Penguin decays of B meson

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Abstract. The theory of loop induced rare B decays is reviewed. Both electromagnetic penguin processes and gluon mediated penguin processes are discussed. After considering $b \rightarrow s\gamma$ and $b \rightarrow se^+e^-$ decays, purely hadronic modes like $B \rightarrow K\phi$ are estimated. Constraints on the Higgs sector of SUSY theory from $b \rightarrow s\gamma$ is discussed. CP violation in charged B Decays is reviewed.

The B meson offers a unique opportunity to study electroweak theory at one loop. Not only are one loop effects senstitive to as yet undetected particles of the Standard Model, like the top quark, but also provide a window on other hypothetical particles like the fourth generation quarks, supersymmetric particles and extra Higgs multiplets. B meson system offers an advantage over the K meson because the long distance effects quite often obscure the quark level physics in the K system. Well known example of [1] this are the one loop decays $K^+ \rightarrow \pi^+e^+e^-$ and $K_L \rightarrow \pi^0e^+e^-$. However, although the quark level calculations are precise, when considering the exclusive decays of the B meson, one is hampered by the lack of knowledge of hadronic form factors or sometimes even a reliable technique to evaluate the interesting modes. In this review we shall resort to quark model calculations based on wave functions following references [2,3]. Many of the estimates can be greatly improved using heavy quark techiques [4] as more data becomes available from the B and the D meson system.

1. Electroweak Penguin Processes

1.1. The process $b \rightarrow s\gamma$

The process $b \rightarrow s\gamma$ proceeds with W boson and u, c, t quarks in the loop as shown in Fig. 1. The purely electroweak result can be obtained from [5,6]. The vertex for a real photon is

$$V_\mu = iG_2m_b\bar{s}_L\sigma_{\mu\nu}q^\nu b_R,$$

where

$$G_2 = \frac{G_F}{\sqrt{2}}\frac{e}{4\pi^2}U^*_{tb}U_{tb}\{F_2(x_t) - F_2(x_c)\}.$$  

Here $x_t = (m_t/m_W)^2$, $x_c = (m_c/m_W)^2$ and

$$F(x) = \frac{3}{2}\frac{x}{x-1}\left[1 + \frac{21}{8}\frac{1}{x-1} + \frac{3}{4}\frac{1}{(x-1)^2} - \frac{9}{4}\left(\frac{x-2}{3}\right)\frac{x\ln x}{(x-1)^2}\right].$$

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This loop diagram, however, has a very large QCD correction as observed in Ref \([7,8]\). The QCD corrections for light quark were first calculated by Ref \([9]\). For heavy top these were done in Ref \([10]\). Since these calculations involved two loop process with presence of $\gamma_5$, there could arise ambiguities depending on regularization procedure. Situation has however been clarified \([11]\) and the procedure used by Ref \([10]\) has been found to be correct. The correction results in replacing the curly bracket in Eqn.(2) by $\bar{F}_2(x_t)$ where

$$
\bar{F}_2(x_t) = \rho^{23/16}
\left\{ (3X/10) \left( \rho^{10/23} - 1 \right) + (3X/28) \left( \rho^{28/23} - 1 \right) + F_2(x_t) \right\}.
$$

Here $\rho = \alpha_s(m_b)/\alpha_s(m_W)$, and $X = 232/81$. The rate for $b \to s\gamma$ can then be found to be

$$
\Gamma(b \to s\gamma) = \frac{G_f^2 m_b^5}{16\pi} \left( 1 - \frac{m_s^2}{m_b^2} \right)^3 \left( 1 + \frac{m_s^2}{m_b^2} \right).
$$

This is plotted as a branching ratio versus $m_t$ in Fig. 2, where we use for the width of $B$, the phase space corrected lifetime

$$
\Gamma_B = 3 |U_{cb}|^2 \left[ \frac{G_f^2 m_b^5}{192\pi^3} \right].
$$

To see the effect of QCD correction we have also plotted the uncorrected rate for $b \to s\gamma$ in Figure 2.

