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Sphalerons and electroweak baryogenesis

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Abstract. It is well known that baryon (lepton) number is not conserved in the standard electroweak theory due to the anomaly. As a consequence, the electroweak phase transition provides plausible ground for producing the baryon number asymmetry of the universe, besides erasing any prexisting asymmetry. This occurs via processes which surmount the instanton potential barrier. The barrier is given by the static saddle point solution of the electroweak theory, called the sphaleron, which we shall briefly expose. This is followed by a pedagogical account of how the asymmetry can be created during the electroweak phase transition including the pitfalls that await the innocent practitioner.

1. Introduction

Originally, in the big bang one had to build in the observed matter antimatter asymmetry of the universe as an initial condition. Grand Unified Theories (GUTS) unified quarks and leptons in the same group multiplet creating vertices that could change baryon number (B) into lepton number (L) and give proton decay mediated by the heavy vector bosons. This providded the possibility of Baryogenesis through interactions at an epoch in the history of the universe when the temperature was 10^{16} GeV.

Lately, has come the realization that the standard electroweak (EW) anomaly in the baryon (lepton) number current can itself be a source of B violation via processes mediated by instanton like configurations in contrast to the B violating vertices in GUTS. At temperatures of the order of the instanton barrier height, which is given by the sphaleron, such processes can occur freely. Baryogenesis does not really require GUTS any more and can happen at a much lower energy scale, the electroweak scale $10^2 \sim 10^3$ GeV, that is, at a much later epoch in the history of the universe when the temperature is of the order of the electroweak scale. To this very lively issue we now turn.

2. The anomaly

The standard electroweak theory – the Weinberg, Salam, Glashow model – has an anomaly in the baryon (B) and lepton (L) number currents which arises from the fact that all quarks and leptons couple only to left handed $SU(2)$ gauge fields in accordance the $V - A$ parity violating nature of the weak interactions.

$$\partial_\mu J^{\mu B(L)} = N_G(g^2/64\pi^2)F\widetilde{F} = N_G\partial_\mu K^\mu,$$

N_G being the number of generations. The $U(1)$ hypercharge abelian gauge field is ignored here as it has no direct role in what follows. The right hand side has been cast as a total derivative of a purely $SU(2)$ left gauge field current K

$$K^\mu = (g^2/32\pi^2)\epsilon^{\mu\nu\rho\sigma}[F^a_{\nu\rho}A^a_\sigma - (2/3)g\epsilon_{abc}A^a_\nu A^b_\rho A^c_\sigma].$$

By an appropriate choice of gauge and some manipulations and on carrying out the space integration we find that

$$\partial_t B(L) = N_G\partial_t T,$$

where

$$B(L) = \int d^3x J_0^{B(L)} \quad \text{and} \quad T = \int d^3x K_0,$$

T being the so called winding number. For a process in which the gauge field configuration changes over time, then,

$$\Delta B(L) = N_G \Delta T.$$

This shows that gauge fields that evolve over time and change the winding number T will necessarily violate B and L!

Now, do we know of any such processes that change T? Of course these are the instantons and t Hoof't [1], in his seminal work, already evaluated the vacuum tunneling from instantons that violates B and L to be

$$\Gamma \sim e^{-(4\pi/\alpha_W)}$$

which is so negligible as to hardly induce any observable B violation during the age of the universe.

3. Digression on instantons

The instanton [2] is the minimum-action Eucidean solution to a Yang Mills gauge theory that changes T by one as it goes from Euclidean time $-\infty$ to $+\infty$. It is scale invariant. In a Yang Mills Higgs theory, as in the standard electroweak model, there is a scale, the Higgs vacuum expectation value. Scale invariance is therefore broken and there are not exact instanton solutions.

