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Threshold and compactification effects in GUTS

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Abstract. We review general results on threshold effects and their implications on GUTs in the context of LEP data. Among the blooming grand-desert models, threshold effects are computed in the presence of a single real scalar $\zeta(3,0,8)$ with $M_\zeta \simeq 10^{10}$ GeV leading to experimentally testable predictions on the proton lifetime τ_p in SU(5) and, in addition, small neutrino masses in SO(10) needed for the solar neutrino flux and the dark matter of the universe . The fine structure constant matching at M_Z is ensured by including threshold effects on the unification coupling. In the minimal SUSY SU(5) such effects at the GUT scale modify the prediction of the supersymmetric mass threshold near the TeV scale and the precision measurments of the Standard Model couplings at M_Z probe into the superheavy mass spectrum. Consequences of theorems proved very useful for threshold, compactification and multiloop effects are discussed. It is noted that in a class of GUTs the highest intermediate scale M_I above which G_{224P} becomes a good symmetry is not affected by the GUT threshold or compactification effects or multiloop contributions in the range $M_I - M_U$. But spontaneous compatification effects can decrease the intermediate scale drastically in models where parity and $SU(2)_R$ breakings are decoupled. Low mass W_R-bsosns are permitted in models with decoupled parity and $SU(2)_R$ breakings.

1. Introduction

Grand unified theories (GUTs) provide an elegant but highly promising extension of physics beyond the Standard Model (SM). Although the SM has many arbitrary unknown parameters its predictive power increases when embedded in a GUT. Recent precision measurements at LEP and improved estimations of the electromagnetic fine structure constant at M_Z has led to very accurate determinations of the three gauge couplings of the SM which,in turn, have revived interests in GUTs [1]. In particular attention has been focussed on predictions including two loop and threshold effects in desert type GUTs with additional degrees of freedom, minimal SUSY SU(5) and SO(10) models with intermediate symmetries [2-10]. Effects of higher dimensional operators on GUT predictions which might arise as a result of spontaneous compactification of the Kaluza-Klein type in higher dimensional theories [10-12], have also been investigated with drastic changes on the predicted values of the mass scales in certain models. We review some of these developments taking into account their consistency with the precision measurements of the Standard Model parameters at the Z-mass.

2. Threshold effects

The matching fuction $\lambda_i(\mu)$ for $\mu \simeq M_U$ modify the boundary condition in the following manner [13-15].

(A) Without SUSY

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_G(\mu)} - \frac{\lambda_i(\mu)}{12\pi},$$

$$\lambda_i(\mu) = \mathrm{Tr}(t_{iv}^2) - 21\,\mathrm{Tr}\left(t_{iv}^2 \ln\frac{M_i^v}{\mu}\right) + \mathrm{Tr}\left(t_{is}^2 \Lambda_s \ln\frac{M_i^s}{\mu}\right) + 8\,\mathrm{Tr}\left(t_{iF}^2 \ln\frac{M_i^F}{\mu}\right).$$

(B) With SUSY

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_G(\mu)} - \frac{\lambda_i(\mu)}{12\pi},$$

$$\lambda_i(\mu) = -2\,\mathrm{Tr}\left(t_{iv}^2 \ln\frac{M_i^v}{\mu}\right) + \mathrm{Tr}\left(t_{is}^2 \Lambda_s \ln\frac{M_i^s}{\mu}\right) + \mathrm{Tr}\left(t_{iF}^2 \ln\frac{M_i^F}{\mu}\right).$$

Here α_G = GUT coupling constant = $g_G^2/4\pi$, i = gauge group index = Y, 2L, 3C when GUT $\rightarrow$ SM; (M_i^v, M_i^s, M_i^F) = masses of superheavy vector bosons (v), scalar (s) and fermion (F) if any in the theory; (t_{iv}, t_{is}, t_{iF}) = generators in the representation of superheavy vector bosons (v), scalar (s) and fermion (F) and Λ_s = projection operator that removes the would be goldstone bosons from the scalar represetation.

Physically threshold effects arise from the modification of light-gauge boson propagators in the effective theory due to the superheavy gauge bosons, scalars, or fermions in the loop. For example when SM(5) $\rightarrow$ SM, i runs over12 light gauge-boson indices of SM whereas M_i^v stand for the masses of $X^{\pm\frac{4}{3}}$ and $Y^{\pm\frac{1}{3}}$ gauge bosons. M_i^s are the superheavy Higgs scalar masses in $\underline{24}$ and $\underline{5}$ of SU(5). We will discuss threshold effects in the particular cases of the following four classes of models.

I. GUT $\xrightarrow{M_U}$ (A, B, C ...) $\xrightarrow{M_\zeta}$ SM $\xrightarrow{M_W}$ $U(1)_{em} \times SU(3)_C$

This is the case where the grand desert blooms with new degrees of freedom A, B, C, ... [1,16,17]. In particular, the most economic model with a single scalar $\zeta(3,0,8)$in the grand desert [8] which we call a colourful bloom in the grand desert will be reviewed in the context of SU(5) and SO(10) with threshold effects.

