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Dark matter: why and what?

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1. Lightning review of the Standard Cosmological Model

On the largest scales, the universe is isotopic and homogeneous. Accordingly we choose the metric to be the maximally symmetric Freidman Robertson metric given by [1]

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{1}$$

where (t, r, θ, ϕ) are comoving coordinates, $R(t)$ is the scale factor and k is a constant which determines whether the universe is open ($k = -1$), closed ($k = +1$) or flat ($k = 0$). Even without going into the dynamics, which are of course given by Einstein equations, we can derive certain features purely from kinematics:

i) Because of the time dependence of $R(t)$, the proper distances scale as $R(t)$.

ii)] Light from galaxies is red-shifted because of the expansion of the universe. Quantitatively, we define

$$1 + z = \frac{\lambda_0}{\lambda_1}, \tag{2}$$

where λ_1 is the emitted wavelength and λ_0 is the detected wavelength.

iii) Hubble's Law: The relationship between the distance and the velocity, or its red shift, can be derived. It is given by

$$d_L(z) = H_0^{-1}\left[z + \frac{1}{2}(1-q_0)z^2 + \ldots\right], \tag{3}$$

where d_L^2 is the luminosity distance given by

$$d_L^2 = \frac{\mathcal{L}}{4\pi\mathcal{F}}, \tag{4}$$

where $\mathcal{L}$ is the absolute luminosity and $\mathcal{F}$ is the measured flux.

H_o is the Hubble constant defined by

$$H_0 = \frac{\dot{R}(t_0)}{R(t_0)} \tag{5}$$

and q_o is the deceleration parameter given by

$$q_0 = -\ddot{R}(t_0)R(t_0)/\dot{R}^2(t_0). \tag{6}$$

H_o determines the rate of expansion of the universe today. Its measured value is

$$H_o = 100\,h\,Km\,s^{-1}\,Mpc^{-1},$$
where h is the uncertainty $0.4 \leq h \leq 1.0$.

Note that this leads to a time and length scale,

$$t \sim H_o^{-1} \sim 10 \times 10^9\,h^{-1} yrs,$$
$$l \sim ct \sim 3000\,h^{-1} Mpc$$

The dynamics of the universe is given by the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \quad (7)$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, G is the gravitational constant and $T_{\mu\nu}$ is the energy momentum tensor. Since the metric is isotropic and homogeneous, we choose $T_{\mu\nu}$ with the same symmetries. In the simplest case this is that of a perfect fluid :

$$T^{\mu}_{\nu} = \mathrm{diag}(\rho, -p, -p, -p). \quad (8)$$

The relationship between p and ρ is given by the equation of state. For simplicity, we take this to be $p = w\rho$, w constant. Then $w = 1/3$ for radiation, $w = 0$ for matter.

For the Friedman-Robertson-Walker metric, the Einstein equation gives us

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho, \quad (9)$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p). \quad (10)$$

Notice that since $(3p + \rho) \geq 0$, $\ddot{R} < 0$. This implies that at some finite time in the past, R was zero. We take this time to be the age of the universe $\sim H_o^{-1} \sim 10^{10} h^{-1}$ yrs. Rewriting the Freidman equation as

$$H^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho \quad (11)$$

$$\frac{k}{H^2 R^2} = \Omega - 1 \quad (12)$$

with

$$\Omega = \frac{\rho}{\rho_c}$$

where

$$\rho_c = \frac{3H^2}{8\pi G}$$

is the critical density. Since $H^2 R^2 \geq 0$, we have

$\Omega > 1$	$k = 1$	(closed universe),
$\Omega < 1$	$k = -1$	(open universe),
$\Omega = 1$	$k = 0$	(flat universe).

If we ignore spatial curvature, then the dependence of R on t can be easily obtained. The conservation of energy gives us

$$d(\rho R^3) = -p d(R^3), \tag{13}$$

which, together with the equation of state $p = w\rho$, gives us

$$\rho \propto R^{-3(1+w)} . \tag{14}$$

Together with the Freidman equation we get

$$R(t) \propto t^{2/3(1+w)} \tag{15}$$

Thus for a radiation dominated universe, $w = 1/3$, $R \sim t^{1/2}$ while for a matter dominated universe, $w = 0$, $R \sim t^{2/3}$. The evolution of R(t) for different values of k is shown in Figure 1.

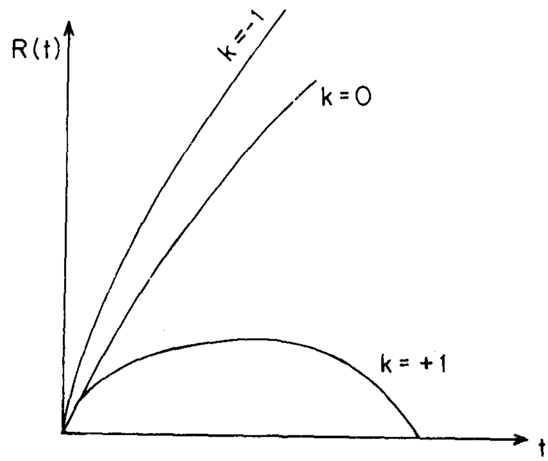

Figure 1. Evolution of R(t) for different values of k

2. Determination of Ω

From the Freidman equations, we saw that the fate of the universe crucially depends on the value of Ω. If $\Omega < 1$, the universe is open and will continue to expand. If $\Omega > 1$, the universe is closed, and will ultimately collapse. Finally, if $\Omega = 1$, the universe is flat corresponding to an Einstein- de Sitter universe. Thus it is important to determine the value of this parameter. There are basically two kinds of methods to determine Ω : dynamical and kinematical [2] as explained below.