However, there are instanton like configurations plotted in Fig. 1. Notice an important feature of the nonabelian instanton. In going from a trivial pure gauge $T(A_i = 0) = 0$ by a so-called large gauge transformation, $U\partial_i, U^{-1}, T(A'_i = U\partial_i U^{-1}) = 1$ one must pass over a hump in the energy axis – the instanton potential barrier (IPB). We can begin with a pure gauge, $A'_i = U\partial_i U^{-1}$, which has $T(A'_i) = 1$ and use a scaling function $\lambda(t_\epsilon)$ of the evolution parameter t_ϵ, to write

$$A_\mu(t_\epsilon) = \lambda(t_\epsilon)U\partial_\mu U^{-1}$$

where $\lambda(t_\epsilon)$ is such that it interpolates from $\lambda(t_\epsilon) = 1$ at $t_\epsilon = +\infty$ to $\lambda(t_\epsilon) = 0$ at $t = -\infty$. The field tensor

$$F^a_{ij} = \partial_i A^a_j - \partial_j A^a_o - g\epsilon_{abc} A^b_i A^c_j,$$
$$F^a_{ij} = \lambda(t_\epsilon)(\partial_i A^1_j - \partial_j A^1_i) - g\lambda^2(t_\epsilon)\epsilon_{abc} A^{1b}_i A^c_j,$$

cannot be pure gauge ($\lambda = 1$ or 0) in the interval $-\infty < t_\epsilon < +\infty$ due to the presence of the non-linear (non-abelian term). Therefore, this configuration must have a nonzero energy when $-\infty < t_\epsilon < +\infty$ while it has zero energy at the extremeties $t_\epsilon = -\infty$ and $t = +\infty$ when we are in a pure gauge. Hence the barrier in Fig. 1. It is this barrier that gives the exponential suppression for the vacuum tunnelling $B(L)$ violating amplitude.

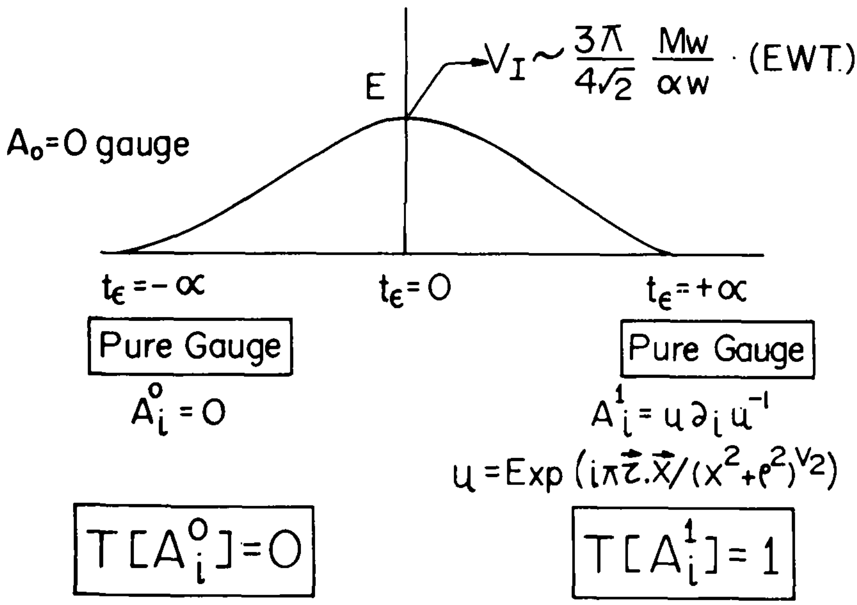

Figure 1. $\Delta T = 1$ Instanton minimum action solution (configuration for E.W. theory).

However, when such a process does go, it gives a $B(L)$ violation

$$\Delta B = \Delta L = N_G \Delta T,$$

where N is the number of generations. We then expect to see an excess of 3 quarks and 1 lepton per generation in an instanton-mediated process.

4. The sphaleron

Now we shall take up the sphaleron. The sphaleron is a static solution for the gauge-Higgs sector of the electroweak theory. This was first found by the author [3] and subsequently by Manton [4] who also showed that it is actually a saddle point of the energy functional that can be identified with the top of the barrier in Fig. 1.