The above rate could also be compared with charmless inclusive $B$ decay into a hard gamma. However, this is quite difficult to measure. Instead the two body mode $B \to K^*\gamma$ provides a clean signature. Using an estimate for hadronic matrix element \([12]\), we find:

$$
\frac{\Gamma(B \to K^*\gamma)}{\Gamma(b \to s\gamma)} \approx 0.06.
$$

Other estimates \([13]\) range from 4% to 40%. Effect of fourth generation \([14]\), super symmetry \([15]\), and extra Higgs boson \([16]\) on the rates have been discussed. The best limit \([17]\) on the neutral $B$ mode is $2.8 \times 10^{-4}$, and on the charged mode is $5.2 \times 10^{-4}$. Recently, Hewett \([18]\) and Barger \([19]\) have independently shown that the inclusive limit $B(b \to s\gamma) \leq 8.4 \times 10^{-4}$ obtained by CLEO puts strong
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Figure 2. $\text{Br}$ for $b \to s\gamma$ and $B \to K^*\gamma$ as functions of $m_t$

constraints on the charged Higgs mass of the SUSY two doublet model. As the top mass becomes larger, charged Higgs mass becomes larger than the top mass. This has important implications for decay of the top. In general, the Higgs mass limit depends on the top mass as well as the ratio of vev's, $\tan\beta$. In Figure 3 we show this dependence following [18].

1.2. The Processes $b \to s\ell^+\ell^-$

The processes $b \to s\ell^+\ell^-$ occurs through one loop diagrams in the Standard Model, and involves photon, $Z$ and $W^+W^-$ exchange. The complete calculation can be extracted from Inami and Lira [5]. The dependence of the inclusive rate as a function of top mass and as a function of fourth generation quark masses and mixing was given in Ref [20]. The QCD correction discussed earlier also enhances the $F_2$ term significantly. The effect of this correction on the inclusive rate is quite substantial. In fact one can ignore QCD corrections to other terms in Eq. (11) because their contribution to the rate is small, although the corrections have been evaluated [21,22]. We show, following Ref [23], in Fig. 4 the $m_t$ dependence of the rate $b \to s\ell^+\ell^-$ including all the QCD corrections. The rates for exclusive modes [24] and resonance backgrounds for the above process through $\psi$ production [25] have also been estimated.

2. Gluon Mediated Penguin Processes

The process at the quark level involving gluons, $b \to sg$ and $b \to sq\bar{q}$ have been considered in Ref [26,27]. However, the experimental signature for such a transition is a charmless exclusive mode like $B \to K\pi$, etc. The estimates for these processes involve matrix elements of four quark operators, and there are difficult to estimate. An added complication here is that charmless hadronic decays can also arise through the tree Hamiltonian with $b \to u$ transition. A careful study of the modes reveals
Figure 3. The excluded region in the $m_{H^+} - \tan \beta$ plane for $m_t$ for values 210, 180, 150, 120, and 90 Gev. respectively, starting from the top curve.

Figure 4. BR of $b \to se^+e^-$ in units of $10^{-6}$ as functions of $m_t$. 

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some where penguin clearly dominates, while others where tree contribution can be significant.

The calculation proceeds in two steps [28]. First we obtain the effective short distance interaction including one-loop gluon mediated diagram. We then use the factorization approximation to derive the hadronic matrix elements by saturating with vacuum state in all possible ways. The resulting matrix elements involve quark bilinears between one meson and vacuum and between two meson states. These are estimated using relativistic quark model wave functions. Such a technique has been used extensively by Ref. [2] for B and D nonleptonic decays and results are in fair agreement with experiment.

