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Standard $SU(5)$ supergravity grand unification model and predictions

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Abstract. A review is given of the Standard $SU(5)$ supergravity model. This model has passed an important check regarding unification of the electro-weak and the strong couplings using high precision LEP data. It is shown that for a significant domain of the parameter space the model also satisfies constraints on the SUSY spectrum from CDF and LEP, as well as proton stability and cosmological relic density constraints.

1. Introduction

The standard $SU(5)$ supergravity grand unification model was proposed [1,2] in 1982. The model generates its own breaking of supersymmetry as well as breaking of the electro-weak symmetry. The most attractive mechanism for the latter is via radiative breaking using renormalization group effects [3]. The recent high precision LEP data appears very encouraging for supersymmetric unification. The LEP data gives the following values for the three coupling constants α_1, α_2 and α_3 of $SU(3)_C \times SU(2)_L \times U(1)$ at the Z-boson mass [4]:

$$\begin{aligned} \alpha_1(M_Z) &= 0.016996 \pm 0.000032, \\ \alpha_2(M_Z) &= 0.03351 \pm 0.00013, \\ \alpha_3(M_Z) &= 0.118 \pm 0.007. \end{aligned} \tag{1.1}$$

In Eq. (1.1) $\alpha_1 = \frac{5}{3}\alpha_Y$ where α_Y is the hypercharge coupling constant. Using renormalization group equations and extrapolating to high energy one finds that the three coupling constants meet to within 1 std for the standard $SU(5)$ SUSY theory [5].

Currently by supersymmetric unification one invariably means supergravity unification since it is only within the framework of supergravity that one can achieve a phenomenologically acceptable breaking of supersymmetry [2]. Historically, the first attempts at unification using supersymmetry were made as early as 1977 [6] and were later extended to include grand unification [7]. However, as mentioned above no phenomenologically viable breaking of supersymmetry can be achieved within globally supersymmetric frameworks of Ref. [6] and [7]. To circumvent this problem it was then proposed to add to globally supersymmetric theories soft supersymmetry breaking terms by hand (see S.D. and H.G., and N.S. in Ref. [7]).

Unfortunately the number of arbitrary soft SUSY breaking terms one can add in globally supersymmetric theories is enormous. For an $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant theory one can add up to 137 arbitrary parameters if one allows complex soft SUSY breaking terms, and up to 87 parameters if one restricts oneself to real terms.

The above drawback is overcome when one extends $N = 1$ global supersymmetry to $N = 1$ local supersymmetry. Here an acceptable breaking of supersymmetry does occur and the number of parameters is drastically reduced leading to a very predictive theory.

2. Supergravity grand unification

We begin by summarizing briefly first the framework of supergravity unification. Here one couples $N = 1$ supergravity with an arbitrary number of $N = 1$ chiral multiplets and an $N = 1$ vector multiplet which belongs to the adjoint representation of a unified gauge group G. The coupling of the above system is accomplished by utilizing techniques of applied $N = 1$ supergravity [2,8]. The Lagrangian is then found to depend on three arbitrary functions. These are the superpotential $W(z_i)$ where z_i are scalar components of the chiral multiplets, the Kähler potential $d(z_i, z_i^\star)$ and the gauge kinetic energy function $f_{\alpha\beta}(z_i, z_i^\star)$. Of these the superpotential $W(z_i)$ and the Kähler potential $d(z_i, z_i^\star)$ appear in a specific combination given by

$$G = -\ln[\kappa^6 W W^\star] - \kappa^2 d, \tag{2.1}$$

where $\kappa = (8\pi G_N)^{1/2} = 0.41 \times 10^{-18}$ GeV and G_N is the Newton constant. The effective (tree) potential of the theory is given by

$$V = \frac{-1}{\kappa^4} e^{-G} \left[(G^{-1})^a_b G_{,a} G^b_, + 3\right] + \frac{g^2}{2} \left(Ref^{-1}\right)^{\alpha\beta} D_\alpha D_\beta, \tag{2.2}$$

where

$$D_\alpha = \frac{-1}{\kappa^4} G_{,a} (T^\alpha)_{ab} z_b. \tag{2.3}$$

$f_{\alpha\beta}$ enters in the gauge kinetic energy as

$$-\frac{1}{4} Ref_{\alpha\beta} F^\alpha_{\mu\nu} F^\beta_{\mu\nu} \tag{2.4}$$

and in the gluino mass term in the form

$$\frac{1}{4} e^{-\frac{G}{2}} (G^{-1})^a b G^a_, f^\star_{\alpha\beta,b} \bar{\lambda}^\alpha \lambda^\beta. \tag{2.5}$$

In supergravity unification, the breaking of supersymmetry is accomplished via a hidden sector using superhiggs effect. One may decompose the scalar fields $\{z_i\}$ so that $\{z_a\} = \{z_i, z\}$ where z_i are the matter fields and z the Polonyi fields which develop a VEV growth $O(M_{Pl})$ and are gauge singlets with respect to the mattter gauge group. Low energy physics is protected from mass growths of $O(M_{Pl})$ by decomposing the superpotential as follows:

$$W(z_a) = W(z_i) + W_P(z). \tag{2.6}$$

In Eq. (2.6), $W(z_i)$ contains interactions of the matter fields while W_P is the superpotential for the Polonyi fields whose VEV growth generates breaking of supersymmetry [9]. Because of the specific form of Eq. (2.6) the interactions between the matter sector and the Polonyi sector are gravitational in nature and as a consequence low energy physics is protected against Planck size mass growths.

In addition to the breaking of supersymmetry, one has in a grand unified theory breaking of the unified group G which we assume reduces to the Standard Model gauge group after appropriate VEV formations at the unification scale i.e. $G \to SU(3)_C \times SU(2)_L \times U(1)_Y$ at a scale $Q = M_G$. Below the scale M_G, one may then generate an effective theory by integrating on the Polonyi fields and on the superheavy fields that arise after the spontaneous breaking of G. The theory below M_G can then be described by the following effective potential [1,10-12]

$$\begin{aligned} V_{eff} \; = \; & \left[\sum_i \left|\frac{\partial W_{eff}}{\partial z_i}\right|^2 + V_D\right] \\ & + \left\{\sum_i m_0^2 z_i z_i + (A_0 W^{(3)} + B_0 W^{(2)} + h.c.)\right\}. \end{aligned} \tag{2.7}$$

In Eq. (2.7) $W_{eff} = W^{(2)} + W^{(3)}$ where $W^{(2)}$ and $W^{(3)}$ are the quadratic and cubic parts of the effective low energy superpotential W_{eff}. In addition to Eq. (2.7) one has SUSY breaking mass term in the gaugino sector which can arise, for example, from Eq. (2.5) if $f_{\alpha\beta}$ has a non-trivial field dependence. Thus we write [13,14]

$$\mathcal{L}^{\rm mass}_{\rm gaugino} = -m_{\frac{1}{2}} \bar{\lambda}^\alpha \lambda^\alpha. \tag{2.8}$$

Eq. (2.7) and (2.8) together give us four SUSY breaking parameters: m_0, $m_{1/2}$, A_0, B_0. This is a drastic reduction in the number of SUSY breaking parameters one can add adhoc to a globally supersymmetric theory, e.g., 87 real parameters that can enter via soft SUSY breaking terms in an $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant theory.

