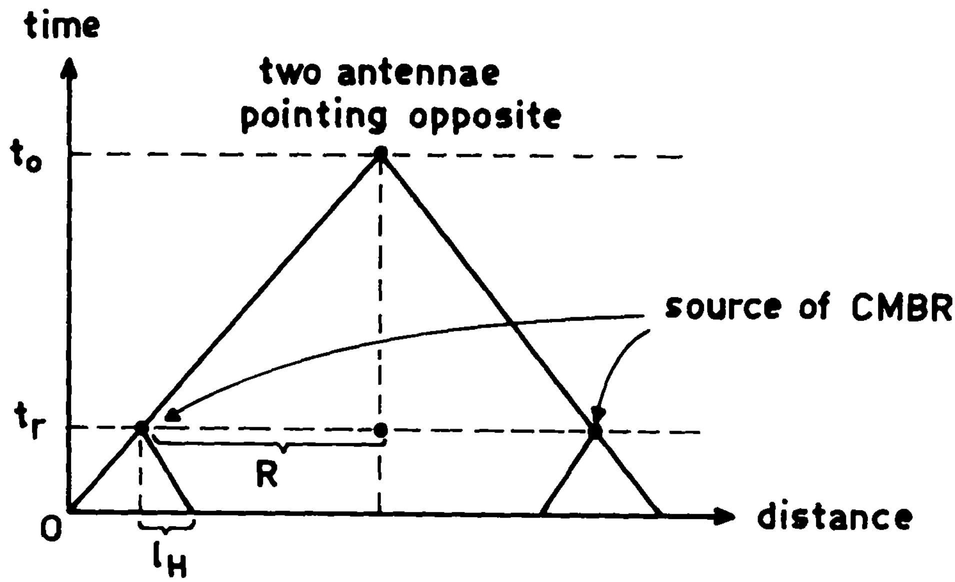

Figure 2. Two dimensional space-time diagram (with non-linear scale) showing the light rays received by the setup discussed in the text.

The ratio of the coordinate distance separating the two sources at time t_R to the horizon distance at that time $= 2R/l_H \sim 75$,assuming that the major contribution to the integrals comes from the matter dominated era, where $a(t) \sim t^{2/3}$. Thus, in the Standard Model, one has simply to assume that these regions were somehow prepared to be at the same temperature, because they did not have any causal contact.

3.3. Density perturbation problem

Although there is large scale homogeneity in the universe, on smaller scales one sees objects like galaxies, cluster of galaxies, etc. To account for this structure one must assume an initial spectrum of density inhomogeneity as perturbation on the uniform background. The Standard Model has no explanation for this perturbation.

Note that none of the three problems mentioned here signify any inconsistency of the Standard Model. They all refer to unnatural initial conditions.

3.4. Monopole abundance problem [5]

Monopoles (like domain walls, cosmic strings) are defects which are produced during the phase transition in the GUT's. They have masses$\sim M_X/(g^2/4\pi)$, where M_X is the symmetry breaking scale and g is the gauge coupling. The density of monopoles is roughly $\sim \xi^{-3}$, where ξ, the correlation length of the Higgs field, is bounded by the horizon distance at the time of phase transition. It can be shown that the monopole-antimonopole annihilation is rather ineffective and the present monopole density would exceed the present critical density by more than 10^{11}. With such a huge density, the universe would have collapsed long ago.

3.5. Cosmological constant problem

An extra piece in $T_{\mu\nu} \sim \rho_\Lambda g_{\mu\nu}$ changes Eq.(4) for $a(t)$ to

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}(\rho + \rho_\Lambda) = \frac{8\pi G\rho}{3} + \Lambda,$$

where Λ is the cosmological constant. Observationally,

$$\rho_\Lambda < \rho_{c,0}, \qquad \rho_{c,0} \sim 10^{-46}(GeV)^4.$$

In GUTs the false vacuum energy density $V(\phi_{false})$ is typically the fourth power of the characteristic mass scale,

$$V(\phi_{false}) \sim (10^{14}GeV)^4 = 10^{56}(GeV)^4. \tag{14}$$

To bring this value down to the magnitude of $\rho_{c,0}$, we need a fine tuning to the accuracy of 112 orders of magnitude! This is the biggest of all fine tunings ever made in Physics.

