PRAMANA
— journal of
physics

Supplement to Vol. 41
December 1993
pp. 261–269

LEP constraints on grand unified theories

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Abstract. Recent developments on grand unified theories (GUTs) in the context of the LEP measurements of the coupling constants will be reviewed. The three coupling constants at the electroweak scale have been measured at LEP quite precisely. One can allow these couplings to evolve with energy following the renormalization group equations for the various groups and find out whether all the coupling constants meet at any energy. It was pointed out that the minimal $SU(5)$ grand unified theory fails to satisfy this test. However, various extensions of the theory are still allowed. These extensions include (i) supersymmetric $SU(5)$ GUT, with some arbitrariness in the susy breaking scale arising from the threshold corrections, (ii) non-susy $SU(5)$ GUTs with additional fermions as well as Higgs multiplets, which has masses of the order of TeV, and (iii) non-renormalizable effect of gravity with a fine tuned relation among the coupling constants at the unification energy. The LEP results also constrain GUTs with an intermediate symmetry breaking scale. By adjusting the intermediate symmetry breaking scale, one usually can have unification, but these theories get constrained. For example, the left-right symmetric theories coming from GUTs can be broken only at energies higher than about $\sim 10^{10} GeV$. This implies that if right handed gauge bosons are found at energies lower than this scale, then that will rule out the possibility of grand unification. Another recent interesting development on the subject, namely, low energy unification, will be discussed in this context. All the coupling constants are unified at energies of the order of $\sim 10^8 GeV$ when they are embedded in an $SU(15)$ GUT, with some particular symmetry breaking pattern. But even in this case the results of the intermediate symmetry breaking scale remain unchanged.

Grand Unified Theories (GUTs) offer the possibility of a simple but unified description of strong and the electroweak interactions. The unified groups break down to the low energy group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ and also explain many puzzles of the Standard Model. All the coupling constants are unified at the unification energy scale, which in turn lead to new consequences like baryon number violation and charge quantization. Early interest in GUTs had died down because of the non-observation of the proton decay, but following the recent measurements of the three coupling constants at the electroweak scale at LEP [1], there has been some renewed interest in GUTs [2-6]. The measurements of the Z mass and width and also the jet cross sections and energy-energy correlations provide very accurate values of $\sin^2\theta_W$ and α_s, respectively, at the scale M_W. These values of $\sin^2\theta_W$ and α_s and the electromagnetic coupling constant α together give very precise values for the three coupling constants at the electroweak scale M_W. These coupling constants, when evolved using the renormalisation group equations, can constrain the unified theories very strongly [2-6].

The uncertainties in the various coupling constants are still considerable, but for the present purpose I shall take the mean values only since the constraints on GUTs arising from these uncertainties will not change any of the conclusions I shall be discussing. In the actual calculations these errors are always incorporated [2-6]. Taking the fine structure constant at the electroweak scale to be $\alpha_{em}(M_Z) = 1/127.9$, and $\sin^2\theta(M_Z) = .2334$, $\alpha_3(M_Z) = .118$ from the high precision LEP data, one can write down the values of the three coupling constants at the electroweak scale (M_Z) to be,

$$\begin{aligned} \alpha_1^{-1}(M_Z) &\equiv \frac{3}{5}\alpha_{em}^{-1}(M_Z)\cos^2\theta_W(M_Z) = 58.83 \ , \\ \alpha_2^{-1}(M_Z) &\equiv \alpha_{em}^{-1}(M_Z)\sin^2\theta_W(M_Z) = 29.85 \ , \\ \alpha_3^{-1}(M_Z) &\equiv \alpha_s^{-1}(M_Z) = 8.47. \end{aligned} \tag{1}$$

We next compute the evolution of the coupling constants [7]. I shall discuss only the one-loop renormalization group equations which have the form,

$$\mu\frac{d\alpha_i(\mu)}{d\mu} = 2\beta_i\alpha_i^2(\mu) \tag{2}$$

where $\alpha_i = (4\pi)^{-1}g_i^2$, the β-functions are defined as, $\beta_i = -(4\pi)^{-1}b_i$, and $b_i = \frac{11}{3}T_g[i] - \frac{4}{3}T_f[i] - \frac{1}{6}T_s[i]$, corresponding to the contributions from gauge bosons, fermions and Higgs scalars, respectively. In actual calculations one also considers the two loop contributions, and for this reason I shall state the results of the two loop calculations as well, which will be covered by the next talk in this meeting [8].