We discuss next a specific $SU(5)$ supergravity theory. The superpotential of this theory can be written as $W = W_Y + W_G$, where W_Y is the Yukawa interaction involving the quark, lepton and Higgs fields and W_G is the interaction governing the GUT sector. We assume that the quarks and the leptons belong to three generations of $10 + \bar{5}$ representations and Higgs to a $5 + \bar{5}$ representation of $SU(5)$. W_Y is then assumed to have the form:

$$W_Y = \epsilon_{uvwxy} H_1^u M_i^{vw} f_{1ij} M_j^{xy} + H_{2x} M_{iy} f_{2ij} M_j^{xy}. \tag{2.9}$$

In Eq. (2.9) $M_i^{xy} = \underline{10}$, $M_{iy} = \underline{\bar{5}}$, $H_1^x = \underline{5}$, $H_{2x} = \underline{\bar{5}}$ and f_1 and f_2 are the Yukawa couplings. The simplest model for W_G is to assume that the GUT sector involves a 24-plet of heavy field Σ_y^x which is responsible for breaking of $SU(5)$ with interactions given by [7,1]

$$W_G = \lambda_1 \left(\frac{1}{3} Tr\Sigma^3 + \frac{1}{2} M Tr\Sigma^2\right) + \lambda_2 H_{2x}\left(\Sigma_Y^x + 3M'\delta_Y^x\right) \cdot H_1^Y. \tag{2.10}$$

$SU(5)$ breaks to $SU(3)_C \times SU(2)_L \times U(1)_Y$ by a VEV growth of the form:

$$\text{diag}\,(\Sigma_Y) = M(2,2,2,-3,-3). \tag{2.11}$$

For this model low energy physics i.e. physics below scales $Q < M_G$ is governed by the effective potential

$$W_{eff} = W^{(2)} + W^{(3)}, \tag{2.12}$$

where

$$W^{(2)} = \mu_0 H_1 H_2$$

and

$$W^{(3)} = \lambda_{ij}^{(e)} \ell_i H_1 e_j^c + \lambda_{ij}^{(u)} q_i H_2 u_j^c + \lambda_{ij}^{(d)} q_i H_1 d_j^c. \tag{2.13}$$

In the model discussed above one needs a fine tuning of M' so that $M' = M + \mu_0/3\lambda_2$ in order to achieve light Higgs doublets H_1, H_2 which are required to complete the breaking of the electro-weak symmetry. The fine tuning mentioned above is technically natural due to the no-renormalization [15] theorem. Recently it has been pointed out that finetuning of the above type may automatically arise in models which possess a higher symmetry $SU(6)$ or $SU(10)$ in the GUT sector [16,17]. Light Higgs may also be achieved via the so called missing partner mechanism which requires the use of $50, \overline{50}$ and 75 dimensional representations [18-20].

3. Radiative breaking of the electroweak symmetry

In the standard $SU(2)_L \times U(1)_Y$ model, electroweak symmetry is broken by addition of a tachyonic mass term $-m^2 H^\dagger H$ to the potential. A remarkable aspect of supergravity GUT is that electro-weak symmetry breaking can be manufactured via renormalization group effects [21-24]. The potential governing electroweak symmetry breaking, assuming conservation of charge and color, is given by

$$V = V_0 + \Delta V_1. \tag{3.1}$$

In Eq. (3.1) $V_0(Q)$ is the renormalization group improved tree potential at scale Q and is

$$\begin{aligned} V_0(Q) = \; & m_1^2(Q)|H_1|^2 + m_2^2|H_2|^2 - m_3^2(Q)(H_1H_2 + h.c.) \\ & + \tfrac{1}{2}(g_2^2 + g_Y^2)(|H_1|^2 - |H_2|^2)^2, \end{aligned} \tag{3.2}$$

where

$$m_i^2(t) = m_{H_i}^2(t) + \mu^2(t); \qquad (i = 1,2), \tag{3.3}$$

$$m_3^2(t) = -B(t)\mu(t), \tag{3.4}$$

$$t = \ln(M_G^2/Q^2) \tag{3.5}$$

and $m_i^2(t)$ obey GUT boundary conditions as follows:

$$m_i^2(M_G) = m_0^2 + \mu_0^2; \qquad (i = 1,2), \tag{3.6}$$

$$m_3^2(M_G) = -B_0\mu_0. \tag{3.7}$$

In Eq. (3.1) $\Delta V_1(Q)$ is the one loop correlation to the effective potential and is given by [25]

$$\Delta V_1(Q) = \frac{1}{64\pi^2} \sum_i (-1)^{2j_i+1} n_i \left[M_i^4 \left(\log \frac{M_i^2}{Q^2} - \frac{3}{2} \right) \right], \tag{3.8}$$

where j_i is the spin and n_i is the multiplicity of particles of type i. We shall discuss the contribution of $\Delta V_1(Q)$ to the extrema equations in somewhat greater detail in Sec. 4.

To generate spontaneous electro-weak symmetry breaking one begins at the GUT scale with supergravity boundary conditions and uses renormalization group equations to compute gauge coupling constants, Yukawa couplings and soft SUSY breaking terms at scales $Q < M_G$. In the analysis of electro-weak symmetry breaking one ensures that the potential is bounded from below both at the GUT scale and at the scale Q where one minimises the potential. This requires (at the tree level)

$$m_1^2(Q) + m_2^2(Q) - 2m_3^2(Q) \geq 0. \tag{3.9}$$

One must also ensure that charge and color invariance are not broken at any scale [26,27]. The on set of electro-weak symmetry breaking is characterized by $D(Q)$ turning negative where

$$D(Q) = m_1^2(Q)m_2^2(Q) - m_3^4(Q). \tag{3.10}$$

Thus electro-weak symmetry is broken for scales $Q < Q_0$ where $D(Q_0) = 0$ and $D(Q) < 0$. Including the loop corrections,the minima of Eq.(3.1) yields

$$\frac{1}{2}M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2\beta}{\tan^2\beta - 1}, \tag{3.11a}$$

$$\sin^2\beta = \frac{2m_3^2}{\mu_1^2 + \mu_2^2}, \tag{3.11b}$$

where

$$\tan\beta = \langle H_2\rangle/\langle H_1\rangle; \quad \mu_i^2 = m_i^2 + \frac{\partial \Delta V_1}{\partial v_i^2}; \quad (i = 1,2). \tag{3.12}$$

4. One loop corrections to radiative breaking

Contributions of one loop corrections to the extrema equations can be summarized by corrections to m_a^2, $a = 1,2$, i.e.

$$\mu_a^2 = m_a^2 + \Sigma^a; \quad (a = 1,2), \tag{4.1}$$

where $\Sigma^a = \partial\Delta V_1/\partial v_a^2$ is given by ($v_a = \langle H_a\rangle$)

$$\Sigma^a = (32\pi^2)^{-1}\sum_i(-1)^{2j_i}n_i M_i^2 \ln\left(\frac{M_i^2}{\sqrt{e}Q^2}\right)\frac{\partial M_i^2}{\partial v_a^2}. \tag{4.2}$$

Loop effects may be quantified by the ratio $R_\mu = \Delta\mu/\mu_t$ where μ_t is the tree value of μ at scale Q and $\Delta\mu = \mu_\ell - \mu_t$ is the one loop correction to the tree. An estimate of $\Delta\mu$ can be obtained from Eq. (3.11a) which gives

$$\Delta\mu^2 = \mu_\ell^2 - \mu_t^2 = (\Sigma^1 - \Sigma^2\tan^2\beta)/(\tan^2\beta - 1). \tag{4.3}$$

It should be noted that there are a large number of contributions individually to $\Sigma^{1,2}$ because of the large number of superparticle states that enter in Eq. (4.2) and thus a priori the one loop contributions can be quite large [28]. Thus if each of the states contributed $\sim 1\%$ of the tree value, one may end up with contributions $\sim 30\%$ or higher to μ_t from one loop effects. In turns out, however, that in much of the parameter space of physical interest the one-loop contributions are small due to internal cancellations among superparticle contributions [29]. We briefly discuss below this cancellation phenomenon.