4. Guth's proposal: old infltionary scenario [6]

Guth made the remarkable observation that if we are willing to make this fine tuning, then all the problems mentioned above, both cosmological and of particle physics, can, in principle, be solved. Let us elaborate this fine tuning by considering the Higgs potential of a single scalar field ϕ,

$$V(\phi) = -\frac{1}{2}m^2 + \frac{\lambda}{4}\phi^4 + \frac{m^4}{4\lambda}. \tag{15}$$

The broken symmetric vacuum (at $T = 0$) which is the present day vacuum in this theory, is given by $\phi_{true} = m/\sqrt{\lambda}$. Here the cosmological constant is chosen to be zero. The one-loop correction to his potential does not significantly alter this situation.

As temperature rises, the (one loop) effective potential [7] gradually changes its shape (Fig.3). At sufficiently high temperature ($m \ll T$), it becomes

$$V^\beta_{eff}(\phi) = \frac{1}{2}(\frac{\lambda T^2}{24} - m^2)\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{m^4}{4\lambda}. \tag{16}$$

leading to a second order phase transition. Above the critical temperature $T_c = (24/\lambda)^{1/2}m$, the symmetry under ($\phi \rightarrow -\phi$) is restored . We then have a constant piece in the effective potential.

Thus we need a constant ($m^4(4\lambda)^{-1}$ in Eq.(15) and (16)), which gives rise to the cosmological costant before the phase transition. This ensures that after the phase transition is over, we have zero cosmological constant.

Returning to the minimal $SU(5)$ theory, Guth considered the effective potential which looks like that in Fig.4. Here we have the situation for a first order phase transition associated with the symmetry breaking $SU(4) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1)$. As the temperature falls below T_c, the universe is trapped in the false vacuum and decays by nucleation of bubbles of the true vacuum in the false vacuum, much

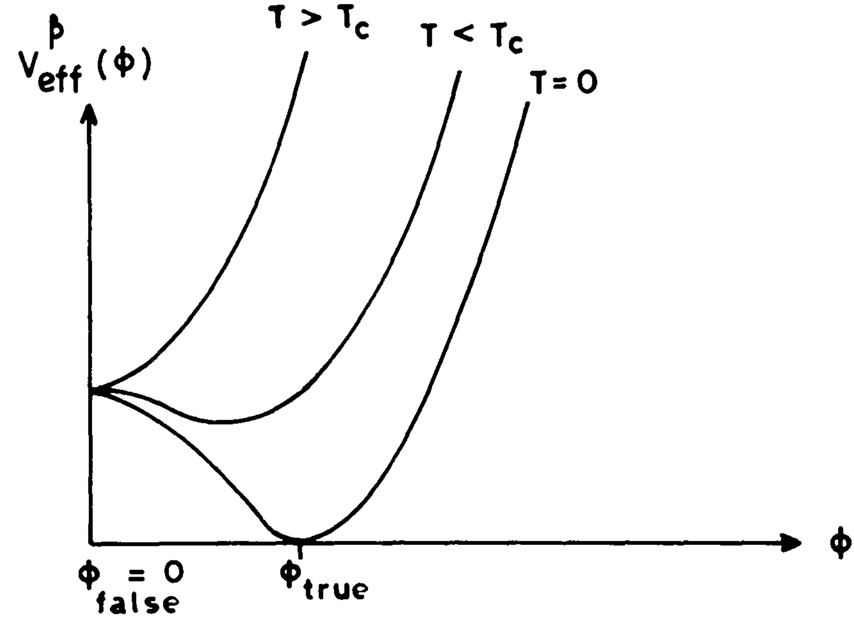

Figure 3. Effective potential at finite temperature for a field theory with a single Higgs field.

in the same way as in a condensed matter system. However, as it involves barrier penetration, this nucleation rate is very slow. So the system will supercool to temperatures near zero, while still remaining near the false vacuum. Then the vacuum energy will dominate the energy density and the equation for the scale factor

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3} V(\phi_{false}) = \chi^2, \tag{17}$$

has the de Sitter solution

$$a(t) \sim e^{\chi t}, \tag{18}$$

where $\chi = 10^{10} GeV$ in a GUT. Thus the scale factor grows exponentially with time with a huge χ.