The fermionic contributions to the various subgroups are the same and are given by $T_f = n_f$, where n_f is the number of generations; these cancel out in the equations of

$$\sin^2\theta_W = \frac{3}{8} - \frac{5}{8}\alpha(\alpha_{1Y}^{-1} - \alpha_{2L}^{-1}) \tag{3}$$

and

$$(1 - \frac{8}{3}\frac{\alpha}{\alpha_s}) = \alpha(\alpha_{2L}^{-1} + \frac{5}{3}\alpha_{1Y}^{-1} - \frac{8}{3}\alpha_{3C}^{-1}). \tag{4}$$

For the calculation of the evolution of the coupling constants one has to take them into consideration.

For the group $SU(n)$, we have $T_g = n$. The Higgs contributions are usually very small. If we consider the minimal Higgs representaions of the Standard Model, and assume that we have only one stage symmetry breaking and there are no other scalars having masses between these scales, then we get, $T_s = \frac{5}{3}, 1$ and 0 for the groups $U(1)_Y, SU(2)_L$ and $SU(3)_C$ respectively.

We can now plot the three gauge coupling constants $\alpha_1^{-1}, \alpha_2^{-1}$ and α_3^{-1} as a function of energy. We see from figure 1, that given the present precise values of the coupling constants as measured at LEP [1], there is no single unification point, i.e. all the three coupling constants do not meet at any point and hence there is no grand unification. Even if one takes into account the errors in the measured coupling constants and also the two loop effect in the renormalization group equation, this conclusion does not change. This result not only rules out the minimal $SU(5)$ grand unified theory, but it rules out any other grand unified theory in which there are no intermediate symmetry breakings and there are not any extra particles. One can look at this problem in a different way also. If we start with the two combinations of equations (3) and (4), then in this class of GUTs we can calculate the unification mass (M_u) from equation (4) and then use equation (3) to calculate $\sin^2\theta_W$. This class of GUTs then predicts $\sin^2\theta_W = .228$, which is completely ruled out by the present precision measurement of $\sin^2\theta_W$.

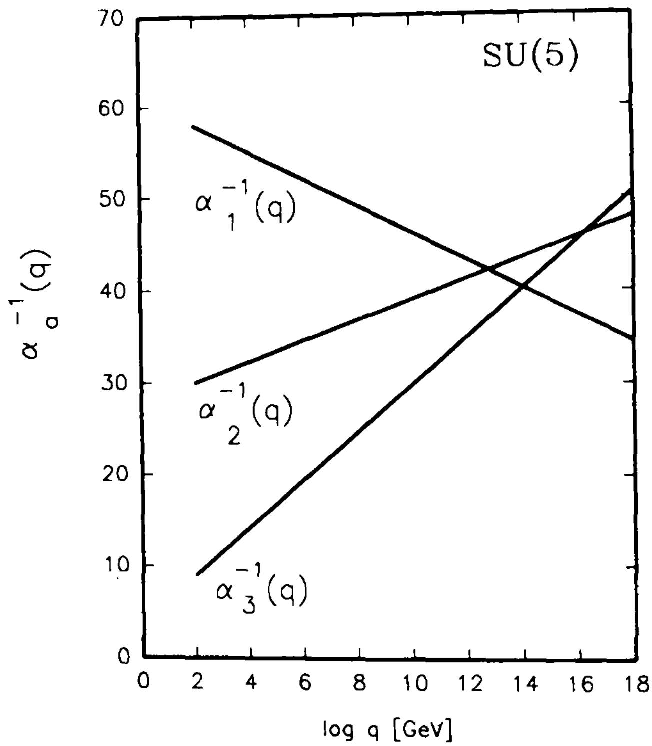

Figure 1. The evolution of the inverse coupling constants α_{3C}^{-1}, $\alpha_{2L,R}^{-1}$, $\alpha_{1(B-L)}^{-1}$ for the $SU(5)$ GUT.