Dynamical methods essentially rely on the observation of the gravitational effects of the mass density and inferring the mass density from these. These include studying the Rotation Curves of galaxies, using the Virial Theorem for clusters of galaxies and studying the local peculiar velocity fields.

The simplest way to detect the gravitational effects of a mass density is to use Kepler's law $G\,M(r) = v^2 r$, where v is the orbital velocity at a distance r and M(r) is the total mass inside of r. Rotation curves, i.e., curves of v vs r have been plotted for various spiral galaxies. They are obtained by measuring the spectral features of stars and also the 21 cm line from HI regions. Here the stars and clouds serve as test particles. The general features of rotation curves are similar; they rise sharply

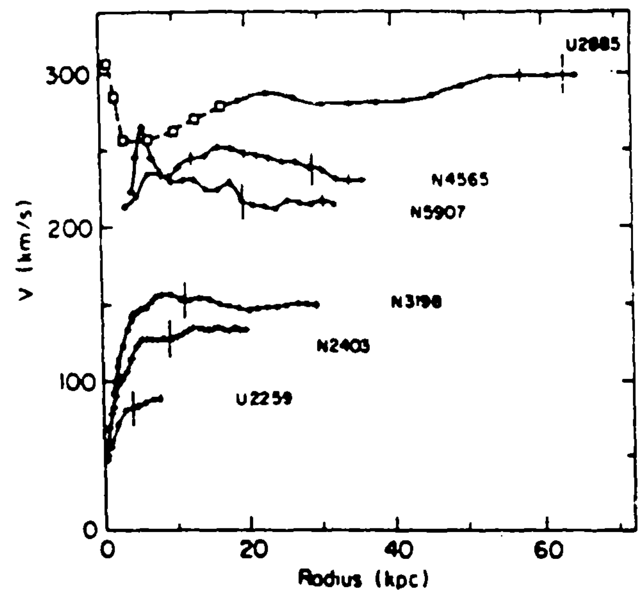

Figure 2. Rotation curves for spiral galaxies (From Ref. [1])

from the center and then become flat and remain flat. In some cases the curves have been plotted up to distances where light is only about a fraction of a percentage of what it is at the center. From these curves we determine the Ω corresponding to the luminous regions of the galaxies. Luminous matter gives

$$\Omega_{\rm LUM} \leq 0.01.$$

For regions where light effectively ceases, the flat part of the curves indicate M(r) $\propto$ r, i.e there is more matter than we see. In fact, the rotation curves of almost all galaxies indicate dark halos which have many times more mass than the bright regions :

$$\Omega_{\rm HALOS} \geq 0.03 - 0.1 .$$

One can also use the statistical form of Kepler's laws, the Virial Theorem for clusters of galaxies :

$$\frac{GM}{r} = \langle v^2 \rangle,$$

where M is the cluster mass, r is the core radius and $\langle v^2 \rangle^{1/2}$ is the velocity dispersion. These estimates give us

$$\Omega_{\rm CLUSTER} \sim 0.1 - 0.3,$$

thereby indicating some nonluminous mass. There are some uncertainties in these estimates which come from the fact that only about 10% of the galaxies are cluster members. Further, the assumptions of virialization and of spherical symmetry may not be valid.

Another dynamical method of determination of Ω is the study of the local peculiar velocity field of galaxies. All galaxies have a velocity because of the Hubble expansion. If there is a local density enhancement, then the galaxy will also have a peculiar velocity, over and above the Hubble velocity. For instance, the Virgo cluster, at a distance of about 20 Mpc has a density enhancement. The peculiar velocity obtained from this indicates

$$\Omega \sim 0.2.$$

A similar analysis of the IRAS catalogue (scales about 100 Mpc) gives us an $\Omega \geq 0.2$, and maybe even 0.6.

In all these dynamical determinations of Ω one must keep in mind that all of them are dominated by local enhancements of density and thus are relatively insensitive to the effects of any smooth distribution of matter.

Kinematic determinations of Ω: these include the Hubble diagram, the galaxy number count vs red shift and the angle vs red shift relationships. All these methods rely on the fact that

$$q_0 = \frac{\Omega_0(1+3p/\rho)}{2}$$

and so a determination of q_o gives us a value of Ω. But these methods have a disadvantage in that the accurate determination of distances involves use of standard candles and so evolutionary effects become very important. The results from these determinations are still very uncertain but some reports have $\Omega \sim 0.9$ (Loh Spillar). At the moment these are inconclusive and uncertain but with improvements these could provide the best estimates of Ω on the largest scales.