II. Minimal Supersymmetric Standard Model (MSSM) embedded in SU(5)

$$SU(5) \times SUSY \underset{\underline{24}}{\overset{M_U}{\longrightarrow}} SM \times SUSY \overset{M_S}{\longrightarrow} SM \underset{\underline{5},\bar{\underline{5}}}{\overset{M_W}{\longrightarrow}} U(1)_{em} \times SU(3)_C\ (\simeq G_{13})$$

This model has been investigated by a number of authors [1,2,18-21].

$$\text{III. SO(10)} \xrightarrow[\underline{210}]{M_U} G_{2213P} \xrightarrow[\underline{126}]{M_R} \text{SM} \xrightarrow[\underline{10}]{M_R} G_{13}$$

Here $G_{2213P} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P$. Threshold effects in III & IV have been recently carried out by Parida et al [22]. These effects in SO(10) with G_{2213} or G_{2214} intermediate symmetry with decoupled parity (P) and $SU(2)_R$ breakings [23,24] have been carried out by Mohapatra and Parida [7].

$$\text{IV. SO(10)} \xrightarrow[\underline{54}]{M_U} G_{224P} \xrightarrow[\underline{126}]{M_C} \text{SM} \xrightarrow[\underline{10}]{M_W} G_{13}$$

Here $G_{224P} = SU(2)_L \times SU(2)_R \times SU(4)_C \times P$ (P=Parity) is the left-right symmetric Pati-Salam gauge group. This belongs to the class of models where two recent theorems on vanishig GUT-threshold effects, compactification effects and multiloop radiative corrections for $\mu = M_C - M_U$ on $\sin^2\theta_W$ are valid [9,10].

The renormalisation group equations (RGEs) are

(a) $\mu = M_Z - M_{th} (= M_S, M_\zeta, M_R, M_C)$

$$\mu\frac{\partial g_i}{\partial \mu} = \frac{a_i g_i^3}{16\pi^2} + \frac{1}{(16\pi^2)^2}\sum b_{ij} g_i^3 g_j^2$$

(b) $\mu = M_{th} - M_U$

$$\mu\frac{\partial g_i}{\partial \mu} = \frac{a_i g_i^3}{16\pi^2} + \frac{1}{(16\pi^2)^2}\sum b'_{ij} g_i^3 g_j^2$$

The combinations of the gauge couplings $\alpha^{-1}(M_Z) - \frac{8}{3}\alpha_2^{-1}(M_Z)$ and $\alpha^{-1}(M_Z) - \frac{8}{3}$ $\alpha_3^{-1}(M_Z)$ give two equations for the cases I-IV whose solutions give analytic expressions for $\ln\frac{M_U}{M_Z}$ and $\ln\frac{M_{th}}{M_Z}$. The RGEs for $\alpha^{-1}(M_Z) = \frac{5}{3}\alpha_1^{-1}(M_Z) + \alpha_2^{-1}(M_Z)$ yields threshold effects on α_G^{-1} [25]. In model I, we obtain

$$\begin{aligned}
\ln\frac{M_U}{M_Z} &= \frac{16\pi}{187\alpha}\left(\frac{7}{8} - \frac{10\alpha}{3\alpha_s} + \sin^2\theta_W\right) + \\
&\frac{5}{187}\left(\frac{8}{3}(P_3^\zeta + P_3^U) - \frac{3}{2}(P_2^\zeta + P_2^U) - \frac{7}{6}(P_1^\zeta + P_1^U)\right) + \\
&\frac{5}{3366}(7\lambda_1 + 9\lambda_2 - 16\lambda_3), \\
\ln\frac{M_\zeta}{M_Z} &= \frac{4\pi}{187\alpha}\left(15 - \frac{23\alpha}{\alpha_s} + 63\sin^2\theta_W\right) + \\
&\frac{3}{187}\left(16(P_2^\zeta + P_2^U) - \frac{23}{3}(P_3^\zeta + P_3^U) - \frac{25}{3}(P_1^\zeta + P_1^U)\right) + \\
&\frac{1}{561}(25\lambda_1 - 48\lambda_2 + 23\lambda_3), \\
\frac{1}{\alpha_G} &= \frac{3}{8\alpha} + \frac{1}{187\alpha}\left(\frac{347}{8} + \frac{466\alpha}{3\alpha_s} - 271\sin^2\theta_W\right) - \\
&\frac{1}{4488\pi}\left(932(P_3^\zeta + P_3^U) - 945(P_2^\zeta + P_2^U) + 1135(P_1^\zeta + P_1^U)\right) + \\
&\frac{1}{13464\pi}(1135\lambda_1 - 945\lambda_2 + 932\lambda_3).
\end{aligned}$$