We shall first discuss the gluon mediated penguin contribution. Dictated by gauge invariance, the effective flavor changing neutral current $J_\mu$ contains two terms. First, proportional to $G_1$, we call the charge radius; while the second one, proportional to $G_2$, is called dipole moment operator.

\[ J_\mu = \left( \frac{g_s}{4\pi^2} \right) \left( \frac{G_F}{\sqrt{2}} \right) \delta^\mu_\nu \lambda_{ij} \left[ G_1 \left( q^2 \gamma_\mu - q_\mu q \cdot \gamma \right) L + iG_2 \omega_{\mu\nu}(m_s L + m_b R) \right] b. \]  

(8)

Here $G_{1,2}$ are functions of $K - M$ angles and quark masses:

\[ G_1 = \sum V_{kb} G_1 \left( x_k \right), V_k = U_{kb} U_{ks}^*, k = u, c, t, \]  

(9)

\[ G_1 = \frac{\left( x_k \right)}{12} \left( \frac{1}{y_k} + \frac{13}{y_k^2} - \frac{6}{y_k^3} \right) + \left[ \frac{2}{3y_k} - \left( \frac{2}{y_k^2} + \frac{4}{y_k^3} - \frac{3}{y_k^4} \right) \right] \ln x_k, \]  

(10)

where $x_k = m_s^2/m_u^2$ and $y_k = 1 - x_k$.

Note that when the gluon is on-shell, (i.e. $q^2 = 0$) the $G_1$ contribution vanishes. In the $q^2 \neq 0$ cases both terms participate. For a gluon exchange diagram (i.e. for the process $b \rightarrow s g q$) we find $G_1$ contribution dominates over $G_2$, and we can neglect $G_2$. We also evaluate $G_1$ at $q^2 = 0$, which is a good approximation. At larger $q^2$, $G_1$ develops a small imaginary part, which is important for discussion of CP violation. We shall discuss CP violation later.

From one-gluon exchange diagram we find the following effective Hamiltonian (local 4-quark operator):

\[ H_{\text{eff}}^P = k^+ k^- \left[ \bar{s}_L^i \gamma_\mu b_{L}^i \bar{d}_L^j \gamma_\mu q_{L}^j - 3\bar{s}_L^i \gamma_\mu b_{L}^j \bar{d}_L^j \gamma_\mu q_{L}^i \right. \]  

\[ \left. + \bar{s}_L^i \gamma_\mu b_{R}^i \bar{d}_R^j \gamma_\mu q_{R}^j - 3\bar{s}_L^i \gamma_\mu b_{R}^j \bar{d}_R^j \gamma_\mu q_{R}^i \right], \]  

(11)

where

\[ k^+ k^- = (\alpha_s/6\pi) \left( G_F/\sqrt{2} \right) G_1, \alpha_s = g^2/4\pi. \]  

(12)

Here $q$ runs over all quark species, although only $u, d$ and $s$ are relevant for our discussion. Charmless decays can also arise from the standard tree level interactions with $b \rightarrow u$ transition. This is given by

\[ H_{\text{eff}}^{\eta} = \eta \bar{s}_L^i \gamma_\mu u_{L}^i \bar{d}_L^j \gamma_\mu b_{L}^j, \]  

\[ \eta = 4U_{ub} U_{us}^* G_F/\sqrt{2}. \]  

(13)
The effects of the tree level interaction could be large in general. We find however that there are certain modes of $B$ decays where penguin clearly dominate. In others tree is comparable or quite large. We evaluate the rates arising from these contributions independently and present our results in Table 1. We shall define $U_{ub}/U_{bc} = 0.1\xi$; where $-1 < \xi < 1$. The modes where we find that penguin contribution clearly dominates are:

\begin{align*}
B^+ &\to K^0\pi^+ , K^*+\rho^0, \quad B^0 \to \phi(K^0, K^{*+}), \\
K^0&\phi(K^0, \pi^+, \rho^0), \quad K^0\phi(K^0, \pi^+, \rho^0).
\end{align*}

(14)