Primordial Nucleosynthesis: one of the most stringent tests of the standard cosmological model is the prediction of the abundances of light nuclei, D, ^{3}He, ^{4}He, ^{7}Li [3]. The predicted abundances agree with the inferred abundances if two parameters are in the correct range. One is the number of light neutrino species, which we now know to be ≤ 3.4 from Z width measurements. The other parameter is the baryon to entropy ratio, $\eta = n_B/s$. For agreement with experiment,

$$3 \times 10^{-10} \leq \eta \leq 5 \times 10^{-10}.$$

Knowing the present photon temperature accurately, this translates into a limit on Ω_B, the critical density contributed by baryons :

$$0.011 \leq 0.011h^{-2} \leq \Omega_B \leq 0.019h^{-2} \leq 0.12.$$

Inflation, which is certainly one of the most attractive ways to solve various fundamental problems plaguing the standard cosmological model, demands, $\Omega = 1$ to a high degree of accuracy. Finally, the age of the universe gives us a constraint $\Omega h^2 \leq 1$.

Summary of Ω values :

From rotation curves for spirals, we get

$$\Omega_{LUM} \leq 0.01,$$

while dynamical methods imply

$$\Omega \sim 0.2.$$

From kinematical evidence, which is somewhat uncertain, one has

$$\Omega \sim 0.9.$$

From nucleosynthesis it follows that

$$0.011 \leq \Omega_B \leq 0.12,$$

whereas inflation gives

$$\Omega = 1.0.$$

The age of the universe implies

$$\Omega h^2 \leq 1.$$

Thus we see that there is certainly some dark matter. Nucleosynthesis bounds also tell us that some of this dark matter is baryonic. If we take the uncertain kinematical evidence into account then there may well be some nonbaryonic dark matter. We would like to survey the various possibilities which exist for non baryonic dark matter. In particular we will consider various candidates for dark matter and try and obtain cosmological bounds on their properties.

3. Thermodynamics in an expanding universe

To study the potentiality of various candidates to contribute to the mass density of the universe, we need to know their abundances and the evolution of their abundances. For this purpose we need to study thermodynamics in the early universe [4].

The basic quantity is the distribution function f(x, p, t). From homogeneity, f will not depend on x. Further we assume that the interactions only serve to thermalize the species. Then, in thermal equilibrium, the form of f is that of an ideal Fermi-Dirac or Bose Einstein gas :

$$f(p) = \frac{1}{\{\exp(E-\mu)/T\} \pm 1}, \tag{16}$$

where $E^2 = p^2 + m^2$, μ is the chemical potential, and the + sign is for Fermi Dirac statistics and − for Bose Einstein.

The number density, energy density and pressure can then be calculated,

$$n = \frac{g}{(2\pi)^3} \int f(p) d^3p \tag{17}$$

$$\rho = \frac{g}{(2\pi)^3} \int E f(p) d^3p \tag{18}$$

$$p = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E} f(p) d^3p \tag{19}$$

where g is the number of internal degrees of freedom. These expressions simplify considerably in the relativistic and non relativistic limits.

The relativistic limit, $T \gg m, \mu$, implies

$$n = \zeta(3)gT^3/\pi^2 \qquad \text{(Bose Einstein)}, \tag{20}$$

$$= 0.75\,(\zeta(3)gT^3/\pi^2) \qquad \text{(Fermi Dirac)}, \tag{21}$$

$$\rho = \frac{\pi^2}{30}gT^4 \qquad \text{(Bose Einstein)}, \tag{22}$$

$$= \frac{7}{8}\frac{\pi^2}{30}gT^4 \qquad \text{(Fermi Dirac)}, \tag{23}$$

where $\zeta(3)$ is the Reimann Zeta function of 3. This expression assumes that all the species in equilibrium are at the same temperature. If they are at different temperatures then we get

$$\rho = \frac{\pi^2}{30}g_*T^4, \tag{24}$$

where

$$g_* = \sum_B g_i(T_i/T)^4 + \frac{7}{8}\sum_F g_i(T_i/T)^4. \tag{25}$$

Clearly g_* is a function of temperature. At different temperatures it has different values.

$T \ll 1\,\text{MeV}$	$g_* \sim 3.36$	(only γ and 3ν),
$T \sim 10\,\text{MeV}$	$g_* \sim 10.75$	(e^+, e, γ and 3ν),
$T \gg 300\,\text{GeV}$	$g_* \sim 106.75$	(all the Standard Model particles).

In contrast, the non relativistic Limit $T \ll m$ implies

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}, \tag{26}$$

$$\rho = mn, \tag{27}$$

$$p = nT \tag{28}$$

Another important quantity is entropy. From the conservation of energy we get that the entropy per comoving volume S is a constant :

$$S = \frac{(\rho + p)\,V}{T}. \tag{29}$$

The entropy density s

$$s = \frac{S}{V} \tag{30}$$

is given, since the entropy density is dominated by relativistic particles, by

$$s = \frac{2\pi^2}{45}g_{*s}T^3, \tag{31}$$

where

$$g_{*s} = \sum_B g_i(T_i/T)^3 + \frac{7}{8}\sum_F g_i(T_i/T)^3; \tag{32}$$

obviously $g_* = g_{*s}$ if all the species are at same temperature. Also since S is conserved,

$$T \propto g_{*s}^{-1/3} R^{-1}. \tag{33}$$

This means that

$$N = n/s = \#\text{ per comoving volume} \tag{34}$$

$$= \frac{45\zeta(3)\, g_*}{2\pi^4\, g_{*s}} \qquad (T \gg m,\ \mu) \tag{35}$$

$$\propto \left(\frac{m}{T}\right)^{3/2} e^{-(m-\mu)/T} \qquad (T \ll m). \tag{36}$$