It can be shown by direct substitution of the sphaleron gauge fields [5] that it carries a winding number $T = 1/2$. Also, from the existence of a zero energy bound state in the sphaleron background (which is charge conjugation symmetric) its induced fermion number is found to be 1/2 [6]. This confirms its identification with the top of the potential barrier in Fig. 1 which is exactly halfway in the instanton process that takes $T(A)$ from 0 to 1. Its significance is related to the fact that, as there are no exact instanton solutions in the EW theory, one cannot exactly compute the IPB. However, from the sphaleron solution one can get not only the exact barrier height but also the exact configuration of the fields at the barrier. This is important as in the early universe at temperatures of the order of the sphaleron energy, $E_{sp} = M_{sp} = 10$ TeV, the processes that give $B(L)$ violation are real time processes with non-zero probability of going over the barrier typically these are controlled by the barrier height in the Maxwell Boltzmann factor $\exp(-M_{sp}/T)$ and the landscape about the barrier which depends on the exact field configuration.

The sphaleron solution considered in [4] is for the $SU(2)$ theory, whereas. Ref. [5] considers $SU(2) \times U(1)$ with $U(1)$ as a perturbation. For some intriguing aspects of non perturbative $SU(2) \times U(1)$ sphaleron configurations in external magnetic fields the second paper in Ref. [3] may be seen. One more point to keep in mind is that these sphaleron mediated processes violate B and L but not $B - L$.

5. Baryogenesis

We are now ready to consider Baryogenesis arising from sphaleron mediated processes. To begin with, let us introduce the subject through the canonical conditions for producing a B asymmetry enunciated by Sakharov [7]. The requirements are :

1) *B violating processes.*

For the electroweak theory these are not point particle processes arising from the interaction vertices as in Grand Unified Theories but proceed by the formation of sphaleron like configurations that take one over the barrier.

2) *CP or T violation.*

This implies that the forward and backward matter $\rightleftharpoons$ antimatter rates are different. The B asymmetry is

$$A_s = (n_q - n_{\bar{q}})/(n_q + n_{\bar{q}}).$$

Under CP, which takes q to $\bar{q}$ and therefore n_q to $n_{\bar{q}}$, A_s goes to $-A_s$. This implies that $A_s = 0$ if CP is unbroken.

3) *Departure from equilibrium.*

A heuristic way to understand this is to realise that even though the forward and backward rates are different, given enough time they will equilibrate finally producing no asymmetry. More formally, the equilibrium distributions of the q and q densities are

$$n_q \sim e^{-(m_q/kT)} \quad \text{and} \quad n_{\bar{q}} \sim e^{-(m_q/kT)}.$$

Under CPT, $m_q = m_{\bar{q}}$ and thus at equilibrium $n_q = n_{\bar{q}}$, giving $A_s = 0$. Recall that the observed asymmetry in our universe, given by the ratio of the baryon density to the photon density, is found to be

$$A_s = 3 \times 10^{-10}.$$

Remember, this is for $B - L = 0$ as the initial condition.

Baryogenesis is justifiably a subject of some confusion and the unwitting reader can easily be led astray. We shall try and correct this by a strict adherence to chronology.

6. Foreplay

Here we shall consider the point of evolution of the universe at which the temperature is above that for the Electroweak Phase Transition (EWPT). When the EW symmetry is not spontaneously broken. There is no length scale and therefore no sphleron solution. However, there are instanton like configurations which typically have an IPB configuration given by

$$A_i^a(t_\epsilon = 0) = \frac{1}{g_W} \frac{\epsilon_{ija} x_j}{(\vec{x}^2 + \rho^2} \qquad \text{and} \qquad IPB \sim 1/(\alpha_{W_e}),$$

where ρ is an arbitrary scale of the configuration. This is the counterpart of the sphaleron at $T > T_c$. However, there is need to be circumspect as there is now a temperature dependent magnetic mass $m_M = \alpha_W T$ which will screen out any configuration of scale $\rho > 1/(\alpha_W T)$. Given this caveat we are now ready to say something about the rate of B violation [see the very useful reviews by Shaposhnikov [8], Dolgov [9] and Dine [10]].