As will be discussed in later sections in much of the parameter space of the standard $SU(5)$ supergravity model consistent with proton stability [30-33] and the current CDF and LEP bounds on SUSY particle spectrum, one has [34,35] $m_0^2 \gg M_Z^2, m_{1/2}^2$. In this limit, one may neglect the left-right $(L-R)$ mixing for the case of the first two generations of squarks and for all the three generations of sleptons. For the first two squark generations one has [29]

$$\Sigma_{\tilde{q}_i}^a = -\frac{s_a}{8\pi\cos^2\theta_W}(\sin^2\theta_W Y_i - \cos^2\theta_W T_{3i})M_{\tilde{q}_i}^2 \ln\left[\frac{M_{\tilde{q}_i}^2}{\sqrt{e}Q^2}\right], \tag{4.4}$$

where $i = u_L, d_L, u_R, d_R, (T_{3i}, Y_i)$ are the $(SU(2)_L, U(1)_Y)$ quantum numbers and $s_a = (1,-1)$ for $a = 1,2$. Further RG analysis gives for $M_{\tilde{q}_i}$

$$M_{\tilde{q}_i}^2 = m_0^2 + \gamma_i m_{1/2}^2 + (\sin^2\theta Y_i - \cos^2\theta_W T_{3i})M_Z^2\cos^2\beta, \tag{4.5}$$

where $\gamma_i = \gamma_{3i} + \gamma_{2i} + \gamma_{1i}$ and $\gamma_{3,2,1}$ are the $SU(3), SU(2)$ and $U(1)$ gaugino contributions and $\gamma_{3i} = \gamma_3$ is independent of the quark label i. From Eq. (4.5) we see that $M_{\tilde{q}_i}^2$ has the form $M_{\tilde{q}_i}^2 = m_0^2 + \Delta_i^2$ where $m_0^2 \gg \Delta_i^2$. Using an expansion in (Δ_i^2/m_0^2) we get [29]

$$\begin{aligned}\Sigma_{\tilde{q}_i}^a = \; & -s_a(8\pi\cos^2\theta_W)^{-1}(\sin^2\theta_W Y_i - \cos^2\theta_W T_{3i}) \\ & \times \left[m_0^2\ln\left(m_0^2/(\sqrt{e}Q^2)\right) + \Delta_i^2\left\{1+\ln\left(m_0^2/(\sqrt{e}Q^2)\right)\right\} + \right. \\ & \left. O\left(\Delta^2/m_0^2\right)\right].\end{aligned} \tag{4.6}$$

It may be easily seen from Eq. (4.6) that trace over the quark states of the first two generations cancels the largest term proportional to $m_0^2\ln(m_0^2/\sqrt{e}Q^2)$. The term left over after the trace in Eq. (4.6) is small. An identical analysis holds for the trace over the three generations of sleptons, where there is again a large cancellation. Thus for the first two generations of squarks and three generations of sleptons one has the following [29]:

$$\begin{aligned}&\Sigma^a(2 \text{ generations } \tilde{q}, 3 \text{ generations } \tilde{\ell}) \cong -(8\pi)^{-1}s_a\alpha_2 \\ &\quad (2.32\cos 2\beta M_Z^2 - 0.26m_{1/2}^2)\left[1+\ln\left(m_0^2/(\sqrt{e}Q^2)\right)\right] + O\left(\Delta^2/m_0^2\right).\end{aligned} \tag{4.7}$$

We note that the cancellations that occur in Eq. (4.7) are not of the usual type that arise between bosonic and fermionic contributions. Rather the cancellations occur among different components of the spin zero spectrum.

In the deduction of Eq. (4.7) we assumed that $L-R$ mixing was negligible for squarks and sleptons. This is a reasonable approximation for the first two generations of squarks and the three generations of sleptons for which Eq. (4.7) was deduced. However, neglect of $L-R$ mixing is not valid for the third generation of squarks due to a large top quark mass. For the third generation the analysis shows only a partial cancellation. Full analysis must also include summation over chargino, neutralino and Higgs as well as over $W^\pm, Z$ and the top quark. Here one finds usual type of SUSY cancellations between the contributions of the neutralinos and neutral Higgs, and between contributions of the charginos and charged Higgs [29]. However, here the cancellations again are partial.

Numerically one finds that over much of the parameter space of physical interest for the standard $SU(5)$ supergravity theory, the one loop correction $\Delta\mu$ is only a few percent of the tree, due to the phenomenon of cancellations discussed above.

5. Constraint of proton stability

Using RG equations a fit to the LEP data of Eq. (1.1) in SUSY $SU(5)$ theory with a single SUSY thresholds at scale M_S and ignoring GUT threshold gives the following for M_S and M_G [for $1\sigma = 0.007$ variation variation in $\alpha_3(M_Z)$]:

$$M_G = 10^{16.185\pm0.34}\ \text{GeV}, \quad M_S = 10^{2.37\pm1.03}\ \text{GeV}. \tag{5.1}$$

Actually M_G and M_S are anti-correlated. One has

$$M_G = 10^{16.185}\left(\frac{10^{2.37}}{M_S}\right)^{0.33}. \tag{5.2}$$

We pause here to recall that in the non-supersymmetric $SU(5)$ theory proton decays into the $e^+\pi^0$ mode via dimension six operators (generated via exchange of lepto-quarks) much more rapidly than the experimental bounds allow. One has [36]

$$\tau(p \to e^+\pi^0) \simeq 4 \times 10^{29\pm2}\ \text{yr}, \tag{5.3}$$

while the current experimental limit from Kamiokande and IMB is [37]

$$\tau(p \to e^+\pi^0)\ >\ 5.5 \times 10^{32}\ \text{yr}\ (90\% CL). \tag{5.4}$$

However, in supergravity $SU(5)$ the scale of unification M_G is higher than in the non-supersymmetric case and consequently the life-time for the decay mode $p \to e^+\pi^0$ is much larger. One has for $SU(5)$ supergravity [see Langacker and Polonski in Ref with $M_V = M_G$]

$$\tau(p \to e^+\pi^0) = 3 \times 10^{37\pm2}\ \text{yr}. \tag{5.5}$$

The result of Eq. (5.5) is consistent with the experimental bound of Eq. (5.4). However, even if the next generation of proton decay experiment can reach a sensitivity of 1×10^{34} yr for this mode, which Superkamiokande aspires to do [38], it is unlikely that this mode will become observable .

The proton decay discussed above arises from baryon number violating dimension six operators generated by exchange of lepto-quarks. However, the dominant proton decay in supergravity $SU(5)$ arises from baryon number violating dimension five operators which arise as a consequence of exchange of the massive Higgs color triplet field H_3 [30-33]. Exchange of this color Higgs triplet field generates dimension five operators of the type $LLLL$ and $RRRR$ where two of the fields are chiral scalars and the remaining two are chiral fermions. These operators when dressed by Wino, Zino and gluino exchange generate dimension six operators of the type $LLLL$, $LLRR$, $RRLL$ and $RRRR$ which generate proton decay. There exist a large number of decay modes for the nucleon from these operators. For the proton one has

$$\bar{\nu}_i K^+,\ \bar{\nu}_i \pi^+,\ e^+ K^0,\ \mu^+ K^0,\ e^+ \pi^0,\ \mu^+ \pi^0,$$
$$e^+ \eta,\ \mu^+ \eta; \qquad i = e, \mu, \tau.$$

For the neutron the possible decay modes are

$$\bar{\nu}_i K^0, \bar{\nu}_i \pi^0, \bar{\nu}_i \eta, e^+ \pi^-, \mu^+ \pi^-; \qquad i = e, \mu, \tau.$$

Among the modes listed above the most dominant modes are $\bar{\nu} K^+$ for the proton and $\bar{\nu} K^0$ for the neutron. We describe now briefly the details of the calculation for $p \to \bar{\nu} K^+$ decay mode. Here $\bar{\nu}_e K^+$ mode is negligible compared to $\bar{\nu}_\mu K^+$ and $\bar{\nu}_\tau K^+$ and one has for $p \to \bar{\nu}_i K^+$ $(i = \mu, \tau)$ using the chiral Lagrangian approach [39] the following result [31]:

$$\begin{aligned}
\Gamma(p \to \bar{\nu}_i K^+) = \; & \beta_p^2 M_{H_3}^{-2} m_N (32\pi f_\pi^2)^{-1} \left(1 - m_K^2/m_N^2\right) \\
& \left|1 + m_N(D + 3F)(3m_B)^{-1} - \tfrac{2}{3}(M_N/m_B)D\right|^2 \\
& \left|\alpha_2^2 (2M_W^2)^{-1} m_{di} m_c V_{i1}^\dagger V_{21} V_{22}\right|^2 A_L^2 (A_S^L)^2 \\
& \left|\textstyle\sum_{A=1,2,3} B_{Ai}\right|^2 .
\end{aligned} \tag{5.6}$$