Thermal equilibrium:
Of course, all the results we have obtained are valid only in thermal equilibrium. What exactly do we mean by thermal equilibrium? Strictly speaking, it is not possible to define equilibrium in an expanding universe. However, we can define it operationally as follows. In an expanding universe, the rate of expansion is given by H, the Hubble constant. The particles have some interactions which serve to thermalize them. If the rate of interactions is Γ, then as long as $\Gamma \gg H$, the interactions can thermalize quickly enough. Now

$$\Gamma \sim n\langle\sigma v\rangle. \tag{37}$$

When $\Gamma < H$, then the interactions do not operate fast enough to thermalize and we do not have thermal equilibrium. Then we say that the particular species has decoupled from the plasma.

What happens to the decoupled particle? Its distribution function retains the same form as for equilibrium. This is easy to see because

$$dN = f\, dV\, d^3p \tag{38}$$

is conserved. dV and d^3p both suffer from red shifting which cancels out and so the form of f must be conserved. Let the distribution function after decoupling be $f_{dec}(p,t)$, $t > t_{dec}$, i.e.

$$f_{dec}(p,t) = f_{equi}\left(\frac{pR(t)}{R(t_d)}, t_d\right). \tag{39}$$

In the relativistic case,

$$f_{dec}(p,t) = \frac{1}{\exp\left(ER(t)/R(t_d)T\right) \pm 1}. \tag{40}$$

This is the same as for a particle in thermal equilibrium but with a temperature

$$T = T_d \frac{R(t_d)}{R(t)}. \tag{41}$$

Note that the temperature falls as R^{-1} and not as $g_{*s}^{-1/3} R^{-1}$ like in the case of thermal equilibrium.

The number density of these decoupled particles is

$$n = g_{eff} \frac{\zeta(3) T_d^3}{\pi^2} \frac{R(t_d)^3}{R(t)^3}, \tag{42}$$

where g_{eff} is g for bosons and .75 g for fermions. Note that this number density is comparable to photons. As the universe evolves, the temperature falls and ultimately these become non relativistic, contributing $\rho = mn$ to the energy density today.

In the case where decoupling takes place when the particle is non relativistic,

$$f_{dec}(p) = \frac{g}{(2\pi)^3} e^{-m/T_d} \exp\left(\frac{p^2 R(t)^2}{2mT_d R(t_d)^2}\right) \tag{43}$$

Here

$$T = T_d \frac{R(t_d)^2}{R(t)^2} \propto R^{-2},$$

i.e. the temperature falls as the inverse square of the distance.

Relic background of particles:
Consider any stable, massive particle. If it remained in thermal equilibrium, its abundance today would be exponentially suppressed and it would not contribute significantly to the energy density. However if it decoupled at some time in the past, it could contribute significantly to the energy density. To find out the relic abundances, we should really integrate the Boltzman equation given by [5]

$$\dot{n} + 3Hn = \text{(interaction terms)}, \tag{44}$$

where the right hand side involves interactions, the 3Hn term is the effect of the expansion.

For a stable, massive particle only annihilation and inverse annihilation channels are open. In this case the Boltzman equation becomes

$$\frac{dN}{dx} = -\frac{x\langle\sigma_\alpha \mid v\rangle s}{H(m)} (N^2 - N_{eq}^2), \tag{45}$$

where $N = n/s$ is the number per comoving volume, x is m/T, N_{eq} is the equilibrium abundance of the species and $H(m)$ is the Hubble constant in terms of m.

There are basically two interesting cases of relics; hot and cold relics :

Hot relics:
Consider the case of particles which decouple while still relativistic, i.e., $T_d \gg m$. In this case the asymptotic value of N, N_∞ is simply the equilibrium value :

$$N_\infty = N_{eq}(T_d) = 0.278 \frac{g_{eff}}{g_{*s}(T_d)}. \tag{46}$$

This means that today their abundance is of the order of that of photons,

$$n = s_0 N_\infty = 2970 N_\infty \text{cm}^{-3} \tag{47}$$

and their contribution to the energy density is

$$\rho = mn = 2970 N_\infty (m/eV)\, \text{eV cm}^{-3} \tag{48}$$

which gives us

$$(\Omega h^2)_{hot} = 0.0783 \left(\frac{g_{eff}}{g_{*s}(T_d)}\right) \frac{m}{eV}. \tag{49}$$

Since $\Omega h^2 \leq 1$, we get

$$m \leq 12.8\text{eV}\left(\frac{g_{eff}}{g_{*s}(T_d)}\right). \tag{50}$$

Massive light neutrinos are good examples of hot relics.