So space continues to supercool and expand exponentially. But the energy density is fixed at $V(\phi_{false})$. Thus the total energy (without the gravitational energy) is increasing. This, however, is no violation of conservation of energy. Eq.(6) shows that the energy of a comoving volume decreases with time when $p > 0$. But, if $p < 0$, as is the case with the false vacuum, the total energy increases with time. If then the phase transition takes place quickly, the exponential expansion will come to an end and the false vacuum energy will reheat the universe to a temperature of about the same magnitude as it had before the inflationary scenario and the latter joins smoothly to the Standard Model. But the original space has expanded by a factor

$$Z = e^{\chi \Delta t}, \tag{19}$$

where Δt is the duration of inflation.

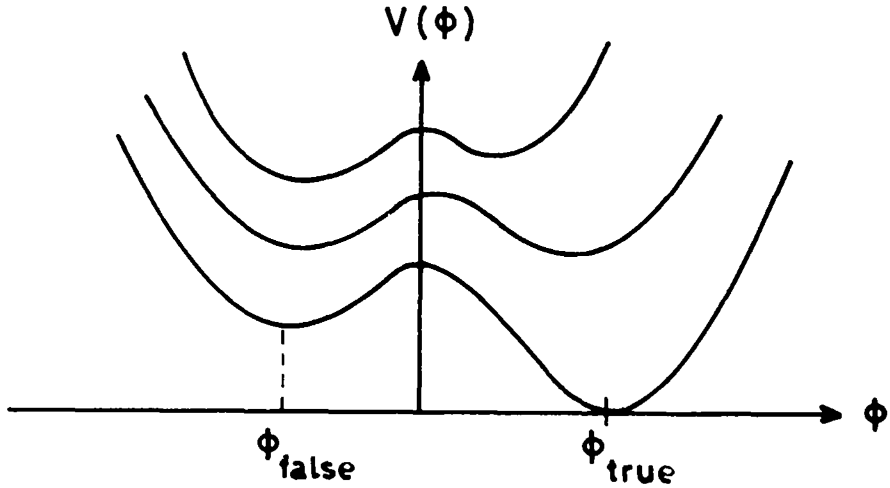

Figure 4. Finite temperature Higgs potential appropriate for the 'old' inflationary scenario.

5. Successes of original inflationary scenario

It is now easy to see how inflation deals with the above mentioned problems of the Standard Model.

5.1. Flatness problem

If the parameter $k = \pm 1$ initially, it will remain so for all times to come. But because of the huge expansion, local measurements cannot distinguish it from the $k = 0$ universe, explaining the present flatness of the universe.

The origin of the big total entropy is the same. Note that the reheating temperature immediately after the inflation is about the same as the temperature immediately before the expansion, so

$$\begin{aligned} s(\text{after}) &= s(\text{before}), \\ \text{but} \quad a(\text{after}) &= Za(\text{before}), \\ \text{so} \quad S(\text{after}) &= Z^3 S(\text{before}). \end{aligned} \tag{20}$$

Thus if $Z > 10^{29}$, S(before) can be of order 1, still giving S(after) $> 10^{87}$.

5.2. Horizon problem

This is also explained as the early universe was very much smaller in the inflationary scenario than in the Standard Model. Let us work out the size of the early universe immediately before inflation which grew into the observable part of the present universe. The first reduction is according to the Standard Model when we move backward from the present temperature $T_0 = 2.7^o K$ to $T_r \sim 10^{14}$ GeV, immediately after or before reheating. It can be calculated using the constancy of the total entropy during this period. The second reduction is brought about by inflation. Thus

$$10^{28} cm \rightarrow 10 cm \rightarrow \frac{10}{Z} cm < 10^{-14} GeV = 10^{4} (\chi GeV)^{-1}.$$

So the observed part of the universe was less than 10^{-4} of the horizon distance just before inflation.

5.3. Density perturbation problem

It is here that the old inflationary scenario fails. If we want sufficient inflation, the first order phase transition must be slow. Then the bubbles of the new phase will form finite sized clusters and they will not percolate [8]. The resulting structure will be highly inhomogeneous having no resemblance with the observed universe.

5.4. Primordial monopole problem

As already noted, the horizon distance increases by a factor of Z after inflation. The density of monopoles is thus diluted by a factor of Z^3 . Thus, at least, the severity of the problem is greatly reduced.