Typically, first let us indicate how the B violating rate can be calculated in the spontaneously broken phase when a sphaleron solution exists, via thermal real time processes. As we pointed out before, this rate depends on a Maxwell-Boltzmann exponential factor $\exp -(M_{sp}/T)$ multiplied by a prefactor that is the small fluctuation determinant about the sphaleron solution. The small fluctations explore the landscape about the sphaleron saddle point: if the saddle point is an undulation, we expect a substantial flow (rate) but if it is constricted the rate is expected to be similarily diminished. This rate calculation [11] has received considerable dedicated effort – we will refer the reader to the literature for further knowledge. Since, no exact calculation of the rate is available in the symmetric phase we shall use the broken phase results but with the configuration given above instead of the sphaleron to estimate the rate.

Consider the exponential factor. The sphaleron energy is substituted by the IPB above and the scale is given by the size of the largest configuration, $\rho \sim 1/(\alpha_W T)$. Putting these together we find

$$e^{-(M_{config.}/T)} \simeq e^{-(1/\alpha_{W_e} T)} \simeq e^{-1}.$$

Thus, we have the rather interesting result that the exponential factor is just a constant and therefore there is no exponential suppression of the rate. Then the prefactor may be guesstimated from the broken phase calculation for the B violation rate <u>per unit volume</u> of Mclerran et al [11] (see also Ref. [8] and [9]):

$$\Gamma = \frac{T^4\omega_-}{M_W(T)}\left(\frac{\alpha_W}{4\pi}\right)^4 N_{Tr}N_{RoT}\left(\frac{2M_W(T)}{\alpha_W T}\right)\exp\left(\frac{-M_{sp.}(T)}{T}\right)K,$$

where ω_- is the frequency of the unstable mode of the sphaleron. Now making the identification $IPB \sim M_{sp} \sim M_W/\alpha_W = 1/(\alpha_W\rho)$, we have $M_W = 1/\rho$ where $1/\rho = \alpha_W T$; also $\omega_- = 1/\rho = \alpha_W T$. Note that the instantaneous B density n_B is proportional to the rate Γ above which is proportional to the sphaleron density $n_{sp} \sim 1/\rho^3 = (\alpha_W T)^3$. It follows that the rate <u>per baryon</u> is

$$R_B = (1/N_B)\frac{dN_B}{dt} = \beta\alpha_W T.$$

The Hubble expansion rate is

$$R_H = \tau_U^{-1} = \dot{R}/R = (T^2/M_{Planck})1.66N_{eff},$$

where N_{eff} is the number of light species existing in the universe at that time. Thus

$$\frac{R_B}{R_H} \simeq \frac{\beta\alpha_W T M_{Planck}}{T^2(1.66N_{eff})} \geq 10^{12}.$$

So there is unsuppressed B violation in the early universe before the EWPT occurs. However, the rate is so much in excess of the Hubble expansion rate that there is no chance of going away from equilibrium and therefore no chance of producing a B asymmetry. The message is then that in the act of creating the B asymmetry one has to lose equilibrium.

Since our assumed initial condition is that the conserved $B - L$ is 0, this has the further effect of washing out any B excess and the equal L excess (to conform to the initial condition) that may have been produced at an earlier epoch as in the GUT scale B-asymmetry scenario. So, ironically, instead of producing any B asymmetry through this very significant B violation via EW instanton mediated processes we have wiped it all out.

The first people to take the sphaleron mediated B violation as a serious candidate for the B asymmetry of the universe were Kuzmin, Rubakov and Shaposhnikov [12] who calculated the rate of B violation for a second order EWPT and found also that all previous B asymmetry would be wiped out. We will not consider the second order EWPT here since we know that the B asymmetry is wiped out even before this and a second order transition does not take one out of equilibrium.

7. The act

We turn now to the to see if the EWPT can be a first order transition (to move one sufficiently out of equilibrium). Also, given some source of CP violation can it bias one in the direction of a net B asymmetry?

The EPWT is approached via a calculation of the effective potential of the electroweak theory at a finite temperature to one loop in the broken phase. Using temperature dependent propagators from finite temperature field theory in the loops, one gets a T dependence in the effective potential (from now on T will designate temperature and not winding number). It should be noted that this one loop calculation is somewhat rudimentary and several improvements are possible, e.g. renormalization group improvement. Also, the infrared effect arising from the spontaneous masses going to zero are nontrivial to include. There is an extended literature on the subject [13]; we also refer the reader to the reviews by Shaposhnikov [8] and Dine [10]. The calculation we exhibit is from Ref. [10].