GUTs with intermediate symmetry breaking chains, however, are not constrained by this analysis and one has the freedom to choose the intermediate symmetry breaking chains to adjust its predictions for $\sin^2\theta_W$. If we now concentrate on only $SU(5)$, then minimal $SU(5)$ is completely ruled out [2]. However, one can now consider various extensions which are not ruled out and may have interesting consequences [2-4]. The obvious extensions are adding new particles to change the evolution of the coupling constants so that unification is achieved. In the minimal $SU(5)$ GUT, if one only adds more number of Higgs doublet scalars, then the prediction of $\sin^2\theta_W$ increases and comes closer to the experimental value, but then the unification scale gets lowered and the proton decays very fast. Similarly, just by adding Higgs scalars or fermions, one cannot get the predicted value of $\sin^2\theta_W$ and simultaneously prevent proton decay [2].

Some authors then introduce one additional scale in the theory, where some new fermions gets masses. By choosing appropriate representations for these particles, one can show that [3] there are many choices which can prevent proton decay and predict $\sin^2\theta_W$ correctly. However, these scales and particles are just put in to save the situation and no natural symmetry reasons can be given for their existence.

The most natural extension of minimal $SU(5)$, which is found to be consistent with the proton decay and the LEP measurements, is the supersymmetric version of the $SU(5)$ GUT [2,4]. In the supersymmetric theory the superparticles contribute to loop corrections and as a result the loop contributions change drastically. The expression for the one loop beta function now becomes, $b_i = 3T_g[i]-T_f[i]-T_s[i]$, and the contributions from the Higgs scalars $T_s[i]$ also change. In the minimal supersymmetric $SU(5)$ model $T_s = 3/10, 1/2,$ and 0 for the groups $U(1)_Y, SU(2)_L$ and $SU(3)_C$ respectively. With this one can find that the coupling constants evolve to a single unification point at an energy $\sim 10^{16}$ GeV, if one assumes that supersymmetry is broken at an energy $\sim$ TeV [figure 2]. This result does not change much with the two loop calculations, but the incorporation of threshold effects may change this result drastically. In fact, one has to calculate the masses of the various fermions and then apply threshold corrections [4]. Since the next talk in this meeting will discuss threshold effects, I shall not go into any further details [8].

Another alternative solution to save the minimal $SU(5)$ GUT, which will also be covered in the next talk, is to include the nonrenormalizable effects induced by gravity. It has been shown that nonrenormalizable terms coming from gravity can be fine-tuned to predict correct value for $\sin^2\theta_W$ and prevent proton decay [5]. In this case instead of adjusting the slope of the curves (as is done with new additional particles), one changes the boundary conditions at the unification scale by fine tuning the nonrenormalizable higher dimensional terms induced by gravity.

I shall next discuss the LEP constraints on the GUTs with intermediate symmetry scales [6]. All these theories can, in principle, predict the correct value of $\sin^2\theta_W$ and prevent proton decay. But if we are interested in some new physics at low energies coming from these GUTs, then such theories are highly constrained. Let us consider the most natural extension of the Standard Model, namely, the left-right symmetric theory [9] of the electroweak interaction. An important class of GUTs accomodates the left-right symmetric models where the Standard Model comes from a larger group with a $SU(2)_L \otimes SU(2)_R$ symmetry. One interesting possibility is that the left-right symmetry surives to relatively low energies ($\sim$ TeV), and would therefore have testable consequences at current experiments or in experiments planned for the near future.