Here $P_i^\zeta = B_{ij}X_j^\zeta$, $P_i^U = B'_{ij}X_j^U$ the repeated index implies summation, $X_j^\zeta = \ln \frac{\alpha_j(M_\zeta)}{\alpha_j(M_Z)}$, $X_j^U = \ln \frac{\alpha_j(M_U)}{\alpha_j(M_Z)}$, $B_{ij} = b_{ij}/a_j$, $B'_{ij} = b'_{ij}/a'_j$. The first (second) term containing α, α_3 and $\sin^2\theta_W$ (P_1, P_2 and P_3) represents one (two)-loop contributions. The third term containing λ_i's represents threshold effects. We use the following values as the input parameters at the Z-mass consistent with the LEP data $\sin^2\theta_W = 0.2333 \pm 0.0008$, $\alpha_3 = 0.113 \pm 0.005$, $\alpha^{-1} = 127.9 \pm 0.2$, leading to $\alpha_1(M_Z) = 0.0171 \pm 0.00006$, $\alpha_2(M_Z) = 0.03351 \pm 0.0002$. At first, ignoring threshold effects but including two loop contributions, we obtained $M_\zeta^0 = 10^{10.2\pm0.6}$ GeV, $M_U^0 = 10^{15\pm0.2}$ GeV, $\alpha_G^{0\,-1} = 39.05 \pm 0.6$.

The graphs of $\alpha_i^{-1}(\mu)$ vs μ have been shown in Fig. 1, demonstrating excellent unification at M_U^0 and slope changes of α_2^{-1} and α_3^{-1} at M_ζ^0. When embedded in SU(5), ζ (3, 0, 8) $\subset \underline{75}$ or SU(5). Thus, for spontaneous symmetry breaking (SSB) of SU(5) to the low energy symmetry through the SM, we need the Higgs representations $\underline{24}$, $\underline{5}$ and also $\underline{75}$ of SU(5). In contrast to other models where more than one irreducible representation of SM have been used, in this case only one additional fine tuning is needed to keep ζ light. The superheavy Higgs components of these representations can be stated in the following manner:

$$\begin{aligned}
\underline{5} &\supset C(1,-\tfrac{2}{3},3),\\
\underline{24} &\supset D_1(3,0,1)+D_2(1,0,8),\\
\underline{75} &\supset E_1(1,\tfrac{10}{3},3)+E_2(2,\tfrac{5}{3},3)+E_3(1,-\tfrac{10}{3},\bar{3})+\\
&\quad E_4(2,-\tfrac{5}{3},\bar{3})+E_5(2,-\tfrac{5}{3},\bar{6})+E_6(2,\tfrac{5}{3},6)+E_7(1,0,8).
\end{aligned}$$

In Table 1 we present threshold effects on the mass scales for the model. Here the factor β that means the superheavy scalar masses are $\beta^{-1}M_U - \beta M_U$.

In the model II the corresponding equations are derived as

$$\begin{aligned}
\ln\frac{M_U}{M_Z} &= \frac{\pi}{19\alpha}\left(\frac{1}{2}+7\sin^2\theta_W-\frac{25\alpha}{3\alpha_s}\right)+\\
&\quad \frac{5}{76}\left[\frac{1}{6}\left(P_1^U+P_1^S\right)+\frac{3}{2}\left(P_2^U+P_2^S\right)-\frac{5}{3}\left(P_3^U+P_3^S\right)\right]-\\
&\quad \frac{5}{76}\left(\frac{1}{3}\lambda_1+3\lambda_2-\frac{10}{3}\lambda_3\right),\\
\ln\frac{M_S}{M_Z} &= \frac{12\pi}{19\alpha}\left(1-5\sin^2\theta_W+\frac{7\alpha}{3\alpha_s}\right)-\\
&\quad \frac{1}{19}\left[5\left(P_1^U-P_1^S\right)-12\left(P_2^U+P_2^S\right)+7\left(P_3^U+P_3^S\right)\right]-\\
&\quad \frac{2}{19}\left(5\lambda_1-12\lambda_2+7\lambda_3\right),\\
\frac{1}{\alpha_G} &= \frac{3}{8\alpha}+\frac{3}{76\alpha}\left(\frac{47}{2}+\frac{250\alpha}{3\alpha_s}-146\sin^2\theta_W\right)-\\
&\quad \frac{1}{76\pi}\left[45\left(P_1^U+P_1^S\right)-51\left(P_2^U+P_2^S\right)+25\left(P_3^U+P_3^S\right)\right]+\\
&\quad \frac{1}{48\pi}\left(45\lambda_1-51\lambda_2+25\lambda_3\right).
\end{aligned}$$