At finite temperatures, particles with spontaneous masses $m(\phi)$ contribute terms proportional to m^2T^2, m^3T and $m^4 ln(m/T)$ to the effective potential $V(\phi, T)$. In what follows it is assumed that the Higgs mass $m_H < m_W, m_Z, m_t$ and therefore the Higgs boson contribution is neglected.

The zero temperature effective potential to one loop is given by

$$V_0^1 = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + 2BV_0^2\phi^2 - \frac{3}{2}B\phi^4 + B\phi^4 ln\left(\frac{\phi^2}{V_0^2}\right),$$

where, $B = 3(64\pi^2 V_0^4)^{-1}(2m_W^4 + m_Z^2 - 4m_t^2)$, $V_0 = 246$ GeV is the VEV of ϕ, $\lambda = \mu^2/V_0^2$ and $m_H^2 = 2\mu^2$. At a finite temperature, the extra contribution to the effective potential is given by

$$V_T = T^4/(2\pi^2)[6I_-(y_W) + 3I_-(y_Z) - 6I_+(y_t)] ,$$

where $y_i = M_i\phi/(V_0T)$ and $I_{\mp}(y) = \pm\int_0^\infty dx\ x^2\ ln(1 \mp \exp(-\sqrt{x^2+y^2})$. In the high temperature limit it is sufficient to use an approximate expression for $V(\phi, T)$

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda_T}{4}\phi^4 .$$

The various terms appearing in the above expression are listed in the Figure 2. Note that E depends only on the gauge couplings. The constants a_B, a_F appearing in the figure are for fermions and bosons respectively, $ln\ a_B = 2\ ln(4\pi) - 2\gamma = 3.91$ and $ln\ a_F = 2ln\ \pi - 2\gamma = 1.14$. This leads to a phase transition (PT) that is weakly first order, basically depending on the coefficient of the term linear in T i.e. E (see [14]).

As shown schemetically in Fig. 2, a second minimum (at nonzero ϕ) in the effective potential appears first at T_1. This new minimum becomes degenerate with the symmetric minimum at $\phi = 0$ at T_c. As we cool down to temperatures below T_c the new minimum becomes the absolute minimum till at T_0 the minimum at $\phi \neq 0$ transits into a maximum. We should note that at T_c the potential barrier between the two minima is $(1/4)(ET_c)^4/\lambda_{T_c}^3$ and the new minimum apears at $\phi = 2ET_c/\lambda T_c$. Note the dependence on λT_c.

It is well known that, for $T_c < T < T_0$, the potential barrier does not allow the universe to transit immediately into the new absolute minimum at $\phi = 2ET_c/\lambda T_c$ due to the small probability of tunneling through the barrier. This makes for supercooling in the range $T_c < T < T_0$, when the universe cools but cannot accomplish the transition. Only at $T = T_0$, when there is no barrier, does the universe roll down to the actual minimum.

In this temperature range the transition proceeds via the formation of bubbles of the broken phase in the symmetric phase [15]. The literature on this rather active

$$V(\Phi,T) = D(T^2-T_0^2)\Phi^2 - ET\Phi^3 + \lambda\Phi^4$$

$$D = \frac{1}{8v_0^2}(2m_W^2 m_Z^2 - 4m_t^2) \qquad E = \frac{1}{4\pi V_0^3}(2m_W^3 + m_Z^3)$$

$$\lambda = \frac{\mu^2}{V_0^2} \qquad m_H^2 = 2\mu^2 \qquad V_0 = 246\ \text{GeV}$$

$$\lambda_T = \lambda - \frac{3}{(16\pi^2 V_0^4)}\left(2m_W^4 \left(\frac{m_W^2}{a_B T^2}\right) + m_Z^4 \left(\frac{m_Z^2}{a_B T^2}\right) - 4m_t^4 \left(\frac{m_t^2}{a_F T^2}\right)\right)$$