Earlier phenomenological studies [9,10] have indicated that it is indeed possible to have a low value of the left-right symmetry breaking scale, M_R, in various grand unified theories with intermediate mass scales and in partially unified theories. In these analyses, however, values of $\sin^2\theta_W$ from as low as 0.21 to as high as 0.28 were considered, which were consistent with other neutral current data. Such a large variation in $\sin^2\theta_W$ was expected from the existence of an extra neutral gauge boson, Z', coming from either a $U(1)_R$ or an $SU(2)_R$ symmetry.

It is now important to study the LEP results [1] to find out how large can $\sin^2\theta_W$ be in the presence of the extra neutral right handed gauge boson. Due to the mixing of the Z with the Z', the usual ρ parameter, defined as

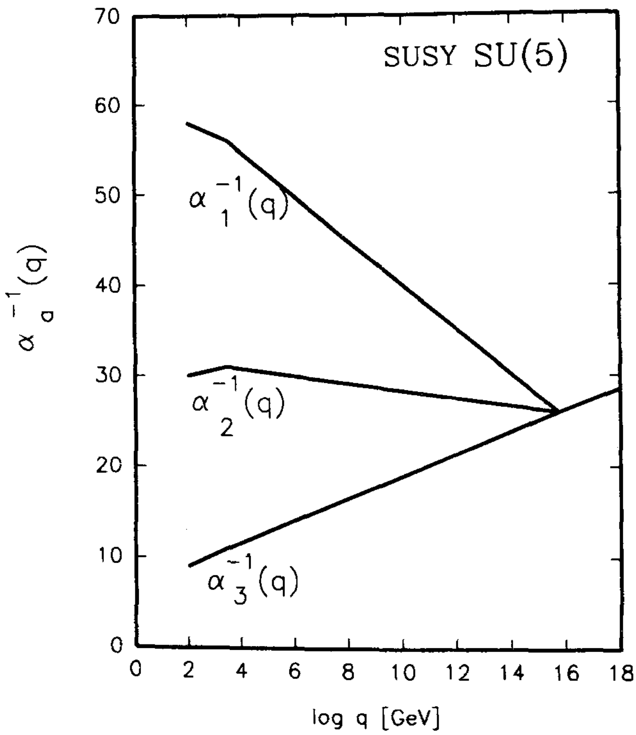

Figure 2. The evolution of the inverse coupling constants α_{3C}^{-1}, $\alpha_{2L,R}^{-1}$, $\alpha_{1(B-L)}^{-1}$ for the supersymmetric $SU(5)$ GUT with $M_{susy} \sim 1$ TeV.

$$\rho \equiv \frac{M_{W\pm}^2}{M_Z^2 \cos^2\theta_W}, \tag{5}$$

changes by a positive quantity, $\Delta\rho_M$, given as [11]

$$\Delta\rho_M = \sin^2\xi\left[\frac{M_{Z'}^2}{M_Z^2} - 1\right], \tag{6}$$

where ξ is the mixing angle. From the definitions of $\sin^2\theta_W$ and ρ one can see that

$$\delta(\sin^2\theta_W) \approx -\frac{\sin^2\theta_W \cos^2\theta_W}{\cos 2\theta_W}\Delta\rho_M \tag{7}$$

The quantity $\Delta\rho_M$ is determined entirely by the measured value of M_W/M_Z, as can be seen clearly from Eq. 6. The results of a recent fit [11] to the LEP data [1] yield the following bound:

$$\Delta\rho_M \leq 0.010 - 0.003\left[\frac{m_t(GeV)}{100}\right]^2 \tag{8}$$

where the dependence on the top mass, m_t, comes through the radiative corrections. Using this value of $\Delta\rho_M$ we get $\delta(\sin^2\theta_W) \approx 3.32 \times 10^{-3}$ which is very small. It is with this very stringently bound value

$$\left[\sin^2\theta_W\right]_{LR} = .2334 \pm .0008 \pm .0033 \qquad (9)$$

that we wish to study whether a low value for the left-right symmetry breaking scale, M_R, is consistent with the unification test.