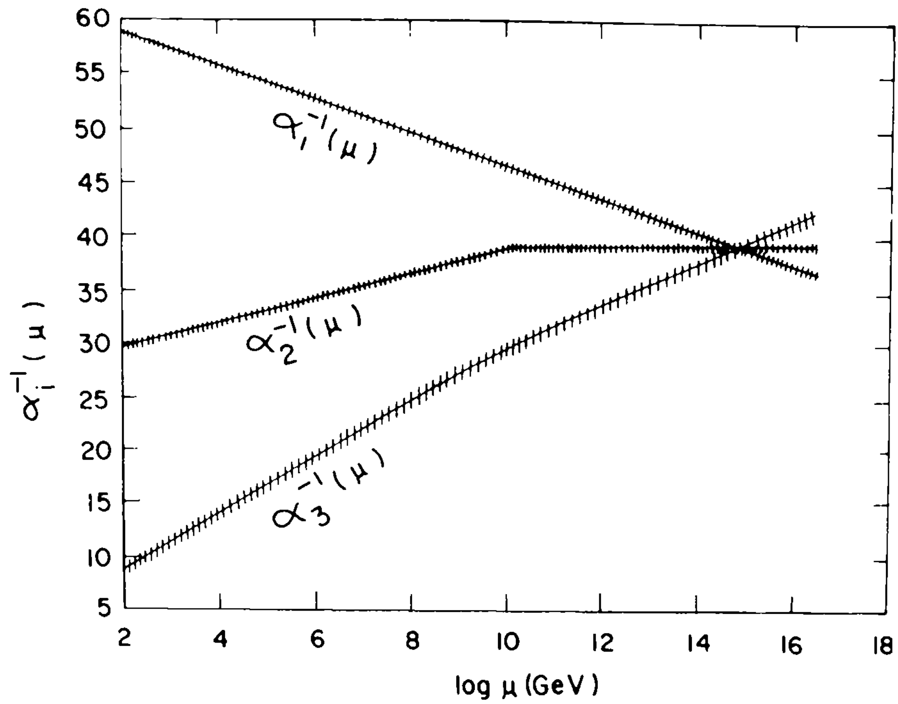

Figure 1. Variation of inverse GUT-couplings in the blooming grand desert model with a real ζ.

Ignoring threshold effects ($\lambda_i = 0$), these equations yield $\ln \frac{M_S}{M_Z} = -1.28 + 3.66 \pm 3.6$; $\ln \frac{M_U}{M_Z} = 32.91 - 0.93 \pm 0.92$ where the first, second and third numbers in the R.H.S. are one-loop, two-loop, and experimental-uncertainty contributions, leading to $M_S^0 = 10^{3\pm1.6}$ GeV, $M_U^0 = 10^{15.9\pm0.4}$ GeV. Noting the superheavy components as $\underline{24} \supset M_\Sigma(3,0,1) + M_\Sigma(1,0,8) + M_V(2,\frac{5}{3},3) + M_V(2,-\frac{5}{3},\bar{3})$, $\underline{5} \supset M_H(1,-\frac{2}{3},3)$, $\bar{\underline{5}} \supset M_H(1,\frac{2}{3},\bar{3})$ and using $M_U = M_V$ we obtain $\Delta \ln \frac{M_S}{M_Z} = -\frac{6}{19}\eta_\Sigma + \frac{18}{19}\eta_H$, $\Delta \ln \frac{M_U}{M_Z} = \frac{4}{19}\eta_H - \frac{5}{19}\eta_\Sigma$ and $\Delta\alpha_G^{-1} = (43\eta_H - 27\eta_\Sigma)/48\pi$ where $\eta_i = \ln \frac{M_i}{M_U}$. Threshold effects for the degenerate case with $\beta = 5 - 10$ are presented in Table 1. In this case we wish to stress that compared to the work of Amaldi et al the experimental uncertainties are evaluated more accurately. For the first time we have separated one and two-loop contributions to the respective scales. Hisano et al. [19] obtain

$$\left(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1}\right)(M_Z) = \frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_H}{M_Z} - 2\ln\frac{M_S}{M_Z}\right),$$

$$\left(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}\right)(M_Z) = \frac{1}{2\pi}\left(12\ln\frac{M_V^2 M_\Sigma}{M_Z^3} + 8\ln\frac{M_S}{M_Z}\right).$$

They also include threshold effects at M_S and obtain

$$2.4 \times 10^{13}\text{GeV} \leq M_H \leq 2.3 \times 10^{17}\text{GeV};$$

$$0.9 \times 10^{16}\text{GeV} \leq (M_V^2 M_\Sigma)^{\frac{1}{3}} \leq 3.1 \times 10^{16}\text{GeV}.$$

Table 1. Threshold effects in SU(5) and SUSY SU(5). All superheavy scalars in the first (last) rows have been taken to be nondegenerate (degenerate) and $M_{th} = M_{\zeta}\,(M_S)$ in the first second model.