For masses < MeV, decoupling occurs when $\Gamma \sim G_F^2 T^5 \sim H$. Here G_F is the Fermi constant. The decoupling occurs at around 1 MeV. Then g_{*s} is about 10.75, and g_{eff} is 1.5 giving us

$$m \leq 92\,\text{eV}.$$

Of course, if the decoupling temperature was much higher, leading to a higher g_{*s}, we get a higher bound on m. For instance, if T_d was $\geq$ 300 GeV, then g_{*s} is about 106 and we get

$$m \leq 910\,\text{eV}$$

Thus neutrinos with masses of order 100 eV and a decoupling temperature of about 1 MeV can provide closure to the universe.

Cold relics: These are particles which decouple when they are nonrelativistic, i.e., $T_d \leq m$. In this case

$$N_{eq} = 0.145\left(\frac{g}{g_{*s}}\right)\left(\frac{m}{T_d}\right)^{3/2} e^{-m/T_d}. \tag{51}$$

Here the equilibrium abundance depends strongly on m and we must determine T_d. Ideally we should integrate the Boltzman equation. However, even using the rough criterion of $\Gamma \sim H$, gives us fairly accurate results.

For nonrelativistic particles, we can assume

$$\langle\sigma v\rangle = \sigma_o \frac{T^k}{m}, \tag{52}$$

where k is a constant depending on the details of the process, i.e whether the process is S wave or P wave etc. It is usually of order 1. Then

$$\Gamma = n\langle\sigma v\rangle \tag{53}$$

$$= \frac{\sigma_o g T^3}{(2\pi)^{3/2}}\left(\frac{m}{T}\right)^{3/2-k} e^{-m/T}. \tag{54}$$

Assuming a radiation dominated universe,

$$H = 1.66 g_*^{1/2}\frac{T^2}{m_{PL}} \tag{55}$$

$$\Rightarrow \quad m/T_d \sim 17.8 + 3\ln\left(\frac{m}{\text{GeV}}\right) \tag{56}$$

and

$$N_\infty \sim \frac{4g_*^{1/2}}{g_{*s} m m_{PL}\sigma_o}\left(\frac{m}{T_d}\right)^{k+1} \tag{57}$$

$$\Rightarrow \quad (\Omega h^2)_{cold} = \frac{10^9}{g_*^{1/2} m_{PL}\sigma_o}\left(\frac{m}{T_d}\right)^{k+1}\text{GeV}^{-1}. \tag{58}$$

An example of cold relics is heavy neutrinos (m $\gg$ MeV). There are two interesting cases, m > m_Z , m < m_Z. For m < m_Z, we can parameterize σ as above and then

$$\sigma_o \sim G_F^2 m^2, \tag{59}$$

where G_f is the Fermi constant. Now

$$N_\infty \sim 6 \times 10^{-9} \left(\frac{m}{GeV}\right)^{-3} \tag{60}$$

and

$$(\Omega h^2) \sim 3 \left(\frac{m}{GeV}\right)^{-2} \tag{61}$$

For m > m_Z, σ_o goes as m^{-2} because of the momentum dependence of the Z propagator and thus we get

$$(\Omega h^2) \sim \left(\frac{m}{TeV}\right)^2 \tag{62}$$
$$\Rightarrow \quad m < 1\,TeV.$$

Thus for neutrinos, relic mass density increases as m up to a few MeV (hot relics), then decreases till about 100 GeV (cold relics) and finally increases as m^2 for large values of m. Closure density is possible for three ranges of m, 100 eV, 1 GeV, 1 TeV.

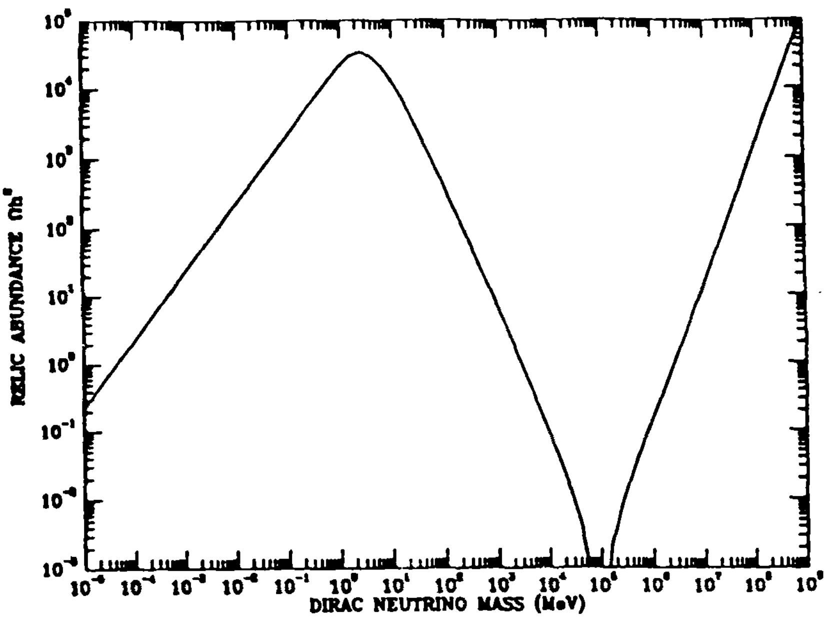

Figure 3. Neutrino mass vs Ωh^2 (From Ref. [6])

The above mentioned analysis holds for stable neutrinos. If the neutrinos had a finite lifetime, then it is possible to evade these limits. The decay products of the neutrino (if relativistic) will have an energy density which goes as R^{-4} as opposed to R^{-3} for the stable neutrino. Thus the energy is $(1 + z_D)$ less than if the neutrino was stable, z_D being the red shift at the time of decoupling [6].