$$T_0 = \frac{1}{2D}(\mu^2 - v_0^2) = \frac{1}{4D}(m_H^2 - 8Bv_0^2)$$

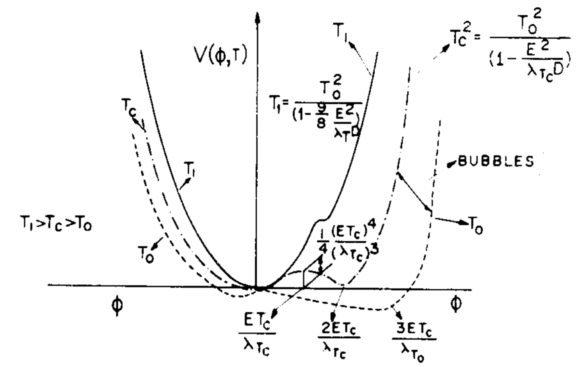

Figure 2. The 1st order phase transition with temeperature dependent 1 loop as effective potential.

subject is vast. We shall not discuss this any further, here, but refer the reader to Ref. [10] and references therein. As a bubble of the broken phase becomes larger it gives out latent heat reheating its surroundings resulting in a loss of equilibrium so crucial to the production of a B symmetry. It is here that all the action is. If the EWPT is strongly first order, that is, when E is large and/or λ_T is small, the chances of going out of equilibrium are brighter both due to the barrier and the latent heat being larger. Since E depends inversely on λ_T, it is λ_T that will effectively control the PT and a strong first order PT is indicated for small enough λ_T that in turn fixes the Higgs mass to be small. This is intimately connected with what happens after the transition. If λ_T and, in turn, the Higgs mass is large, the transition becomes only nominally first order and the post transition scenario is very similar to a second order transition which allows for equilibrium and subsequent washout of the asymmetry. We shall see in the next but one section that this happens for a Higgs mass above $\sim$ 50 GeV. This is clearly unrealistic.

There are several important but unsettled issues in the subject of nucleation of bubbles and the dynamics of bubble wall formation. The inter-bubble distance must be larger than bubble size for any significant first order PT. For details and the lively debate on the subject we refer the reader to Ref. [10].

8. The baryon asymmetry

The first order phase transition, coupled with some CP nonconserving interaction, guarantees 1) non-equilibrium from the former and 2) a direction for B asymmetry from the latter and therefore a nonzero B asymmetry. The question is one of a quantitative estimate.

We begin with a CP symmetric situation at $T = T_c$ and $A_s = 0$. Then cnsider, generically, some source of CP violation: the anomaly itself, the CKM matrix or some physics from scales above the standard model like supersymmetry, technicolour or a two Higgs model which can induce a CP violating effective interaction. We shall, first, consider an effective CP violating operator at the weak scale obtained by integrating out the physics at some higher scale and then comment on the individual sources above.

1) We shall consider the simplest situation when the effective theory at the weak scale, the scale of the EWPT, has only a single Higgs doublet (see Ref. 10 and the third paper in [15]). The lowest dimension CP violating operator that one can write down in this case is

$$S_{eff}^{CPV} = \int d^4x (g^4 \sin(\delta)|\phi|^2 F\widetilde{F})/64\pi^2,$$

where $\sin\delta$ is a measure of the CP violation and M is the scale of the new physics that has been integrated out

Using the anomaly $\partial_\mu J^{\mu B} = (3g^2/64\pi^2)F\widetilde{F}$, this S_{eff}^{CPV} can be recast into

$$S_{eff}^{CPV} = \int d^4x \ \frac{g^2 \sin(\delta)}{4M^2}(\partial_t|\phi|^2)n_B = \int d^4x(\mu)n_B,$$

where

$$\mu = (g^2/4M^2)\sin(\delta)\partial_t|\phi|^2 \ .$$

Here, a further assumption has been made that $\phi(T)$ is spatially uniform (this may be relaxed without much change).

As $\phi(T)$ changes in time, with the growth of a bubble in a first order transition, μ acts as a chemical potential for the baryon density. Now, if we write down the free energy for the massless quarks

$$F = E - \mu n_B,$$
$$E = \left(\tfrac{7}{8}\pi^2 N_D/30\right)T^4, \ n_B = \tfrac{1}{3}n_q = (3/4)(1.302/\pi^2)N_D T^3,$$

with N_D as the degeneracy factor and minimize the free energy with respect to n_B, we can find the preferred baryon density at the minimum of the free energy to be

$$n_B^0 \simeq \mu \, T^2 \ .$$

The equilibrium value of $\phi(T)$ is controlled by the temperature in a second order transition. The Hubble expansion rate relates the time to the temperature. Thus, the time rate of change of ϕ is set by the Hubble rate.