If $U_{ub}$ turns out to be very small and penguin dominates all the two body decays, then we would have $\Delta I = 0$ rule. We find, assuming $\Delta I = 0$, the following isospin relations between the decay modes:

\begin{align*}
\Gamma(B^+ &\to K^0\pi^+) = \Gamma(B^0 \to K^+\pi^-) \\
&= 2\Gamma(B^+ \to K^+\pi^0) = 2\Gamma(B^0 \to K^0\pi^0), \\
\Gamma(B^+ &\to K^+\phi) = \Gamma(B^0 \to K^0\phi), \\
\Gamma(B^+ &\to K^0\rho^+) = \Gamma(B^0 \to K^+\rho^-) \\
&= 2\Gamma(B^+ \to K^+\rho^0) = 2\Gamma(B^0 \to K^0\rho^0), \\
\Gamma(B^+ &\to K^{*0}\pi^+) = \Gamma(B^0 \to K^{*+}\pi^-) \\
&= 2\Gamma(B^+ \to K^{*+}\pi^0) = 2\Gamma(B^0 \to K^{*0}\pi^0), \\
\Gamma(B^+ &\to K^{*+}\phi) = \Gamma(B^0 \to K^{*0}\phi), \\
\Gamma(B^+ &\to K^{*0}\rho^+) = \Gamma(B^0 \to K^{*+}\rho^-) \\
&= 2\Gamma(B^+ \to K^{*+}\rho^0) = 2\Gamma(B^0 \to K^{*0}\rho^0).
\end{align*}

(15)

Any departure from these relations will clearly show the importance of doubly suppressed Cabbibo transitions.

We have considered a large number of two body charmless decay modes of the B meson. Although in principle they can receive contributions from the penguin or the tree diagram, we found a number of modes where one of the contribution clearly dominates. Many of the branching ratios are in the interesting range of a few times $10^{-5}$, which should be accessible very soon, as seen from the present experimental limits. We further find a few modes that are highly suppressed, receiving negligible contribution from both the Hamiltonians. Lack of observation of these would test our method of computation. When the contributions are comparable, one of course should add the amplitudes and square. We have not done so because we do not know the magnitude and sign of $U_{ub}$, and there might be different final state phases on these amplitudes. A clean test is provided by those decays where one of the Hamiltonian dominates. In the case where the tree Hamiltonian dominates, one could even estimate $U_{ub}$. As our knowledge of these decays improves, we believe more ambitious methods might be used to calculate the amplitudes.

3. Theory of CP Violation in $B^\pm$ Decays

The basic theory follows from the work of Bander, Silverman and Soni [29]. The
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Table 1. Branching ratios for charless two-body decay modes of the B meson. Decay rates from the penguin and tree Hamiltonians are contributed separately. Some of the zeros stand for extremely small rates.

<table>
<thead>
<tr>
<th>Branching ratio mode</th>
<th>Contribution from $H_{eff}^T$ ($10^{-5}$)</th>
<th>Contribution from $H_{eff}^P$ ($10^{-5}$)</th>
<th>Present experimental limit at 90% C.L. ($10^{-5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \rightarrow K^+\pi^0$</td>
<td>0.06$\xi^2$</td>
<td>0.53</td>
<td>9.0</td>
</tr>
<tr>
<td>$K^0\pi^+$</td>
<td>0</td>
<td>1.06</td>
<td>8.0</td>
</tr>
<tr>
<td>$K^+\phi$</td>
<td>0</td>
<td>1.12</td>
<td>7.0</td>
</tr>
<tr>
<td>$K^0\rho^+$</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$K^{*+}\pi^0$</td>
<td>0.05$\xi^2$</td>
<td>0.29</td>
<td>13.0</td>
</tr>
<tr>
<td>$K^{*0}\pi^+$</td>
<td>0</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>$K^{*+}\phi$</td>
<td>0</td>
<td>3.12</td>
<td></td>
</tr>
<tr>
<td>$K^{*+}\rho^0$</td>
<td>0</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>$K^{*0}\rho^+$</td>
<td>0</td>
<td>1.24</td>
<td></td>
</tr>
<tr>
<td>$B^0 \rightarrow K^0\pi^0$</td>
<td>0</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>$K^+\pi^-$</td>
<td>0.06$\xi^2$</td>
<td>1.06</td>
<td>49.0</td>
</tr>
<tr>
<td>$K^0\phi$</td>
<td>0</td>
<td>1.12</td>
<td>58.0</td>
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<tr>
<td>$K^0\rho^0$</td>
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<td>0</td>
<td></td>
</tr>
<tr>
<td>$K^+\rho^-$</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$K^{*0}\pi^-$</td>
<td>0</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>$K^{*+}\pi^-$</td>
<td>0</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>$K^{*0}\phi$</td>
<td>0</td>
<td>3.12</td>
<td>44.0</td>
</tr>
<tr>
<td>$K^{*0}\rho^0$</td>
<td>0</td>
<td>0.62</td>
<td>67.0</td>
</tr>
<tr>
<td>$K^{*+}\rho^-$</td>
<td>0.01$\xi^2$</td>
<td>1.24</td>
<td></td>
</tr>
</tbody>
</table>