In Eq. (5.6) V_{ij} are the $K-M$ matrix elements, and D, F, f_π, m_N, m_B are the chiral Lagrangian factors for which we take the values $D = 0.76$, $F = 0.48$, $f_\pi = 139$ MeV, $m_N = 938$ MeV, $m_B = 1154$ MeV. $A_L(A_S^L)$ in Eq. (5.6) are the long (short) range suppression factors which we evaluate at the two loop level to be [34.35]

$$A_L = 0.283, \quad A_S^L = 0.883. \tag{5.7}$$

β_p in Eq. (5.6) is the three-quark matrix element of the proton defined by

$$\epsilon_{abc}\epsilon_{\alpha\beta}\langle 0|d_{aL}^\alpha u_{bL}^\beta u_{cL}^\gamma|p\rangle = \beta_p U_L^\gamma, \tag{5.8}$$

where U_L^γ is the proton spinor. Recent lattice gauge calculations of β_p (using the quenched approximation) give [40]

$$\beta_p = (5.6 \pm 0.8) \times 10^{-3}\ \mathrm{GeV}^3. \tag{5.9}$$

Finally B_{Ai} in Eq. (5.6) are the dressing loop functions which depend on the sparticle masses. Here the first generation contributions B_{1i} are essentially negligible i.e. $B_{1i} \ll B_{2i}, B_{3i}$. For the case of the second generation contribution B_{2i}, the left-right mixing is negligible and neglecting this mixing one may write B_{2i} in the form [31,34,35]

$$B_{2i} = (\sin 2\beta)^{-1}[F_2(\tilde{c}, \tilde{d}_i, \tilde{W}) + \tilde{d}_i \to \tilde{\ell}_i], \tag{5.10}$$

where $F_2(\tilde{c}, \tilde{d}_i, \tilde{W})$ are defined by

$$F_2(\tilde{c}, \tilde{d}_i, \tilde{W}) = \sin\gamma_+ \cos\gamma_- f(\tilde{c}, \tilde{d}_i, \tilde{W}_1) + \cos\gamma_+ \sin\gamma_- f(\tilde{c}, \tilde{d}_i, \tilde{W}_2). \tag{5.11}$$

In Eq. (5.11) one has $\gamma_\pm = \beta_+ \pm \beta_-$ where

$$\sin(2\beta_\pm) = (\mu \pm \tilde{m}_2)/[4\nu_\pm^2 + (\mu \mp \tilde{m}_2)^2]^{\frac{1}{2}} \tag{5.12}$$

and

$$\nu_\pm = \frac{1}{\sqrt{2}} M_W (\sin\beta \pm \cos\beta). \tag{5.13}$$

For the third generation dressing loop contribution B_{3i} there is a considerable amount of L_R mixing due to a large top quark mass. B_{3i} is given by

$$B_{3i} = \frac{P_3}{P_2}\left(m_t m_c^{-1} V_{31} V_{32} V_{21}^{-1} V_{22}^{-1}\right)(\sin 2\beta)^{-1} \quad \left(F_3(\tilde{t}, \tilde{d}_i, \tilde{W}) + \tilde{d}_i \to \tilde{\ell}_i\right); \quad (i = 2, 3), \tag{5.14}$$

where

$$\begin{aligned} F_3(\tilde{t}, \tilde{d}_i, \tilde{W}) = & \Big[E \cos\gamma_- \sin\gamma_+ \tilde{f}(\tilde{t}, \tilde{d}_i, \tilde{W}) + \cos\gamma_+ \sin\gamma_- \\ & \tilde{f}(\tilde{t}, \tilde{d}_i, \tilde{W})\Big] - \tfrac{1}{2}\sin 2\delta_t \left[m_t / \left(\sqrt{2}\sin\beta M_W\right)\right] \\ & \Big[\Big\{f(\tilde{t}_1, \tilde{d}_i, \tilde{W}_1) E \sin\gamma_- \sin\gamma_+ - f(\tilde{t}_1, \tilde{d}_i, \tilde{W}_2)\cos\gamma_- \cos\gamma_+\Big\} \\ & -(\tilde{t}_1 \to \tilde{t}_2)\Big] \end{aligned} \tag{5.15}$$

and where

$$\tilde{f}(\tilde{t}, \tilde{d}_i, \tilde{W}) = \sin^2\delta_t f(\tilde{t}_1, \tilde{d}_{iL}, \tilde{W}) + \cos^2\delta_t f(\tilde{t}_2, \tilde{d}_{iL}, \tilde{W}) \tag{5.16}$$

and

$$\sin 2\delta_t = -2m_t(A_t + \mu\cot\beta)(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^{-1}. \tag{5.17}$$

In Eqs. (5.15) – (5.17) $\tilde{t}_{1,2}$ are the stop 1 and stop 2 mass eigen-states with stop masses given by eigen-values of the matrix

$$\begin{pmatrix} m_{\tilde{t}_L}^2 & m_t(A_t + \mu\cot\beta \\ m_t(A_t + \mu\cot\beta) & m_{\tilde{t}_R}^2 \end{pmatrix}, \tag{5.18}$$

where

$$m_{\tilde{t}_L}^2 = m_Q^2 + m_t^2 + \left(-\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta \tag{5.19}$$

$$m_{\tilde{t}_R}^2 = m_U^2 + m_t^2 + \left(-\tfrac{2}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta\ . \tag{5.20}$$

In Eqs. (5.19) and (5.20) m_U^2 and m_Q^2 are defined as in Ref. [23]. The factor E that appears in Eq. (5.15) is defined so that

$$E = (-1)^\theta\ ,$$

$$\theta = 0; \quad (\sin 2\beta\ >\ \mu\tilde{m}_2/M_W^2), \tag{5.21}$$

$$= 1; \quad (\sin 2\beta\ <\ \mu\tilde{m}_2/M_W^2). \tag{5.22}$$

As may be seen from Eq. (5.6), proton decay depends on both GUT physics which appears via the Higgs triplet mass M_{H3} as well as on low energy SUSY mass spectrum via the dressing loop function B_{Ai}. It is useful to introduce an effective dressing loop function B such that [34,35]

$$\begin{aligned} B = & \left((B_{2\mu} + B_{3\mu})^2 + \left(m_b V_{31}^\dagger (m_s V_{21}^\dagger)^{-1}\right)^2 (B_{2\tau} + B_{3\tau})^2\right) \\ & \times \left(M_S(\text{GeV})/10^{2.4}\right)^{0.33} \times 10^6\ \text{GeV}^{-1}. \end{aligned} \tag{5.23}$$

The current experimental bound for this decay mode is [41]

$$\tau(p \to \bar{\nu}K^+)\ >\ 1 \times 10^{32}\ \text{yr}. \tag{5.24}$$

Using the above experimental bound, one obtains for B the following limit:

$$B < \frac{127}{(M_S)^{0.0286}}\left(\frac{M_{H3}}{M_G}\right) \text{GeV}^{-1}. \tag{5.25}$$

For variation of M_S from 200 GeV - 1200 GeV, $B_0 = 127(M_S)^{-0.0286}$ varies from 109 to 104. Thus B_0 is not very sensitive to the variation of M_S. Regarding the Higgs triplet mass Refs. [33] and [42] have recently advocated an upper limit for M_{H3} as large as $10M_G$. Perhaps a conservative upper limit is $6M_G$.