To keep matters simple at first, we consider a symmetry breaking scheme without any intermediate mass scale other than the left-right symmetry $[SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \xrightarrow{M_R} SU(2)_L \otimes U(1)_Y]$ breaking scale [6] and neither do we specify the GUT group, G. We take $M_R \approx 1$ TeV and try to determine if there is any unification point below the Planck scale ($\approx 10^{19}$ GeV). The three coupling constants are evolved to the scale M_R. Using the matching conditions of the coupling constants at M_R

$$\begin{aligned} \alpha_{1Y}^{-1}(M_R) &= \frac{3}{5}\alpha_{2R}^{-1}(M_R) + \frac{2}{5}\alpha_{1(B-L)}^{-1}(M_R), \\ \alpha_{2L}^{-1}(M_R) &= \alpha_{2R}^{-1}(M_R), \end{aligned} \qquad (10)$$

the renormalisation group equations can be used to plot the evolution of the various coupling constants with energy. We have taken the number of fermion families, $n_f = 3$. In figure 3 we have plotted the gauge coupling constants $\alpha_{1(B-L)}^{-1}$, $\alpha_{2L,R}^{-1}$ and α_{3C}^{-1} as a function of energy, taking $M_R = 1$ TeV. We see from the figure that, as a result of choosing a low value for M_R, there is no unification point even when we evolve upto a scale as high as 10^{19} GeV. The $SU(3)_C$ and $SU(2)_{L,R}$ lines do intersect at about 10^{17} GeV, but the $U(1)$ scale remains much too high for any possibility of unification to exist. In this calculation it has been assumed that the D-parity is not broken. If the D-parity is broken then the situation improves, since the g_L and the g_R can now be different at M_R. But before comparing with the LEP data, one has to analyze the LEP data with $g_L \neq g_R$, which has not been done so far.

One can now proceed in the other way we discussed earlier. From the expression for $(1 - \frac{8}{3}\alpha\alpha_s^{-1})$, one can calculate the unification scale with $M_R = 1$ TeV and then calculate $\sin^2\theta_W$. It comes out to be about .28 and hence way outside the present LEP result for $[\sin^2\theta_W]_{LR}$. We then try to find out for what value of the M_R can we have right prediction for $\sin^2\theta_W$, and it is found to be $M_R \sim 10^{10}$ GeV. Two loop corrections change the number by a negligible amount. Thus if right handed gauge bosons are observed in ongoing or planned experiments, then the LEP result will imply that there cannot be any grand unified theory [6]. This lower bound is independent of the number of intermediate symmetry breaking scales as well as any other physics in the grand desert and is extremely general.

I shall now discuss the LEP constraints on a new GUT paradigm [12-14], which has started in the last few years. One of the most important predictions of GUTs is proton decay. In models such as SU(5) or $SO(10)$, the coupling constants for the $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ groups evolve to $M_u \sim O(10^{14})$ GeV or higher before they get unified. Proton decay in these models is predicted to occur with a rate of the order of the present experimental limit for such M_u. Recently, it has been observed that at least one symmetry breaking chain of a GUT based on the group SU(15) can be unified at a very low energy $M_u \sim O(10^7)$ GeV [12]. Because baryon number B is a gauge symmetry in this model, proton decay can be suppressed, and certain possible Higgs structure has been proposed to this end. This is true for SU(16) GUT [15] also, where is also known that the proton lifetime can be large.