Model	β	M_{th}/M^0_{th}	M_U/M^0_U	α_G^{-1}
SU(5) with	10	$10^{{-2.6 \atop +3.0}}$	$10^{{+0.38 \atop -0.35}}$	$39.05^{+1.6}_{-1.4}$
$\zeta(3,0,8)$	5	$10^{{-0.17 \atop +0.22}}$	$10^{\pm 0.23}$	$39.05^{+1.08}_{-0.92}$
SUSY	10	$10^{\pm 0.7}$	$10^{\pm 0.47}$	25.7 ± 0.31
SU(5)	5	$10^{\mp 0.44}$	$10^{\pm 0.33}$	25.7 ± 0.21

They identify $(M_V^2 M_\Sigma)^{\frac{1}{3}} \simeq M_{GUT}$ as an effective unification scale.

In the case with G_{2213P} intermediate symmetry, we have

$$\begin{aligned}
\ln\frac{M_U}{M_Z} &= \frac{\pi}{147\alpha}\left(\frac{15}{2} + 33\sin^2\theta_W - \frac{53\alpha}{\alpha_s}\right) + \\
&\quad \frac{1}{392}\left(-5P^U_{2R} - 27P^U_{2L} - \frac{10}{3}P^U_{BL} + \frac{106}{3}P^R_Y - 27P^R_{2L} + \frac{106}{3}P^R_{3C}\right) \\
&\quad + \frac{1}{1176}\left(27\lambda^U_{2L} + 5\lambda^U_{2R} + \frac{10}{3}\lambda^U_{BL} - \frac{106}{3}\lambda^U_{3C} + 27\lambda^R_{2L} + \frac{25}{3}\lambda^R_Y\right. \\
&\quad \left. -\frac{106}{3}\lambda^R_{3C}\right), \\
\ln\frac{M_R}{M_Z} &= \frac{8\pi}{7\alpha}\left(\frac{1}{4} - \sin^2\theta_W + \frac{\alpha}{3\alpha_s}\right) - \\
&\quad \frac{1}{14}\left(3P^U_{2L} - P^U_{2R} - \frac{4}{3}P^U_{3C} - \frac{2}{3}P^U_{BL} - \frac{5}{3}P^R_Y + 3P^R_{2L} - \frac{4}{3}P^R_{3C}\right) - \\
&\quad \frac{1}{42}\left(3\lambda^U_{2L} - \lambda^U_{2R} - \frac{2}{3}\lambda^U_{BL} - \frac{4}{3}\lambda^U_{3C} + 3\lambda^R_{2L} - \frac{5}{3}\lambda^R_Y - \frac{4}{3}\lambda^R_{3C}\right), \\
\frac{1}{\alpha_G} &= \frac{3}{8\alpha} - \frac{11}{42\alpha}\left(\frac{3}{4} + \frac{\alpha}{\alpha_s} - 3\sin^2\theta_W\right) - \frac{3}{224\pi}\left(10P^U_{2L} + \right. \\
&\quad \left. 5P^U_{2R} - \frac{4}{3}P^U_{3C} + 4P^U_{BL} + 10P^R_Y + 10P^R_{2L} - \frac{4}{3}P^R_{3C}\right) \\
&\quad - \frac{1}{224\pi}\left(10\lambda^U_{2L} + 5\lambda^U_{2R} + 4\lambda^U_{BL} - \frac{4}{3}\lambda^U_{3C} + 10\lambda^R_Y - \frac{4}{3}\lambda^R_{3C}\right).
\end{aligned}$$

Excluding threshold effects, $M^0_R = 10^{10.0\pm0.4}$GeV, $M^0_U = 10^{15.5\pm0.3}$ GeV, $\alpha_G^{0\,-1} = 43.6 \pm 0.4$.

In case IV with G_{224P} intermediate symmetry, we obtain

$$\begin{aligned}
\ln\frac{M_U}{M_Z} &= \frac{3\pi}{550\alpha}\left(\frac{27}{2} + 17\sin^2\theta_W - \frac{53\alpha}{\alpha_s}\right) + \frac{9}{4400}\left(-9P^U_{2R} - \right. \\
&\quad \left. \frac{61}{3}P^U_{2L} + \frac{88}{3}P^U_{4C} + 15P^C_Y - \frac{61}{3}P^C_{2L} + \frac{106}{3}P^C_{3C}\right) -
\end{aligned}$$

$$\frac{3}{4400}\left(-\frac{61}{3}\lambda^{U}_{2L}-9\lambda^{U}_{2R}+\frac{88}{3}\lambda^{U}_{4C}+\frac{106}{3}\lambda^{C}_{3C}-\frac{61}{3}\lambda^{C}_{2L}-15\lambda^{C}_{Y}\right),$$

$$\ln\frac{M_C}{M_z} = \frac{3\pi}{11\alpha}\left(\frac{1}{2}-\sin^2\theta_W-\frac{\alpha}{3\alpha_s}\right)+ \frac{3}{88}\left(P^{U}_{2L}-P^{U}_{2R}+\frac{2}{3}P^{C}_{3C}-\frac{5}{3}P^{C}_{Y}+P^{C}_{2L}\right)- \frac{1}{88}\left(\lambda^{U}_{2L}-\lambda^{U}_{2R}+\frac{2}{3}\lambda^{C}_{3C}+\lambda^{C}_{2L}-\frac{5}{3}\lambda^{C}_{Y}\right),$$