6. New inflationary scenario

In an attempt to overcome the problems of the 'old' model, the 'new' inflationary model was proposed by Linde and by Albrecht and Steinhardt [9]. The effective potential of this model is chosen to be of the Coleman-Weinberg type (Fig.5) in the minimal $SU(5)$ theory. It corresponds effectively to a second order phase transition, where the scalar field, originally in the false vacuum, slowly 'rolls over' the plateau of the effective potential, obeying the differential equation,

$$\frac{d^2\phi}{dt^2}+3\chi\frac{d\phi}{dt}=-\frac{dV_{eff}}{d\phi}. \tag{22}$$

The inflation then produces huge regions of homogeneous space. There is no problem with bubble collision, since the observable part of the universe lies deep within one such bubble. When the field reaches the steep part of the potential, it falls quickly to the minimum and executes damped oscillations about it, leading to particle production and reheating of the universe.

It should be noted that quantum field theory is generally formulated around a stable minimum of the potential, which, however, is not available during the slow 'rollover' regime. The proper way here [10,11] is to start from a thermal state at high enough temperature so that the stable minimum of the effective potential is situated at the zero of the thermally averaged field and thermal equilibrium prevails. Once the theory is formulated with the system in such a state, its later evolution can be traced on the basis of this formulation, even though these conditions cease to hold any more.

We now describe several requirements for the inflationary scenario to work.

(a) There are several temperatures in the description [12]. We have the temperature T_{deS} at which the geometry of space-time makes a transition from one of radiation dominance to that of vacuum energy dominance(de Sitter space-time). Then there is the temperature T_{eq} at which thermal equilibrium just gets lost. It is easy to see why thermal equilibrium cannot be maintained during inflation. For thermal equilibrium the collision rate ($\sim g^4T$) must exceed the expansion rate ($\sim \chi$). But as inflation proceeds, the collision rate rate falls off exponentially fast. Thus within a few time intervals of order χ^{-1}, the

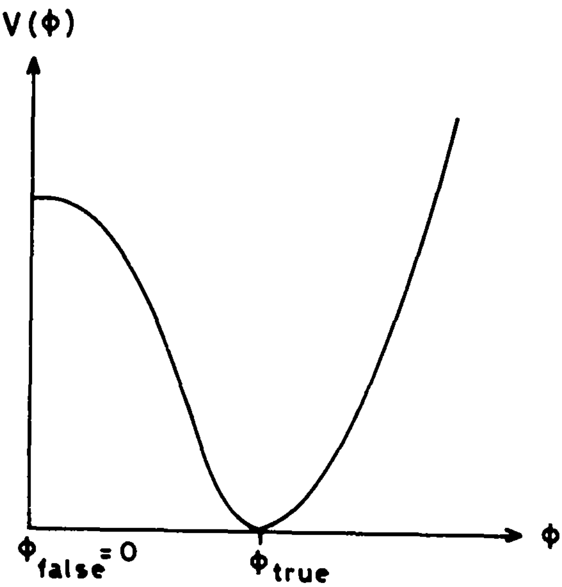

Figure 5. Effective potential appropriate for the 'new' inflationary scenario.

collision rate becomes insignificant compared to the expansion rate and the system goes out of thermal equilibrium. Finally we have the critical temperature T_c for the system to undergo phase transition in thermal equilibrium. Clearly we must have

$$T_c < T_{eq} < T_{deS} < M_p,$$

where we assume that all temperatures must lie below the Planck mass M_p, as we are treating gravity at the classical level. These inequalities will constrain the parameters of the theory.

(b) The effective potential $V_{eff}(\phi)$ at finite temperature must have the following properties [1] : (i) It must have a minimum away from $\phi = 0$. (ii) It must be reasonably flat near $\phi = 0$. (iii) At high temperature, the thermal average of ϕ must be zero.

(c) The fluctuation in ϕ must also be restricted [13]. For, if it is large, the field may settle locally in the true vacuum already before the 'rollover' begins. This will form domains of true vacuum whose subsequent dynamics of growth and coalescence will resemble that of the old inflationary scenario.

Clearly, all the successes of the old inflationary scenario are preserved in the new scenario. In addition, the problem of primordial monopoles is just eleminated. But the most spectacular result of the new scenario is the possibility of predicting the observed density inhomogeneity in terms of calculable quantum fluctuation of the energy density in the de Sitter epoch. We now turn briefly to explain this possibility.