6. Analysis and results with CDF, LEP and proton stability constraints

The analysis involves use of renormalization group equations for gauge and Yukawa couplings and soft SUSY breaking terms with supergravity boundary conditions at the GUT scale. The parameter space of the theory consists of m_0, $m_{1/2}$, A_0, B_0, μ and m_t. To accomplish electro-weak symmetry breaking, one may eliminate μ to fix the Z-mass and trade B_0 in favor of $\tan\beta$. Thus after electro-weak symmetry breaking the parameter space of the theory consists of the five dimensional space parametrized by

$$m_0, m_{1/2}, A_0, \tan\beta, m_t. \tag{6.1}$$

This five dimensional space will reduce to a four dimensional space once the top quark mass is determined which one hopes will happen in the near future. Using the parameter space of Eq. (6.1) the masses of 32 new particles (12 squarks, 9 sleptons, 2 charginos, 4 neutralinos, 1 gluino and 4 Higgs) are then determined. The analysis is subjected to CDF [43] constraints as follows:

Top :	$m_t \geq 90$ GeV	[CDF],
Gluino :	$m_{\tilde{g}} \geq 130$ GeV	[CDF;BTW [44]],
Squark :	$m_{\tilde{q}} \geq 150$ GeV	[CDF;BTW]

and LEP [45] constraints of

Sleptons :	$m_{\tilde{e}}, m_{\tilde{\mu}}, m_{\tilde{\tau}} > 45$ GeV	[LEP],
	$m_{\tilde{\nu}} > 32$ GeV	[LEP],
Stop :	$m_{\tilde{t}_i} \geq 45$ GeV	[LEP],
Charginos :	$m_{\tilde{Z}_1} > 20$ GeV; $\tan\beta > 3$	[LEP]
Neutralinos :	$m_{\tilde{Z}_1} > 20$ GeV; $\tan\beta > 3$,	
	$m_{\tilde{Z}_2} > 51$ GeV; $\tan\beta > 3$	[LEP],
Higgs (SUSY) :	$m_h > 43$ GeV,	
	$m_A > 20 - 44$ GeV,	
	$m_{H^\pm} > 42$ GeV	[LEP]. (6.2)

There exist many recent analyses of radiative breaking of electro-weak symmetry in $SU(5)$ supergravity which have been carried out without proton-stability constraints [46-50]. However in the following analysis we also subject the SUSY spectrum to proton stability constraint as discussed in Sec. 5. We discuss next the results of this analysis.

No-Scale $SU(5)$ Supergravity Model [51-53]: In this model one has $m_0 = 0 = A_0$ and here aside from m_t, one has just the parameter $m_{\frac{1}{2}}$ and $\tan\beta$. In this case it was shown in Ref. [54] that under the restrictions that $m_{\tilde{q},\tilde{g}} \leq 1$ TeV and $M_{H3} \leq 8\ M_G$, proton stability makes this model not viable.

Standard $SU(5)$ Supergravity Model [1]: Here one investigates the full parameter space of the theory given by Eq. (6.1). One imposes proton stability constraint discussed in Sec. V and the fine tuning constraint of $m_{\tilde{q},\tilde{g}} \leq 1$ TeV. Under these restrictions one generally finds [34,35] that $\mu^2 \gg M_Z^2$, $\tilde{m}_2^2$ in much of the parameter space of the theory and some interesting mass relations result. We discuss these below. First we discuss the chargino masses. These are given by the relation [13]

$$m_{\tilde{W}_{1,2}} = \frac{1}{2}\left|\left[4\nu_+^2 + (\mu - \tilde{m}_2)^2\right]^{1/2} \mp \left[4\nu_-^2 + (\mu + \tilde{m}_2)^2\right]^{1/2}\right|, \qquad (6.3)$$

where $\nu_\pm$ is defined in Eq. (5.13). For $\mu^2 \gg M_Z^2$, $\tilde{m}_2^2$ one has [34,35]

$$m_{\tilde{W}_1} \cong \tilde{m}_2 - M_W^2 \sin 2\beta/\mu; \quad m_{\tilde{W}_2} \cong \mu + M_W^2/\mu. \qquad (6.4)$$

We note that $\tilde{m}_2 = (\alpha_2/\alpha_3)m_{\tilde{g}} \cong 0.294 m_{\tilde{g}}$. Thus for $\mu > 0$, $m_{\tilde{W}_1}$ is approximately $\sim \frac{1}{4}m_{\tilde{g}}$ while for $\mu < 0$, $m_{\tilde{W}_1}$ is reduced approximately to $\sim \frac{1}{3}m_{\tilde{g}}$. Next regarding neutralinos, one finds that the Zino masses correspond to the four roots of the quartic eigen-value equation [13,35]

$$f(\lambda) = 0, \qquad (6.5)$$

where

$$\begin{aligned} f(\lambda) \;=\;& \lambda^4 - (m_{\tilde{\gamma}} + m_{\tilde{Z}})\lambda^3 - (M_Z^2 + \mu^2 + m_{\tilde{\gamma}\tilde{Z}}^2 - m_{\tilde{\gamma}} m_{\tilde{Z}})\lambda^2 \\ &+ \left[(m_{\tilde{\gamma}} - \mu \sin 2\beta)M_Z^2 + (m_{\tilde{\gamma}} + m_{\tilde{Z}})\mu^2\right]\lambda \\ &+ \left(\mu^2(m_{\tilde{\gamma}\tilde{Z}}^2 - m_{\tilde{\gamma}} m_{\tilde{Z}}) + \mu m_{\tilde{\gamma}} M_Z^2 \sin 2\beta\right) \end{aligned} \qquad (6.6)$$

with

$$\begin{aligned} m_{\tilde{\gamma}} &= \tilde{m}_2 \sin^2\theta_W + \tilde{m}_1 \cos^2\theta_W, \\ m_{\tilde{Z}} &= \tilde{m}_2 \cos^2\theta_W + \tilde{m}_1 \sin^2\theta_W, \\ m_{\tilde{\gamma}\tilde{Z}} &= \sin\theta_W \cos\theta_W(\tilde{m}_1 - \tilde{m}_2). \end{aligned} \qquad (6.7)$$

In the limit $\mu^2 \gg M_Z^2$, $\tilde{m}_2^2$ one finds that the four Zino masses separate into two small roots and two large roots. The small roots are [34,35]

$$m_{\tilde{Z}_1} \cong \tilde{m}_1 - (M_Z^2/\mu)\sin^2\beta \sin 2\theta_W, \qquad (6.8)$$

$$m_{\tilde{Z}_2} \cong \tilde{m}_2 - (M_Z^2/\mu)\sin 2\beta. \qquad (6.9)$$

While the two large roots are [34,35]

$$m_{\tilde{Z}_{3,4}} \cong \left|\mu + \frac{1}{2}(M_Z^2/\mu)(1 \pm \sin^2\beta)\right|. \qquad (6.10)$$