$$\frac{1}{\alpha_G} = \frac{1}{11\alpha}\left(\frac{91}{25}+\frac{181}{75}\frac{\alpha}{\alpha_s}-\frac{28}{25}\sin^2\theta_W\right)-\frac{1}{1100\pi}\left(63P^{U}_{2L}+ 91P^{U}_{2R}+121P^{U}_{4C}+63P^{C}_{2L}+\frac{455}{3}P^{C}_{Y}+\frac{181}{3}P^{C}_{3C}\right)+\frac{1}{3300\pi} \left(63\lambda^{U}_{2L}+91\lambda^{U}_{2R}+21\lambda^{U}_{4C}+63\lambda^{C}_{2L}+\frac{455}{3}\lambda^{C}_{Y}+\frac{181}{3}\lambda^{C}_{3C}\right).$$

Ignoring threshold effects, we obtain $M^0_C = 10^{13.6\pm0.2}$GeV, $M^0_U = 10^{15\pm0.25}$ GeV, $\alpha_G^{0-1} = 40.6\pm0.2$. The uncertainties here are due to those in the experimental values of $\sin^2\theta_W$, α_s and also $\alpha^{-1}(M_Z)$. Threshold effects on M_U, intermediate scales M_I (= M_C or M_R), and α_G for the case III and IV are shown in Table 2 where the effects on M_U have been extremized. The superheavy scalar components belonging to a given GUT-representation have been taken to be degenerate near M_U, but all other components near M_I haven been taken to be degenerate while respecting the parity restoration constraints for $\mu \geq M_I$. For $\beta = \frac{M_i}{M_U} = \frac{1}{10}$–10 ($M_i$ = superheavy scalar masses), we obtain

$$\begin{aligned} M_U &= 10^{15\pm.45\pm.25}(10^{15.5^{+.30}_{-.45}\pm.30})\ \text{GeV},\\ M_C &= 10^{13.6\pm0.4\pm0.2}\ (M_R = 10^{10.0\pm0.1\pm0.4})\ \text{GeV},\\ \alpha_G^{-1} &= 40.6^{+0.2}_{-2.0}\pm0.2\ (43.61\pm1.0\pm0.4) \end{aligned}$$

for the model with G_{224P} (G_{2231P}) as the intermediate symmetry. Thus the prediction for τ_p by model IV saturates the Super-Kamiokande limit while model III yields $\tau_p \leq 10^{36}$ yrs when $\beta \simeq 10$. Threshold effects in SO(10), with G_{2213} or G_{224} as single intermediate symmetries with parity broken at M_U, have been carried out recently by Mohapatra and Parida [7] under the assumption of degenerate Higgs scalars also at M_I. If these scalar are taken as nondegenerate, an additional threshold uncertainty $M_I/M^0_I \simeq 10^{\pm1}$ is introduced as shown in the last row of Table 2. Comparing this with the present analysis, it is clear that threshold uncertainties are reduced due to the presence of parity at the intermediate scale even though the superheavy scalar masses could be nondegenerate. But we will demonstrate in the following sections that in case IV $M_C = 10^{13.6\pm0.4\pm0.2}$ GeV in a number of GUTs for β=10 although M_U might change due to different threshold or compactification effects in various models.

3. Compactification effects

In the usual Kaluza-Klein type higher dimensional theories the Einstein-Hilbert action in 4+D dimensions leads to the corresponding action in 4-dimensions and the

Table 2. Threshold effects in SO(10) with G_{2213P} and G_{224P} as single intermediate symmetries. $M_I = M_R(M_C)$ for the first (next) two rows. The effects for G_{2213} and G_{224} have been derived from formulas of Mohapatra and Parida [7].

Model	β	M_I/M_I^0	M_U/M_U^0	α_G^{-1}
G_{2213P}	10	$10^{\pm 0.10}$	$10^{\overset{+.30}{-.43}}$	$44.0^{+1.0}_{-0.8}$
	5	$10^{\pm .07}$	$10^{\overset{+.21}{-.30}}$	44.0 ± 0.63
G_{224P}	10	$10^{\overset{-.27}{+.36}}$	$10^{\overset{-.44}{+.40}}$	$40.6^{-2.0}_{+0.2}$
	5	$10^{\overset{-.20}{+.26}}$	$10^{\overset{-.30}{+.27}}$	$40.6^{-1.4}_{+0.2}$
G_{2213}	10	$10^{\overset{+.30}{+.10}}$	$10^{\overset{+0.4}{-0.5}}$	46.2
G_{224}	10	$10^{\overset{+3.6}{-2.5}}$	$10^{\overset{+1.6}{-2.7}}$	45.9