7. Density inhomogeneity in new inflationary scenario

All pre-existing inhomogeneities in the universe disappear due to the tremendous expansion of space. It is the arbitrarily small wavelengths, contained in the zero point quantum fluctuations in the density and streched to macroscopic and astronomical dimensions by inflation, which give rise to the observed small scale inhomogeneties.

One conveniently analyses such perturbations in a Fourier series, the Fourier amplitudes describing the perturbation spectrum as a function of the wave number. The crucial distinction of this scenario from the standard one may be understood by following the growth of a physical wavelength $\lambda = 2\pi a(t)/k$ of the perturbation belonging to a given co-moving wave number k and comparing it with the appropriate horizon distance, which is H^{-1}, $H = \dot{a}/a$. It is easy to see that a physical distance l remains in causal contact if $l < H^{-1}$.

Starting from a time when $T \sim 10^{17}$, after which the classical theory of gravitation should be valid, we have the radiation dominated phase, $a(t) \sim \sqrt{t}$ and $H^{-1} \sim t$. Then, according to the inflationary scenario, it enters the de Sitter era at a time t_i and lasts till a time t_f. During this era $a(t) \sim e^{\chi t}$ but H^{-1} remains constant.

Consider a physical wavelength λ in the spectrum of the energy density perturbation, which is very small compared to the causal horizon at time t_i (Fig.6). λ grows exponentially and eventually crosses the causal horizon at time t_1,say. It continues to expand exponentially untill about time t_f, when the inflationary period terminates. After reheating it ends up in a radiation dominated era again wih the scale factor $a(t) \sim \sqrt{t}$. So now the causal horizon expands faster than the wavelength and eventually the latter reenters the causal horizon at time t_H,say.

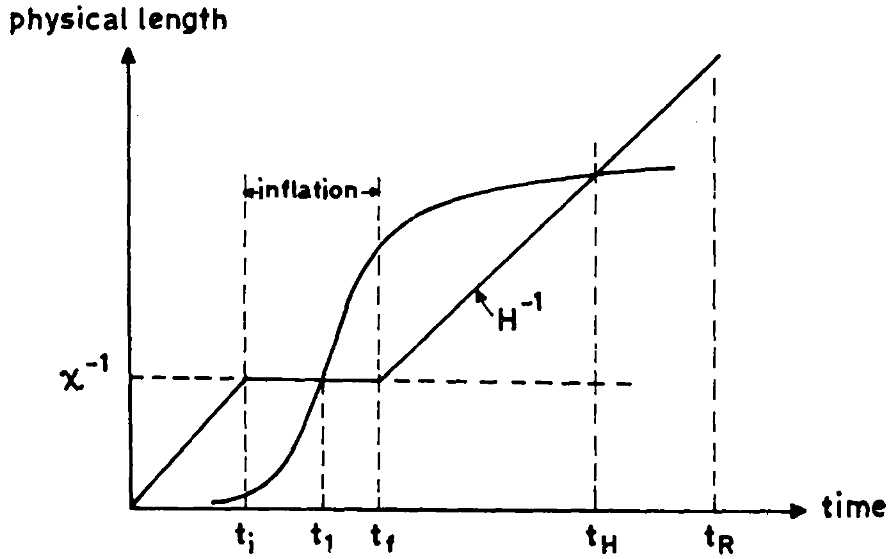

Figure 6. Evolution of a wavelength of the density perturbation.

One can calculate the density perturbation $\delta(t) \equiv \delta\rho(t)/\rho(t)$ around time t_1 in terms of the two-point function of field theory in the de Sitter space [12]. Its evolution can then be traced by the linearised Einstein equation for $\delta(t)$ to the time t_H [14]. It remains constant until the universe becomes matter dominated. This prediction for $\delta(t_H)$ can then be compared with the presently observed anisotropy in CMBR, which gives $\delta(t_R)$ around the time of hydrogen recombination and hence $\delta(t_H)$.

In standard Cosmology the physical wavelength would cross the horizon only once at t_H. At all times earlier to this time, the physical wavelength would be bigger than the horizon length. Thus it is even difficult to imagine a mechanism, within the standard Cosmology, by which the density perturbations could be determined by physical processes at these early times.

8. Concluding remarks

It turns out that the minimal $SU(5)$ model gives too large a value for the density inhomogeneity. Subsequently, several models [15] have been constructed which give the right magnitude for this quantity. In addition, there is the chaotic inflationary model [16] which is left out of our review. It remains to be seen which of these models are natural from the particle physics point of view.

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