Thus

$$\Omega_{decay} h^2 = \Omega_\nu h^2 (1 + z_D)^{-1} \tag{63}$$

giving us bounds on $(1 + z_D)$:

$$\begin{aligned} 1 + z_D &\geq (m/21.8\text{eV}) && 21.8 \text{ eV} \leq m \leq 1 \text{ MeV}, \\ &\geq (5\text{GeV}/m)^2 && 1 \text{ MeV} \leq m \leq 5 \text{ GeV (Dirac } \nu\text{s)}, \\ &\geq (18\text{GeV}/m)^2 && 1 \text{ MeV} \leq m \leq 18 \text{ GeV (Majorana } \nu\text{s)}. \end{aligned}$$

These can be translated into bounds on neutrino lifetimes as in the figure 3. These

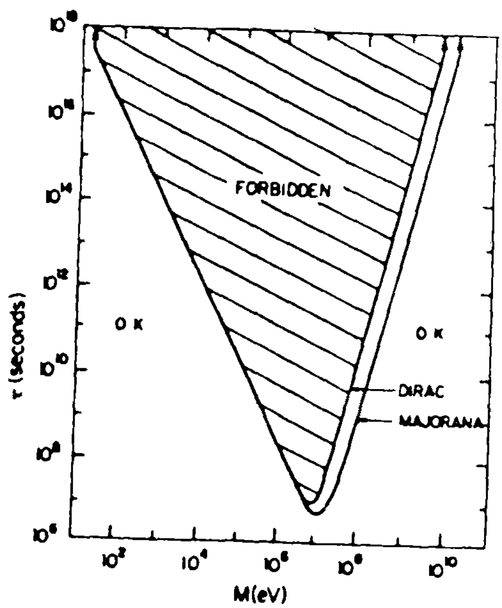

Figure 4. Neutrino lifetime vs mass (From Ref. [6])

bounds are valid if the decay products are all invisible. If some of the decay products are visible (photons, electrons etc) then much more stringent bounds exist from CMBR and nucleosynthesis.

4. Neutralino dark matter

Supersymmetric theories [7] predict the existence of a lightest supersymmetric particle (LSP), which is stable in most models because of R parity. R parity is a discrete symmetry which is enforced to obtain a long enough proton lifetime. It is given by

$$\begin{aligned} R &= +1 && \text{for known particles,} \\ &= -1 && \text{for supersymmetric particles,} \end{aligned}$$

$$= \quad (-1)^{L+3B+2S}.$$

The favored LSP is a neutralino, which is linear combination of the Bino, Wino and the Higgsinos :

$$\tilde{\chi} \;=\; Z_{11}\tilde{B} + Z_{12}\tilde{W} + Z_{13}\tilde{H}_1 + Z_{14}\tilde{H}_2. \tag{64}$$

The masses are completely determined by three following parameters :

	$\tan\beta = v_2/v_1$,	where v's are the vevs of the Higgs,
	M, M′	the soft supersymmtery breaking parameters
and	μ	the supersymmetric Higgs mass.

For $\mu \ll M$, the neutralino is essentially a Higgsino with a mass μ. For $\mu \gg M$, the neutralino is essentially a gaugino with a mass given by M′.

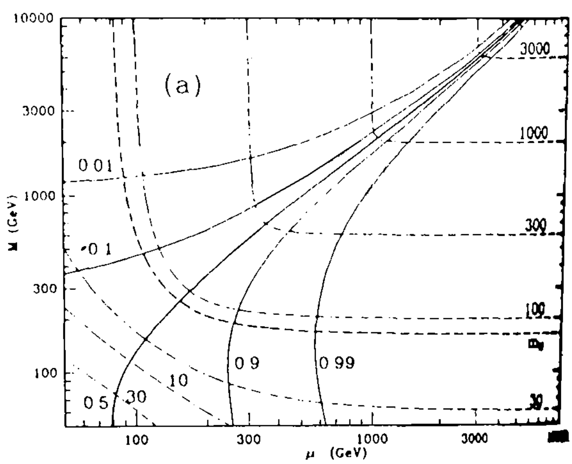

Figure 5. Neutralino mass and composition (From Ref. [8])

To determine the relic abundance we need masses and cross sections. The mass is undetermined though accelerator searches have pushed up the mass of the neutralino to be > 20 GeV. We also know that if SUSY is to solve the hierarchy problem then, m_{susy} should be about m_{weak}. In this case the interaction rates are comparable to weak rates and the neutralino behaves qualitatively like neutrinos. The relic abundances can be calculated. However because of the large number of parameters, the abundances depend on μ, $\tan\beta$ and M.