From Eq. (6.4), (6.8) – (6.10) we find the following approximate mass relations [34,35]:

$$m_{\tilde{Z}_1}/m_{\tilde{Z}_2} \cong \tilde{m}_1/\tilde{m}_2 = 0.508\ , \qquad (6.11)$$

$$m_{\tilde{W}_1} \cong m_{\tilde{Z}_2}\ , \qquad (6.12)$$

$$m_{\tilde{W}_2} \cong m_{\tilde{Z}_3} \cong m_{\tilde{Z}_4}\ , \qquad (6.13)$$

We turn now to a discussion of the limits on the top and Higgs mass. First with $M_{H_3} < 3M_G$, one finds that proton decay constrains $\tan\beta$ so that $\tan\beta \leq 4.7$. As discussed in Sec. V, the stop masses depend critically on $L-R$ mixing through the off diagonal term $m_t(A_t + \mu\cot\beta)$. Thus the off diagonal term becomes large as m_t increases and the mass of the lighter stop decreases. Thus for large enough m_t, the lighter stop becomes tacheonic. Actually the current LEP data places a lower bound of 45 GeV for the lighter stop. Along with proton decay constraint this bound places a limit on the mass of the top quark. One has [35] $m_t \leq 180$ GeV which is consistent with the result on m_t extracted from radiative corrections using LEP data [55] (assuming SM)

$$m_t = 130^{+25}_{-28}\ \text{GeV}\ [M_H = M_Z]. \tag{6.14}$$

Regarding the Higgs mass, one loop corrections are known to be significant [56] for the lightest Higgs especially for a heavy top and we have taken these corrections into account in computing the Higgs mass. There is an interesting correlation that emerges between the Higgs mass m_h and the lighter chargino mass [35] $m_{\tilde{W}_1}$. One finds [35] that under the current proton lifetime constraints with $M_{H_3} < 3M_G$,then $M_{\tilde{W}_1} < 100$ GeV when $m_h > 95$ GeV for the case $m_t < 140$ GeV. This means that either the Higgs mass is less than 95 GeV, and is thus accessible at LEP2 if one can achieve an integrated luminosity is excess of 500 pb^{-1} and an efficient b-tagging [57], or the Higgs mass is greater than 95 GeV but then $m_{\tilde{W}_1} < 100$ GeV and hence the $\tilde{W}_1$ is accessible at LEP2. The above correlation would become independent of the top quark mass if the lower limit on the proton partial lifetime on $p \to \bar{\nu}K^+$ mode increases so that $\tau(p \to \bar{\nu}K^+) > 1.3 \times 10^{32} yr$, with all other assumptions the same as before. Thus there is strong likelihood that either the lightest Higgs or the lighter chargino and perhaps both may be observed at LEP2.

7. Dark matter in SU(5) supergravity GUT

A remarkable feature of the particle spectrum that emerges in the standard SU(5) Supergravity GUT under the constraints discussed is that the lightest neutralino is the lightest supersymmetric particle [34,35] (LSP). Thus the lightest neutralino is a possible candidate for cold dark matter in the universe. There have been several analyses over the recent years which have investigated this possibility [58-61]. However, in all of these works an expansion on the thermally averaged quantity $\langle\sigma v\rangle$ has been used, where σ is the annihilation cross–section of the two neutralinos and v is their relative velocity. However, it is known that the usual expansion $\langle\sigma v\rangle = a + bv^2$ used in previous works breaks down when one is either close to thresholds or poles [62]. Recently, it was pointed out [63] that for the parameter space where one satisfies the constraints of proton stability and constraints of CDF and LEP data, significant annihilation of neutralinos to satisfy relic density constraint can only occur in the vicinity of the Higgs and Z-Poles in the s-channel of annihilating neutralinos. For this situation the expansion $\langle\sigma v\rangle = a + bv^2$ breaks down and one must use the rigorous thermal averaging on the pole [63]. An analysis of the dark matter in SU(5) supergravity GUT using the rigorous analysis was given recently [64]. We give here a symmary of results of that analysis.

In addition to the constraints on SU(5) supergravity GUT discussed already, we now explore the implications of the constraint [65] that the neutralino relic density not overclose the universe i.e.

$$\Omega_{\tilde{z}_1} h^2 \leq 1, \tag{7.1}$$

where

$$\Omega_{\tilde{z}_1} = \rho_{\tilde{z}_1}/\rho_c \tag{7.2}$$

and where $\rho_{\tilde{z}_1}$ is the relic density due to the lightest neutralino $\tilde{z}_1$ and ρ_c is the critical density :

$$\rho_c = 3H_0^2/(8\pi G_N). \tag{7.3}$$

In Eq.(7.3) G_N is Newton constant and H_0 is the Hubble parameter

$$H_0 = h \times 100 km/sMPC, \tag{7.4}$$

where $\frac{1}{2} \leq h \leq 1$.

The equation governing the number density n of neutralino $\tilde{z}_1$ at time t is [66]

$$\frac{dn}{dt} = -3\frac{\dot{R}}{R}n - \langle\sigma v\rangle(n^2 - n_0^2), \tag{7.5}$$

where R is the cosmic scale which enters in the line element of the Robertson–walker universe, $n_0(T)$ is the number density of neutralinos in thermal equilibrium at temperature T and v is related to the center–of–mass energy $\sqrt{s}$ by the relation

$$\sqrt{s} = 2m_{\tilde{z}_1} + \frac{1}{4}m_{\tilde{z}_1}v^2. \tag{7.6}$$

We can write Eq.(7.5) in terms of scaled variables

$$f = n/T^3, x = kT/m, \tag{7.7}$$

where k is the Boltzman constant, in the following form :

$$\frac{df}{dx} = \nu\langle\sigma v\rangle(f^2 - f_0^2), \tag{7.8}$$

where $f_0 = n_0/T^3$ and

$$\nu = \frac{m_{\tilde{z}_1}}{k^3}\left(\frac{8\pi^3 N_F G_N}{45}\right)^{-\frac{1}{2}}. \tag{7.9}$$

In Eq.(7.9) N_F is the effective number of degrees of freedom at temperature T.

We shall follow the standard semi–analytic approach of Lee and Weinberg [66]. Here one assumes that $f(x) \approx f_0(x)$ up to the freeze–out temperature x_f where the neutralinos go out of thermal equilibrium. The freeze–out temperature for neutralinos is determined by the approximate relation

$$\frac{df_0(x)}{dx} = \nu\langle\sigma v\rangle f_0^2(x); \text{ at } x = x_f. \tag{7.10}$$

Eq.(7.10) gives

$$x_f^{-1} \approx \ln\left[x_f^{\frac{1}{2}}\nu\langle\sigma v\rangle\right], \tag{7.11}$$

where $\langle \sigma v \rangle$ in Eq.(7.11) is evaluated at x_f. Eq.(7.11) is to be solved interatively to compute x_f. Below the freeze–out temperature one assumes that $f(x)$ is given by

$$\frac{df}{dx} = \nu \langle \sigma v \rangle f^2 \tag{7.12}$$

with the boundary condition $f(x_f) = f_0(x_f)$. The integration of Eq.(7.12) gives [66,59, 64]

$$\rho_{\tilde{z}_1} = 4.75 \times 10^{-40} \left(\frac{T_{\tilde{z}_1}}{T_\gamma}\right)^3 \left(\frac{T_r}{2.75 \circ K}\right)^3 N_F^{\frac{1}{2}} \left(\frac{GeV^{-2}}{J(x_f)}\right) \frac{g}{cm^3}, \tag{7.13}$$

where $(T_{\tilde{z}_1}/T_\gamma)^3$ is a reheating factor, T_γ is the current photon temperature, N_F is the effective degrees of freedom computed at the freeze–out temperature and $J(x_f)$ is given by

$$J(x_f) = \int_0^{x_f} \langle \sigma v \rangle dx \tag{7.14}$$

and where

$$\langle \sigma v \rangle = \int_0^\infty dv (\sigma v) e^{-v^2/4x} / \int_0^\infty dv v^2 e^{-v^2/4x}. \tag{7.15}$$

For the case of the neutralino annihilation one has

$$J = J_{Higgs} + J_{Zpole} + J_{exch}, \tag{7.16}$$

where J_{Higgs} is the contribution of the s-channel Higgs pole, J_{Zpole} is the contribution of the s-channel Z–pole and J_{exch} is the contribution of the s–fermion t–channel exchange diagram. For the case of J_{Higgs} and J_{Zpole} we shall carry out a rigorous thermal averaging over the poles, while for J_{exch} we shall use the conventional expansion approximation of a $x_f + \frac{1}{2} b x_f^2$.