The easiest way to generate an absorptive part, at least at the quark level is to consider one loop diagrams involving a gluon or a photon, usually referred to as penguin diagrams. The loop diagram generates an absorptive part when the virtual gluon four-momentum $q^2$ is greater than $4m_q^2$ where $m_q$ is mass of the intermediate quark. For b quark decay, the intermediate quarks are u,c,t, so that $q^2 > 4m_u^2$ or $4m_c^2$ generates the absorptive part. The two amplitudes necessary for the effect can be a tree diagram and a penguin diagram, or if the tree diagram is absent, two
penguin diagrams with different quark intermediate states suffice. The asymmetries are easiest to calculate at the quark level, but since the measurements will be made with a definite hadronic mode, the question of the reliability of such a calculation has to answered. We shall comment on this point in the conclusions.

Let us review the calculation with \( b \rightarrow uu\bar{s} \) as a definite quark channel for illustrative purposes. The following diagrams contribute to this process:

![Tree Diagram](image1)  
![Penguin Diagram](image2)

Figure 5. Diagrams for \( b \rightarrow uu\bar{s} \)

We can write the amplitude for this process as

\[
M (b \rightarrow uu\bar{s}) = V_u A_u + \left( \frac{\alpha_s}{\pi} \right) \sum_i V_i P_i,
\]

where \( i \) runs over \( u,c,t \) and \( V_i \) is defined in terms of K-M angles as

\[
V_i = U_{bi} U_{si}^*.
\]

Unitarity requires that

\[
\sum_i V_i = 0.
\]

\( A_u \) refers to the contribution from the tree diagram, while \( P_i \) refers to the contribution from the penguin diagram. We have factored \( (\alpha_s/\pi) \) so that the remaining penguin operator contributes roughly the same as the tree operator. Using unitarity we can remove \( V_i \) dependence, and we have

\[
M = V_u (A_u + (\alpha_s/\pi) P_{u-t}) + (\alpha_s/\pi) V_c P_{c-t}.
\]

We now define \( \tilde{M} \) as the amplitude for \( \bar{b} \rightarrow \bar{u}u\bar{s} \). This is given by

\[
\tilde{M} = V_u^* (A_u + (\alpha_s/\pi) P_{u-t}) + (\alpha_s/\pi) V_c^* P_{c-t}.
\]

The asymmetry is then given by

\[
A = \frac{|M|^2 - |\tilde{M}|^2}{|M|^2 + |\tilde{M}|^2}.
\]

In the denominator we may safely assume that \( (\alpha_s/\pi) V_c \gg V_u \), and obtain

\[
A = \frac{2 Im (V_u V_c^*) Im [(A_u^* + (\alpha_s/\pi) P_{u-t}^*) (\alpha_s/\pi) P_{c-t}]}{|V_c|^2 (\alpha_s/\pi)^2 |P_{c-t}|^2}.
\]
This expression is clearly zero unless the three conditions we mentioned above are satisfied. The maximum asymmetry occurs when tree amplitude is present and \( P_{c-t} \) has absorptive part, which requires \( q^2 > 4m^2 \). If \( q^2 < 4m^2 \), the formula generates absorptive part due to \((\alpha_s/\pi)^2\) contribution, however, to this order there are other contributions which must be evaluated [30]. In pure penguin process, like \( b \rightarrow s\bar{s}s \), we have the expression

\[
A = \frac{2Im[V_u V_s^*] Im[P_{c-t}^A P_{c-t}^A]}{|V_{c}^2 P_{c-t}^A|}. 
\]  

(23)

The asymmetry is smaller, but the estimates are less model dependent in this case.