Let us summarize some of the results for neutralinos [8], [9]. As mentioned above, neutralinos behave qualitatively like massive neutrinos and are cold relics. Over most of parameter space, the abundances give $\Omega h^2 \sim O(1)$. (This implies fixing some model parameters of course) In fact, for the neutralino mass in the 100 GeV and TeV range, the value of Ωh^2 is 1. For large neutralino masses, the cross section goes as m^{-2} and so like neutrinos gives us an upper bound on the neutralino mass. For a pure Higgsino state, $m_\chi < 3000$ GeV. For a pure gaugino state, $m_\chi < 550$ GeV.

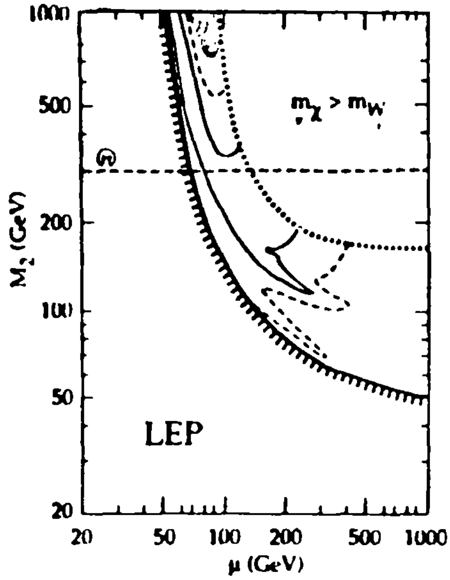

Figure 6. Relic density of $\tilde{\chi}$ (From Ref. [9])

5. Axion

Another attractive candidate for dark matter is the axion. The axion arises as the Nambu Goldstone boson (pseudo) when the Peccei-Quinn symmetry is spontaneously broken [10]. The PQ symmetry is an elegant way to solve the strong CP problem in QCD. The properties of the axion are :

Mass: The mass of the axion is related to the PQ scale f_{PQ}

$$m_a \sim 0.62\,eV\frac{10^7\,GeV}{f_{PQ}/N}, \tag{65}$$

where N is the color anomaly and f_{PQ} the Peccei Quinn symmetry breaking scale which is unknown.

Interactions: The axion interacts with nucleons, photons, electron etc. with all couplings proportional to m_a.

Finite lifetime: The axion decays primarily into 2 photons through the triangle graph

$$\tau \sim 6\times 10^{24}\,secs\left(\frac{m_a}{1\,eV}\right)^{-5}. \tag{66}$$

The mass of the axion and hence its interactions are not determined by theory. There are, however, bounds on the mass. From rare kaon and quarkonium decays ($K^+ \to \pi + a$, $J/\psi \to a + \gamma$) we get

$$m_a < 10\,KeV \text{ or } f_{PQ} > 1000\,GeV.$$

There are even more stringent bounds which arise from stellar evolution. Since axions interact very weakly, they can rapidly transfer energy from the stellar interior to the surface. From observations of stellar lifetimes, one can put the following bounds on the axion mass :

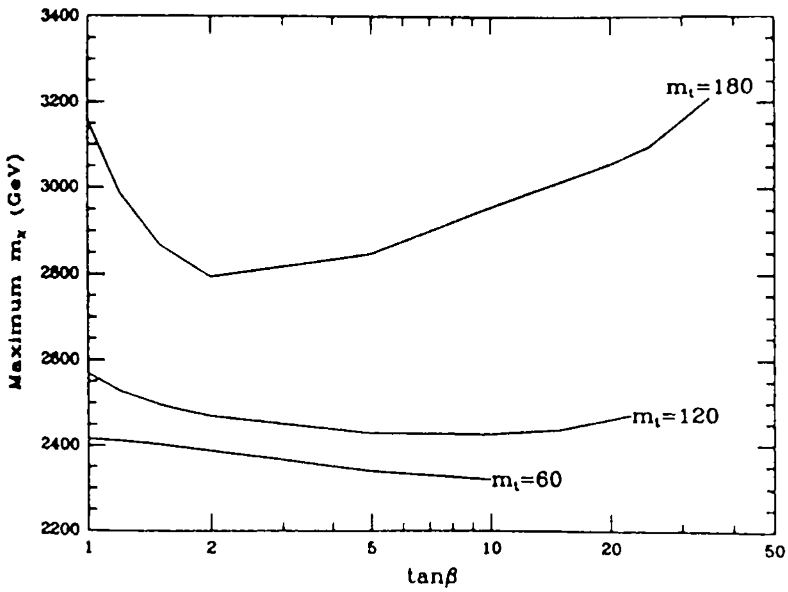

Figure 7. Limits on M_χ (From Ref. [8])

$$
\begin{array}{lll}
m_a & < 1\,\mathrm{eV} & \text{(Main Sequence Stars)},\\
m_a & \sim (10^{-2} - 2)\,\mathrm{eV} & \text{(Red Giants)},\\
m_a & \sim (10^{-3} - 1)\,\mathrm{eV} & \text{(SN 1987A)}.
\end{array}
$$

Relic abundances:

There are three mechanisms for axion production in the universe:

Thermal production:
Axions are produced in interactions like

$$
\begin{aligned}
N + \pi &\rightarrow N + a,\\
\gamma + Q &\rightarrow Q + a.
\end{aligned}
$$

Assuming the axions to be relativistic, we get

$$n_{eq} = \frac{\zeta(3)\Gamma^3}{\pi^2} \tag{67}$$

We can calculate Γ and compare with H to get decoupling temperature. The result for the relic abundance today is

$$(\Omega h^2)_{axion} = \frac{m_a}{130\mathrm{eV}}\left(\frac{10}{g_*(T_d)}\right). \tag{68}$$

Since $\Omega h^2 < 1$, we get $m_a < 100$eV. Thus only axions of about 100 eV can provide closure. But this mass range is excluded from astrophysical considerations. In any case, if $m_a \sim 100$ eV, the lifetime is smaller than the age of the universe.