kinetic energy of the gauge fields with the additional presence of 5-dimensional operators when extra D dimensions are compactified [26]. The origin of such effects has been related to the geometry of the compact space by a number of authors [27]. In 1984 it was noted tht SUSY or non-SUSY SU(5) or SO(10) with G_{224P} might have drastic changes in GUT predictions due to such opertors. However in non-SUSY SU(5) the presence of 5-dim. operators $\eta \mathrm{Tr}(F_{\mu\nu}\phi_{(24)}F^{\mu\nu})/2M_G$ (M_G = compactification scale, $\eta = 0(1)$) decreases $\sin^2\theta_W$ below 0.21 for $M_U \geq 10^{15}$ GeV [28,29]. Further if the effects of a 6-dim. operator of the type $\eta' \mathrm{Tr}(F_{\mu\nu}\phi^2_{(24)}F^{\mu\nu})/2M_G^2$ is also considered in the Lagrangian, it has been shown [30] that the SU(5) GUT with such gravity induced effects could have $\tau_p \geq 10^{38}$ yrs. for $\sin^2\theta_W$, α_s and $\alpha(M_Z)$ consistent with the present LEP measurements. Effects of other forms of 6-dim. operators have also been investigated [31]. Effects of 5-dim. operators in SO(10) with an $SU(2)_L \times U(1)_R \times SU(4)_C$ intermediate symmetry has been shown to make the model accessible to experimental tests by rare kaon decays [11]. With G_{2213} intermediate symmetry and parity broken at the GUT-scale, low mass W_R-boson within the TeV scale are also possible in SO(10) when compactification effects are included [12] through the 5-dim. operators $\eta \mathrm{Tr}(F_{\mu\nu}\phi_{(24)}F^{\mu\nu})/2M_G$. But in constrast to the observations [28] that M_C and $\sin^2\theta_W$ change in case IV due to compactification effects, Patra and Parida [32] have found that neither of these quantities changes due to the 5-dim. operator $\eta \mathrm{Tr}(F_{\mu\nu}\phi_{(24)}F^{\mu\nu})/2M_G$. On the other hand, it is possible to enhance the unification scale M_U from 10^{15} to 10^{19} GeV depending upon the values of M_G and η leading to the possibility of a very stable proton. Compactification and threshold effects on superstring inspired GUTs are subjects of current investigation by a number of authors [33,34].

4. Implications of theorems on vanishing threshold effects, Compactification effects and multiloop radiative corrections at high mass scales

We now discuss the implications of theorem [9,10] mentioned abovr and show how they are very useful in estimating threshold or compactification effects, or multiloop contribution at high mass scales. We also mention a theorem due to Mohapatra [35] where the vanishing of the threshold contribution to the mass scales or $\sin^2\theta_W$ has been proved in the case of a degenerate representation. In fact, the vanishing of GUT-threshold or compactification effects or two loop contributions for $\mu = M_C - M_U$ on the intermediate scale M_C in case IV has been shown to follow from the following theorems [22],

Theorem-1: In all GUTs where the G_{224P} symmetry breaks at the highest intermediate scale (M_I), the GUT-threshold contribution to $\sin^2\theta_W$ vanishes.
Theorem-2: In all GUTs where the G_{224P} symmetry breaks at the highest intermediate scale, the gauge-boson renormalization to every m-loop order ($m \geq 2$) has vanishing contributions to $\sin^2\theta_W$.

Since the proof of Theorem-1 does not depend upon any specific form of $\lambda_i(M_U)$, i = 2L, 2R, 4C, it holds to cancel the compactification effects or any other nonrenormalizable or nonperturbative contributions to the corrected boundary condition. Theorems 1 and 2 lead to vanishing GUT-threshold and compactifications effects and multiloop radiative corrections arising from $\mu = M_I\ (M_C) - M_U$ to the intermediate scale M_C in model IV. It is because of this reason that M_C receives threshold corrections only from the intermediate threshold and no contribution from the GUT-threshold or any compactification effect even though M_U is modified by all such contributions as has been noted recently [9,10,32]. As against the conclusion by Dixit and Sher [36] that all SO(10) models are liable to large threshold-effect uncertainties, these theorems reveal the existence of a class of GUTs including model IV where precise predictions on M_C are possible. That the GUT-threshold effect, compactifications effects and the relevant two-loop contribtuions vanish at M_C can be checked directly from the expression for $\ln\frac{M_C}{M_Z}$ where left-right symmetry for $\mu \geq M_C$ demands $\lambda^U_{2L} = \lambda^U_{2R}$ and $P^U_{2L} = P^U_{2R}$. As a consequence of the second theorem, the two-loop contribution on $\ln\frac{M_C}{M_Z}$ exists only from the integration in the range $\mu = M_Z - M_C$ contributing only 0.2%.