We discuss now the details of the thermal averaging over the Higgs pole. Here $(\sigma v)_{Higgs}$ is given by

$$(\sigma v)_{Higgs} = \frac{A_{Higgs}}{m_{\tilde{z}_1}^4} \frac{v^2}{((v^2 - \epsilon_h)^2 + \gamma_h^2)}, \tag{7.17}$$

where

$$\epsilon_h = (m_h^2 - 4 m_{\tilde{z}_1}^2)/m_{\tilde{z}_1}^2 \tag{7.18}$$

and

$$\gamma_h = m_h \Gamma_h / m_{\tilde{z}_1}^2. \tag{7.19}$$

In Eqs.(7.18) and (7.19) m_h is the Higgs mass and Γ_h is the Higgs width. A_{Higgs} is given by

$$\begin{aligned} A_{Higgs} = {} & (8\pi)^{-1} \left[g_2 (2 M_W)^{-1} \sin\alpha (\cos\beta)^{-1} \right]^2 g_2^2 (\cos^2\theta_W)^{-1} \\ & (n_{11} \cos\theta_W - n_{12} \sin\theta_W)^2 (n_{13} \sin\alpha + n_{14} \cos\alpha)^2 \\ & \textstyle\sum_i C_i m_{fi}^2 \left(1 - m_{f_i}^2 / m_{\tilde{z}_1}^2\right)^{\frac{1}{2}}. \end{aligned} \tag{7.20}$$

In Eq.(7.20) $n_{1i}(i=1-4)$ are components of the eigen-vector of $\tilde{Z}_1, C_i = (3,1)$ for (quarks, leptons) and α is related to β by

$$\sin 2\alpha = -(m_A^2 + m_Z^2)(m_H^2 - m_h^2)^{-1} \sin^2\beta, \tag{7.21}$$

where m_H is the mass of the heavier CP even Higgs and m_A is the mass of the CP odd Higgs. It is then shown in Ref.[64] that $J_{Higgs}(x_f)$ is given by

$$J_{Higgs}(x_f) = \frac{A_{Higgs}}{2\sqrt{2}m_{\tilde{z}_1}^4}\left[I_{1h} + \frac{\epsilon_h}{r_h} I_{2h}\right], \tag{7.22}$$

where I_{1h} and I_{2h} are defined by

$$I_{1h} = \frac{1}{2}\int_0^\infty dy y^{-\frac{1}{2}} e^{-y} \ln\left[\frac{(4yx_f - \epsilon_h)^2 + r_h^2}{\epsilon_h^2 + r_h^2}\right], \tag{7.23}$$

$$I_{2h} = \int_0^\infty dy y^{-\frac{1}{2}} e^{-y}\left[\tan^{-1}\left(\frac{4yx_f - \epsilon_h)}{r_h}\right) + \tan^{-1}\left(\frac{\epsilon_h}{r_h}\right)\right]. \tag{7.24}$$

Eq.(7.22) is very useful in that it produces a smooth result for integration over the Higgs pole. A similar analysis can be carried out for the $J_{Zpole}(x_f)$. Here one gets [64]

$$J_{Zpole}(x_f) = \frac{1}{2\sqrt{\pi}m_{\tilde{z}_1}^4}\left[A_z \frac{I_{1z}}{\epsilon_z} + \frac{\epsilon_z}{r_z} B_z (I_{1z} + I_{2z})\right]. \tag{7.25}$$

In Eq.(7.25) A_z is given by

$$\begin{aligned} A_z = \; & \tfrac{\pi}{8}\,\alpha_2^2 \cos^{-4}\theta_W (n_{13}^2 - n_{14}^2)^2 \left[1 - \left(4m_{\tilde{z}_1}^2/M_Z^2\right)\right]^2 \times \\ & \left(3m_b^2\left[1 - (m_b^2/m_{\tilde{z}_1}^2)\right]^{\frac{1}{2}} + m_\tau^2\left[1 - (m_\tau^2/m_{\tilde{z}_1}^2)\right]^{\frac{1}{2}} + \right. \\ & \left. 3m_c^2\left[1 - (m_c^2/m_{\tilde{z}_1}^2)\right]^{\frac{1}{2}}\right), \end{aligned} \tag{7.26}$$

where we have retained only the dominant b, c and τ contributions, and B_z (in the zero–fermion mass approximation) is given by

$$B_z = \frac{\pi}{6}\frac{\alpha_2^2}{\cos^4\theta_w} m_{\tilde{z}_1}^2 (n_{13}^2 - n_{14}^2)^2 \left[\frac{21}{2} + \frac{80}{3}\sin^4\theta_w - 20\sin^2\theta_w\right], \tag{7.27}$$

while I_{1z}andI_{2z} are defined analogous to I_{1h} and I_{2h} with $(m_h, \Gamma h)$ replaced by $(M_z, \Gamma z)$.

The details of the rest of the analysis are the same as in Sec.VI except that now the additional constraint of Eq.(7.1) is imposed. The analysis shows that the solutions with $\mu > 0$ are cosmologically preferred and for this case it is possible to satisfy all of the desired constraints i.e. CDF, LEP, proton stability and relic density constraints. However, the parameter space is now much more constrained in that only the space where $2m_{\tilde{z}_1} \approx m_h$ and sometimes also when $2m_{\tilde{z}_1} \approx M_z$ is admissible. In fact the allowed parameter space is a shell approximately 5–20 GeV wide in the gluino mass in the five dimensional space defined by $m_0, m_{\tilde{g}}, A_t, \tan\beta$ and m_t. Further details can be found in Refs. [63-64].

8. Conclusion

The standard SU(5) supergravity model has passed an important first check i.e. the renormalization group analysis using high precision LEP data on the three coupling constants $\alpha_3, \alpha_2, \alpha_1$ of $SU(3)_C \times SU(2)_L \times U(1)$ shows that these couplings meet to within 1 std. for the standard SU(5) theory. In this talk it is shown that the model can also simultaneously satisfy all of the current constraints from accelerator and proton decay experiments and constraints of cosmology. Predictions are made regarding the SUSY particle spectrum of the model which should be testable by experiment in the near future.

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References

[1] A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. **49** (1982) 970.

[2] For a review see P. Nath, R. Arnowitt and A.H. Chamseddine in "Supersymmetry, Supergravity/Nonperturbative QCD", Springer Verlag 1984, edited by P. Roy and V. Singh; H. Nilles, Phys. Rev. **110** (1984) 1.

[3] L.E. Ibanez, Phys. Lett. **B118** (1982) 73;
J. Ellis, D.V. Nanopoulos and K. Tamuakis, Phys. Lett. **B121** (1983) 123;
L. Alvarez–Gaume, J. Polchinski and M. Wise, Nucl. Phys. **B221** (1983) 495;
J. Ellis, J. Hageliv, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. **B134** (1984) 429;
L.E. Ibanez and C. Lopez, Phys. Lett. **B126** (1983) 54.

[4] XXVI International Conference on High Energy Physics, Dallas, August 1992; The value of α_3 is taken from analysis of G. Altarelli presented at the Erice School on "From Superstrings to the Real Superworld", 14–22 July, 1992.

[5] P. Langacker, Proc. PASCOS 90 – Symposium, p 231: Eds. P. Nath and S. Reucroft (World Scientific, Singapore 1990);
P. Langacker and M. Luo, Phys. Rev. **D44** (1991) 817;
J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. **B249** (1990) 441; **B260** (1991) 131;
U. Amaldi, W.de Boer and H. Furstenau, Phys. Lett. **260** (1991) 447;
F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, Nuov. Cimento **A115** N.4 (1992) 581; Nuov. Cimento **A104** N.12 (1991) 1817.

[6] P. Fayet, Phys. Lett. **B69** (1977) 489; **B70** (1977) 461;
M. Sohnius, Nucl. Phys. **B122** (1977) 291.

[7] E. Witten, Nucl. Phys. **B177** (1981) 477; **B185** (1981) 513;
S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981) 150;
N. Sakai, Zeit. F. Phys. **C11** (1981).

[8] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. **B116** (1982) 231; Nucl. Phys. **B212** (1983) 413;
E. Witten and J. Bagger, Nucl. Phys. **B222** (1983) 125.