Misalignment mechanism:
For axion masses $> 10^{-2}$ eV, the thermal production of axions is not dominant, rather the production by misalignment is dominant. Instanton effects are very

highly temperature dependent. At very high temperatures, the axion is massless. When the PQ symmetry breaks, there is a random choice of initial Θ, which is different from its zero temperature value and thus misaligned. Around $T \sim \Lambda_{QCD}$, instanton effects turn on and the field develops a mass. It relaxes towards $\Theta = 0$, overshoots and oscillates behaving like nonrelativistic matter. The equation of motion is

$$\ddot{\Theta} + 3H\dot{\Theta} + m_a^2(T)\Theta = 0 \tag{69}$$

With this we can solve for n, and get Ωh^2. Taking into account all the details, we get

$$\Omega h^2 \sim \left(\frac{m_a}{10^{-5}\,eV}\right)^{-1.18} \Theta_1^2 \Lambda_{200}^{-0.7}, \tag{70}$$

where Λ_{200} is $\Lambda_{QCD}/200$ MeV. If Θ_1, the initial value is of order 1, then we can get closure for m_a between 10^{-6}eV and 10^{-4} eV. Note that these axions are produced cold, i.e. their effective mass is more than the temperature at which they are produced.

Axionic string decay:
The spontaneous breaking of $U(1)_{PQ}$ leads to the production of one dimensional defects or axionic strings. These axionic strings radiate axions and are another source of production of axions. The calculations of the energy of the axions thus produced are fairly uncertain, but basically one gets

$$\Omega h^2 \sim \left(\frac{m_a}{10^{-3}\,eV}\right)^{-1.18}. \tag{71}$$

Thus we can get closure for a mass range of m_a between 10^{-3} eV and 10^{-4} eV.

Summary:[11]

1. Only two mass windows are allowedfor axions : $m_a \sim 10^{-5}$ eV, 1 eV.
2. Thermally produced axions cannot provide closure density. Axions produced by misalignment or axionic string decay can but there are many uncertainties in the calculations.

6. Monopoles

Superheavy magnetic monopoles are another candidate for dark matter. These arise generically in most Grand Unified Theories and have masses of $\mathcal{O}(M_{gut})$. The production of topological defects in the expanding universe is by the Kibble mechanism. This is basically related to the fact that particle horizons exist and thus there is a finite volume which is causally related. Thus the Higgs field will be uncorrelated at distances larger than the horizon volume. This implies that there is at least one monopole per horizon volume. If the annihilations are ineffective (which they indeed are), then we will have a significant relic density in monopoles.

$$n_m \sim d_H^{-3}, \tag{72}$$

$$d_H \sim H^{-1}, \tag{73}$$
$$n_M \sim H^3 \sim T_c^6/m_{PL}^3, \tag{74}$$
$$s \sim T^3, \tag{75}$$
$$N_m \sim n_M/s \sim \left(\frac{T_c}{m_{PL}}\right)^3. \tag{76}$$

Today the contribution to the energy density would be

$$\Omega h^2 \sim 10^{24}\left(\frac{n_M}{s}\right)\left(\frac{m}{10^{16}\,\mathrm{GeV}}\right) \tag{77}$$

Typically,

$$m \sim 10^{16}\,\mathrm{GeV},\ T_c \sim 10^{14}\,\mathrm{GeV}$$
$$\Rightarrow \quad \Omega h^2 \gg 1,$$

which is unacceptable. This is the monopole problem which was one of the original motivations for inflation. Inflation could solve this problem. Also, we could conceive of more complicated symmetry breaking patterns which could lead to an acceptable relic density of monopoles.

7. Summary

We can confidently say that the amount of luminous matter in the universe is much less than the amount of dark matter. From an analysis of primordial nucleosynthesis, it is reasonable to assume that some of the dark matter is baryonic. This could be in the form of jupiters, brown dwarfs and other such objects. Amongst the nonbaryonic dark matter candidates, there are three attractive possibilities. A stable, 100 eV neutrino which would be a hot relic. An axion of mass 10^{-5} eV which would be a cold relic and would be produced nonthermally. A neutralino of mass between 100 GeV and 3 TeV would also be a cold relic. Given that none of these three attractive dark matter candidates have been seen, it is clear that future lab searches and experiments will be important in determining the interactions and masses of these. Thus the clue to the ultimate fate of the universe may well be found in the laboratories!

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Discussion

V.Soni : Are GUT monopoles or other heavier ones observationally ruled out?

S. Mahajan : Probably. At least they are not favoured.