In a first order transition we are out of equilibrium and there is no such Hubble connection between the time and the temperature – the temperature may be almost constant over the time taken to accomplish the transition. $\phi(T)$ can change much faster than the Hubble rate, conforming to the picture of a discontinuous change in the order parameter $\phi(T)$ for a first order transition.

Let us fix our attention on a growing bubble wall of the broken phase in a bath of the symmetry phase. The exponential suppression factor that governs the rate of sphaleron mediated transitions is $\exp -(M_W/\alpha_W T)$ where $M_W = g\phi(T)/2$. When $g\phi(T) < \alpha_W T$, the suppression is small and $\phi(T)$ can respond to the B violating rate and move the system towards minimising the free energy by moving the B density towards n_B^0. However, this being a first order transition, equilibrium is not reached. As $\phi(T)$ increases simultaneously with time, the reaction rate cuts off sharply when $g\phi(T) > \alpha_W T$ due to large exponential suppression. The assumption of adiabaticity is made for the time that the rate suppression is small i.e. $g\phi(T) < \alpha_W T$ so that $\phi(T)$ can keep tuning itself with the rate.

Since baryon number is violated, the system tries to get to the minimum. The rate equation is obtained by detailed balance

$$\frac{dn_B}{dt} = -18\Gamma(t)T^{-3}(n_B - n_B^0).$$

This result makes sense. The rate vanishes at the minimum, it is proportional to the number of quark doublets and the suppression factor $\Gamma(t)$ which is determined by the instantaneous value of $\phi(T)$.

A crude estimate is gained by using the following parametrization:

$$\Gamma(t) = K(\alpha_W T)^4 \text{ for } M_W(T) < \sigma\alpha_W T,$$

$$\Gamma(t) = 0 \text{ for } M_W(T) > \sigma\alpha_W T,$$

where σ is an unknown constant that controls the temperature at which the reaction shuts off – the larger σ is the larger is the T interval over which an appreciable rate is maintained.

We emphasize, again, that in equilibrium n_B would be equal to n_B^0. In a first order transition equilibrium is not reached and $\phi(T)$ changes fast in comparision to the Hubble rate. In this case one neglects n_B in the RHS of the rate equation (remember that n_B starts off from zero at T_c). It is now easy to integrate the rate equation to get

$$n_B(T) \simeq \frac{3}{2}K(\alpha_W)^4\frac{\sin(\delta)}{M^2}T^3\phi^2(T)g^2,$$

where

$$\phi^2(T) = \frac{4M_W^2(T)^4}{g_W^2},$$

It follows that

$$A_s = \frac{n_B}{n_\gamma} \sim \frac{K\alpha_W^6 3\sigma^2 T^2 \sin(\delta)}{M^2 N_{eff}},$$

which is simply too small indicating that a single Higgs doublet model will not do (see Refs. [9] and [10]) regardless of the scale of the new physics.

2) The review by Shaposhinikov [8] contains the value of the B asymmetry that is calculated without any higher scale physics, strictly in the standard EW model with the CKM matrix as the only source of CP violation. This follows from computing higher order Feynman diagrams and effectively induces terms like in 1). The result is

$$A_s \sim \frac{\alpha_W^4}{N_{eff}}\left[\frac{g_W^2}{2M_W^2}\right]^7 S_1^2 S_2 S_3 \sin(\delta) m_t^6 m_b^4 m_c^2 m_s^2 \sim 10^{-22}.$$

3) There seems to exist the possibility that the anomaly by itself can be a viable CP violating source. In the symmetric phase the anomaly can induce an effective $L^{eff} \sim \theta F\tilde{F}$, as in QCD, where θ is the vacuum angle. However, if there are massless fermions (neutrinos) θ can be rotated away. If the neutrinos acquire a mass and θ changes during the EWPT, a scenario similar to 1) may be allowed. We do not consider this any further.