The largest observable asymmetry is expected to arise in processes where tree diagram and penguin contribute approximately the same amount. At the quark level the process is \( b \rightarrow s\bar{u}u \), and the asymmetry is given by (22), which can be written as

\[
A = \eta (A_u/|P_{c-t}|) (P_{c-t}^A/|P_{c-t}|). 
\]  

(24)

We need the ratio of tree and penguin contributions, as well as the absorptive to dispersive ratio. The former can be calculated in from the lagrangian if factorization approximation is used. A large number of processes have been evaluated in [28] using this approximation. We present below the best modes with the corresponding ratio \( R = A_u/|P_{c-t}| \). To calculate the absorptive part we need to know the value of \( q^2 \) to be used. If the argument used in the previous section can be applied, \( 12(GeV)^2 \) might be suitable. The value of \( P_{c-t}^A/|P_{c-t}| \approx 0.25 \) in this case.

| Mode        | \( R = A_u/|P_{c-t}| \) |
|-------------|-------------------------|
| \( B^- \rightarrow K^- \pi^0 \) | 1.1                     |
| \( B^- \rightarrow K^* \pi^0 \) | 1.4                     |
| \( B^0 \rightarrow K^+ \pi^- \) | 0.8                     |
| \( B^0 \rightarrow K^* \pi^- \) | 1.4                     |

The asymmetry then is given by

\[
A \cong \eta R/4. 
\]  

(25)

This could be as high as 3.5% for \( \eta = 0.1 \), where \( \eta \) is the CP violating parameter in the mixing matrix.

The above estimate are based on the applicability of the one loop quark diagram to a hadronic decay, a procedure that has been criticized recently by Wolfenstein [31]. In my opinion the use of one loop diagram is clearly possible only if the quark diagram is a short distance operator and long distance effects are negligible. This might be true for a heavy quark decay where large momenta are involved in the decay products. A more troubling question of the appropriate \( k^2 \) to be used will have to deferred until a better understanding emerges regarding hadronic decays. Importance of detecting direct CP violation outweighs the fact that the theoretical calculations are not at present very reliable. We expect the situation to improve as the penguin processes are measured.

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References

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[31] L. Wolfenstein, preprint, NSF-ITP-90-29

Discussion

P.N. Pandita: Does the lower bound on \(m_{H^+}\) for any value of \(\tan \beta\) in minimal SUSY model translate into a bound on other Higgs masses in the model.

N.G. Deshpande: Yes. The limit translates into a limit on \(m_A\), but it is a function of \(\tan \beta\). The curves are given in Fig.3 in Hewett's paper. For \(m_t = 150\) GeV, and \(\tan \beta\) between 1 and 10, \(m_A > 80\) GeV.

K.V.L. Sarma: This is a general question. Which is the best place to look for the evidence for Penguin mechanism (given the present rates)?

N.G. Deshpande: The best processes are \(B \rightarrow K^* \gamma\) and \(B \rightarrow K\phi\).

Sreerup Raychaudhuri: You showed us that the charged Higgs in the MSSM can be heavier than the top quark from \(b \rightarrow s \gamma\) data. This was for \(m_t = 150\) GeV. What is the situation for \(m_t = 100\) GeV or 200 GeV?

N.G. Deshpande: The answer can be found in the figure of Hewett's paper. For \(m_t \approx 200\) GeV the limit is \(m_{H^+} > 200\) GeV. For \(m_t \approx 100\) GeV the only limit comes from LEP, \(m_{H^+} > 45\) GeV.