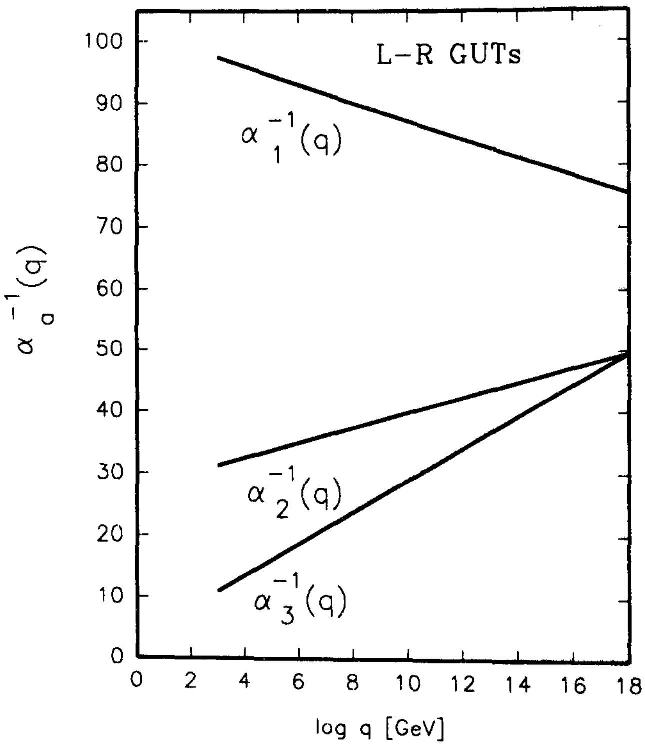

Figure 3. The evolution of the inverse coupling constants α_{3C}^{-1}, $\alpha_{2L,R}^{-1}$, $\alpha_{1(B-L)}^{-1}$ for the left-right symmetric model with $M_R \sim 1$ TeV.

Low energy unification makes these models free from problems of grand unified monopoles [13] and the gauge hierarchy problem is also much less severe.

This scenario of symmetry breaking of the $SU(15)$ GUT has some more interesting features. It is essential for the low energy unification scenario to have the chiral colour $SU(3)_{CL} \otimes SU(3)_{CR}$ group and the quark-lepton un-unified group $SU(2)_L^q \otimes SU(2)_L^l$ survive till very low energy, for the gauge coupling constants to evolve very fast and get unified at an energy scale $M_u \sim O(10^8)$ GeV. Thus these groups and the lepto-quarks are some of the essential features of low energy unification, which can be tested in the laboratory in the near future [16]. Furthermore, any signature of supersymmetry will rule out the possibility of low energy unification [14]. If we find signatures of both low energy unification and supersymmetry, then our understanding of grand unification has to be revised completely. Because of these interesting features of the $SU(15)$ GUT, we shall now discuss how the present LEP data restrict our previous conclusions in this scenario of GUT.

As far as the one stage unification is concerned, there is no difference between this scenario and any other, since it does not depend on the unification group. Low energy unification does not take place unless the low energy groups are enlarged to let the coupling constants evolve fast. But the existence of low energy left-right symmetry in this theory (it now has to be only $SU(16)$ since $SU(15)$ will not accomodate left-right symmetry) can in principle change things. Since all the groups get unified at a lower energy ($\sim 10^8$ GeV), it may be expected that left-right symmetry will also exist at a lower energy ($< 10^{11}$ GeV). But, surprisingly, it is found that even in this theory left-right symmetry does not survive at lower energy and the existence of the left-right symmetry pushes up the unification scale to a higher scale ($> 10^{11}$ GeV).

To summarize, the LEP precision data have ruled out the minimal $SU(5)$ GUT conclusively, though some unnatural extensions are still allowed. However, supersymmetric $SU(5)$ is favoured, although the supersymmetry breaking scale is not uniquely fixed due to uncertainties in the threshold calculations. The LEP data constrain the symmetry breaking scales in GUTs with intermediate symmetry breaking. In the case of left-right symmetric GUTs, the left-right symmetry cannot exist at low energies. The $SU(15)$ GUT seems to have many nice features including very low energy unification, but fails to change any of the above conclusions.

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Discussion

Probir Roy : Do you have extra nondecoupled heavy particles compatible with LEP constraints on S, T parameters?

U. Sarkar : No, there are not any extra nondecoupled heavy particles in SU(15) or SU(16) GUT. There are only usual fermions and mirrors, but new higgses are there.