4) The same review also indicates that in a two Higgs model [16] the possibility does exist of obtaining a B asymmetry of the required order [15]

$$A_s = \frac{n_B}{n_\gamma} = \frac{\alpha_W^4}{N_{eff}} \left(\frac{m_t}{\pi T_0}\right)^2 \lambda_{CP} \sim 10^{-10}.$$

There is yet another possibility suggested in this review, that differs in the initial assumption that there is no CP violation and asymmetry before the EWPT i.e. $T > T_c$. This proposal is based on the presence of a net winding or Cherns-Simons number density in the symmetric phase itself, which converts to a net baryon number density once even a small CP violation is turned on, as this sets the direction or sign of the asymmetry. It is claimed by Shaposhnikov that this asymmetry can be large. However, its survival after the phase transition is subject to the Higgs mass being unrealistically small, 45 GeV, in the standard EW theory (see following section).

It has been pointed out in [17] that time varying scalar fields can couple to hypercharge and induce nonzero hypercharge density which in turn biases B production and can produce a substantial B asymmetry. For these other scenarios that can generate a realistic B asymmetry we refer the avid reader to the reviews by Dine and Doglov.

9. Postgenesis or afterplay

Shaposhinkov [18] has shown and emphasized that we are not done once we have created the asymmetry during the phase transition. In the standard EW theory there exists the very real possibility that the afterplay may wipe it all out. We indicate how this conclusion is arrived at.

First, we write the expression for M_{sp} just after the EWPT has been completed i.e. at $T = T_0$:

$$M_{sp}(T_0) = \frac{1}{2}\frac{g\phi(T_0)}{\alpha_W} = \frac{1}{2} g \frac{3ET_0}{\lambda T_0}\frac{1}{\alpha_W} = \zeta T_0, \qquad \zeta = \frac{1}{2}\frac{g}{\alpha_W}\frac{3E}{\lambda T^0}.$$

This shows that if λT_0 is small and in turn the Higgs mass is small, there will be a strong suppression of the rate of B asymmetry production. The rate equation is

$$\frac{dA_s}{dt} = -A_s \beta_0 T \exp(-\zeta T_0/T).$$

Since we are now in equilibrium, it follows that the Hubble expansion rate that relates the time and temperature applies. Thus

$$\tau_U^{-1} = \dot{R}/R - \frac{dT}{dt}\frac{1}{T} = \frac{N_{eff}^{1/2}}{M_{Planck}} T^2 = \alpha T^2, \; \alpha = \frac{N_{eff}^{1/2}}{M_{Planck}}.$$

Using this in the rate equation and integrating, we find

$$A_s \begin{pmatrix} t \to \infty \\ T \to 0 \end{pmatrix} \simeq A_s(T_0) \exp\left(-\frac{\beta_0}{\alpha}\frac{1}{\zeta T_0}\exp(-\zeta)\right),$$

which shows that the asymmetry has a very sensitive dependence on λT_0 through a double exponential.

It is found on plotting this function that this post-transition rate is very sensitive to the Higgs mass. If the Higgs mass exceeds 45 GeV, the rate is large and all previous asymmetry is washed out. If the Higgs mass is less than 45 GeV, the rate is suppressed and the previous asymmetry is preserved. This is bad news for the standard electroweak theory, as such a small Higgs mass is ruled out by LEP. There are enough uncertainties in the calculations performed using the standard EW model to allow raising this bound to present LEP limits $\sim$ 65 GeV.

Let us add that by introducing a new and additional scalar (see the fourth paper in Ref. [13]) that interacts with the Higgs, this bnound can be evaded while preserving a strongly first order transition. This is achieved by increasing the coefficient E and changing the relationship between λ_T and the Higgs mass through this mechanism.

In this talk we have attempted to provide a didactic summary of a most interesting and currently lively subject which can sometimes be a trifle convoluted. Apologies are due for incomplete crediting and referencing. More of this useful literature can be found in the excellent reviews referred to in the text. Though the standard electroweak theory has all the ingredients of providing the baryon asymmetry of the universe, it seems to require modifications to yield the right measure.

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