[9] J. Polonyi, University of Budapest Report No.KFKI–1977–93, 1977 (unpublished).

[10] R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett. **B119** (1982) 343.

[11] L. Hall, J. Lykken and S. Weinberg, Phys. Rev. **D27** (1983) 2359.

[12] P. Nath, R. Arnowitt and A.H. Chamseddine, Nucl. Phys. **B227** (1983) 121.

[13] P. Nath., R. Arnowitt and A.H. Chamseddine, HUTP-83/A077-NUB#2588.

[14] L. Alvarez-Gaumo, J. Polchinski and M. Wise, Nucl. Phys. **B221** (1983) 495.

[15] M.T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. **B159** (1979) 420.

[16] K. Inoue, A. Kakuto and T. Tankano, Prog. Theor. Phys. **75** (1986) 664;
A. Anselm and A. Johansen, Phys. Lett. **B200** (1988) 331;
A. Anselm, Sov. Phys. JEPT **67** (1988) 663.

[17] R. Barbieri, G. Dvali and A. Strumia, CERN-TH-6526/92.

[18] G. Grinstein, Nucl. Phys. **B206** (1982) 387.

[19] H. Georgi, Phys. Lett. **B108** (1982) 283.

[20] A. Masiero et. al., Phys. Lett. **B115** (1982) 380.

[21] L.E. Ibanez, Phys. Lett. **B118** (1982) 73;
J. Ellis, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. **B121** (1983) 123.

[22] L. Avarez-Gaume, J. Polchinski and M. Wise, Nucl. Phys. **B221** (1983) 495;
J. Ellis, J. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. **B125** (1983) 275.

[23] L.E. Ibanez and C. Lopez, Phys. Lett. **B126** (1983) 54;
L.E. Ibanez, C. Lopez and C. Munos, Nucl. Phys. **B256** (1985) 218.

[24] J. Ellis, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. **B134** (1984) 429.

[25] S. Coleman and S. Weinberg, Phys. Rev. **D7** (1973) 1888;
S. Weinberg, Phys. Rev. **D7** (1973) 2887.

[26] J.M. Frere, D. Jones and S. Raby, Nucl. Phys. **B222** (1983) 1;
M. Claudson, L. Hall and I. Hinchliffe, Nucl. Phys. **B228** (1983) 501.

[27] M. Drees, M. Gluck and K. Brassie, Phys. Lett. **B157** (1985) 164.

[28] G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. **B331** (1990) 331.

[29] R. Arnowitt and P. Nath, Phys. Rev. **B46** (1992) 3981.

[30] S. Weinberg, Phys. Rev. **D26** (1982) 287;
N. Sakai and T. Yanagida, Nucl. Phys. **B197** (1982) 533;
S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. **B112** (1982) 133;
J. Ellis, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. **B202** (1982) 43;
B.A. Campbell, J. Ellis and D.V. Nanopoulos, Phys. Lett. **B141** (1984) 229;
S. Chadha, G.D. Coughlan, M. Daniel and G.G. Ross, Phys. Lett. **B149** (1984) 47.

[31] R. Arnowitt, A.H. Chamseddine and P. Nath, Phys. Lett. **B156** (1985) 215;
P. Nath, R. Arnowitt and A.H. Chamseddine, Phys. Rev. **D32** (1985) 2348.

[32] M. Matsumoto, J. Arafune, H. Tanaka and K. Shiraishi, Phys. Rev. **D46** (1992) 3966.

[33] H. Hisano, H. Murayama and T. Yanagida, Tohuku University Preprint TU-400-July, 1992.

[34] R. Arnowitt and P. Nath, Phys. Rev. Letts. **69** (1992) 725.

[35] P. Nath and R. Arnowitt, Phys. Lett. **B289** (1992) 308.

[36] H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32** (1974) 448.

[37] R. Becker-Szendy et. al., Phys. Rev. **D42** (1990) 2974.

[38] Y. Totsuka, Proc. of XXIV Conf. on High Energy Physics, Munich, 1988, Eds. R. Kotthans and J.H. Kuhn (Springer Verlag, Berlin, Heidelberg, 1989).

[39] S. Chadha and M. Daniels, Nucl. Phys. **B229** (1983) 105.

[40] M.B. Gavela, et. al., Nucl. Phys. **B312** (1989) 269.

[41] Particle Data Group, Phys. Rev. **D45** (1992).

[42] A.E. Faraggi, B. Grinstein and S. Meshkov, SSCL-Preprint-126 (1992).

[43] See M. Davier at the Lepton-Photon High Energy Physics Conference (LP-HEP 91), Geneva, 1991.

[44] H. Baer, X. Tata and J. Woodside, Phys. Rev. Lett. **63** (1989) 352 and FSU-HEP-910211/UH-511-721-91/IS-J4185.

[45] Lepton-Photon High Energy Physics Conference (LP-HEP 91) Geneva, 1991.

[46] M. Drees and M.M. Nojiri, Nucl. Phys. **B369**, 54 (1992); Phys. Rev. **D45** (1992) 2482.

[47] G.G. Ross and R.G. Roberts, Nucl. Phys. **B377** (1992) 571.

[48] S. Kelley, J.L. Lopez, D.V. Nanopoulos, H. Pois and K. Yuan, Phys. Lett. **B273** (1991) 423.

[49] K. Inoue, M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. **D45** (1992) 328.

[50] H. Arason, D. Castano, B. Keszthelyi, S. Mikaelian, E. Picard, P. Ramond and E. Wright, Phys. Rev. Lett. **67** (1991) 2933.

[51] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, Phys. Lett. **B133** (1983) 61;
N.P. Chang, S. Ouvry and X. Wu, Phys. Lett. **51** (1983) 327.

[52] J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. **B241** (1984) 405; Nucl. Phys. **B247** (1984) 373.

[53] For a review of No-Scale models see A.B. Lahanas and D.V. Nanopoulos, Phys. Rep. **145** (1987) 1.

[54] P. Nath and R. Arnowitt, Phys. Lett. **B287** (1992) 3282.

[55] J. Ellis, G.L. Fogli and E. Lisi, Phys. Lett. **B274** (1992) 456; ibid **B285** (1992) 238; ibid **B288** (1992) 85, BARI-TH-111-92.

[56] Y. Ikada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. **85** (1991) 1;
J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. **B257** (1991) 83;
H.E. Haber and R. Hampling, Phys. Rev. Lett. **66** (1991) 1815.

[57] J. Hilgart et. al., Z. Phys. **C35** (1987);
K.J.F. Gaemers and N. de Groot, Z. Phys. **C49** (1991) 267;
H. Haber, SCIPP 92/10 (1992).

[58] H. Goldberg, Phys. Rev. Lett. **50** (1983) 1419.

[59] J. Ellis, J.S. Hagelin, D.V. Nanopoulos, K. Olive and M. Srednicki, Nucl. Phys. **B238** (1984) 453.

[60] J. Lopez, D.V. Nanopoulos and K. Yuan, Nucl. Phys. **B370** (1992) 445;
J. Lopez, D.V. Nanopoulos and H. Pois, CERN-TH-6628/92 - CTP-TAMU-61/92 - ACT-19/92.

[61] M. Drees and M.M. Nojiri, DESY 92 - 101 (1992).

[62] K. Griest and D. Seckel, Phys. Rev. **D43** (1991) 3191;
P. Gondolo and G. Gelmini, Nucl. Phys. **D360** (1991) 145.

[63] R. Arnowitt and P. Nath, Phys. Lett. **B** (to appear).

[64] P. Nath and R. Arnowitt, NUB-TH-3056/92 – CTP-TAMU-66/92 – TIFR/TH/92-69.

[65] See R. Kolb and M. Turner, "The Early Universe", Addison–Wesley Publishing Company, 1990.

[66] B.W. Lee and S. Weinberg, Phys. Rev. Lett. **39** (1977) 165.