Mohapatra [35] has proved a theorem which is very useful in estimating threshold effects. According to this theorem threshold corrections to unification and intermediate scales as well as $\sin^2\theta_W$ and α_s vanish for a Higgs multiplet of the grand unified group which does not acquire vacuum expectation values provided that all its submultiplets are degenerate in mass after symmetry breaking. An important outcome of this theorem is that threshold related uncertainties in the relevant physical quantities are the same even if several Higgs multiplets are used to break the gauge symmetry at any stage.

5. Discussion and conclusion

With the analytic expressions available for the mass scales and the GUT-coupling constants, besides separating out one-loop, two-loop and threshold effects, we have a better estimation of experimental uncertainties compared to the numerical methos in the corresponding cases [1]. Ignoring threshold effects, these equations can yield the mass scales and the GUT-coupling for given values of $\sin^2\theta_W$, α_s and $\alpha(M_Z)$. Our one-loop term for $\ln \frac{M_S}{M_Z}$ in model II is -1.3 as compared to -2.1 by Barbieri and Hall with $\alpha_s = 0.118$. These analytic expressions are essential for threshold effects which are necessary before ruling out a model on the basis of experimental data (e.g. lower bound on τ_p). The models I and IV predict proton lifetimes saturating the Super-Kamiokande limits. Models I and III-IV predict Majorana neutrino masses as per the diagram shown in Fig.2, $m_{\nu_i} \simeq \lambda m_{q_i} \langle\phi^0\rangle\langle\Delta_R^0\rangle/M_{\Delta_L}^2$, where $\Delta_R(\Delta_L)$ is the right (left)-handed triplet carrying two units of lepton number and contained in $\underline{126}$ of SO(10), and $\langle\phi^0\rangle$ is the standard Higgs vacuum expectation value [24,37]. The masses of ν_e and ν_μ are consistent with values needed for solar neutrino flux in the MSW mechanism. The mas of ν_τ is nearly $\simeq$ 10 eV in model I and IV. In the SO(10) model with G_{2213} or G_{224} intermediate symmetry, the neutrino massees are given by the usual see-saw mechanism. If the nonadiabatic MSW solution to solar neutrino puzzle is accepted, then SO(10) with G_{2213} intermediate symmetry is severely constrained [7].

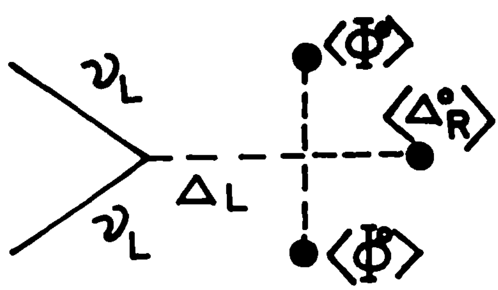

Figure 2. Feynman diagram for the Majorana mass of the left-handed neutrino in models I, III and IV.

In all GUTs having G_{224P} [38] as intermediate symmetry, the prediction of the intermediate scale is precise inspite of unknown masses of superheavy particles and poorly known compactification effects. However, low mass $W_R^\pm$ gauge bosons around the TeV scale are permitted in models with decoupled parity and $SU(2)_R$-breakings with larger neutrino masses [24,39,40].

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Discussion

V. Soni : In one loop the gauge couplings and their β functions are independent of the Yukawa and Higgs couplings. In two loops they are not at the symmetry breaking scale as the Yukawa coupling is not asymptotically free - they will increase affecting the gauge coupling and $\sin^2\theta_W$. If the scalars are composite, then Yukawa couplings would diverge at the symmetry breaking scale! So, how is it clear that this will not affect the gauge couplings and $\sin^2\theta_W$?

M.K. Parida : In nonsupersymmetric GUTs, even with an intermediate scale, none of the couplings is large. Even if some of the couplings grow with the mass scale, the theorems are derived with the assumption of the validity of perturbation theory upto the GUT-scale at the level of elementary particles. However, if the correction at the GUT threshold is nonperturbative or non-renormalizable, such effects cancel out from $\sin^2\theta_W$ and M_C in case of the G_{224P} intermediate symmetry. The apprehension of couplings being large below the GUT scale in composite models is a separate issue.

Probir Roy : Are you sure about the numerical disagreement with Amaldi et al. as well as Hall and Barbieri? I believe, others have checked it.

M.K. Parida : The statement is that the one-loop term for $\ln(M_S/M_Z) \simeq -1.3$ which is in disagreement with the Barbieri-Hall number -2.1. We have checked it and can be verified from the formulas given. Since Amaldi et al do not have analytic formulas for $\ln(M_U/M_Z)$ and $\ln(M_S/M_Z)$ they might have overestimated the experimental uncertainties on the relevant mass scales.

M. Drees : Why do we need to know the intermediate scale so precisely?

M.K. Parida : A precise knowledge of intermediate scale is necessary for accurate prediction on Majorana neutrino masses in the models.