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Decay-time asymmetries at the B-factories

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1. Introduction

The primary motivation for proposing to build a B-Factory is to explore the signals of CP violation in B meson decays. The idea is to establish a rate asymmetry which is implied by unequal rates for B to decay into a state f and $\bar{B}$ to decay into the CP-conjugate state $\bar{f}$,

$$\Gamma(B \to f) \neq \Gamma(\bar{B} \to \bar{f}). \tag{1}$$

One might look for such a violation in the neutral mesons B_d and B_s, and in the charged ones B_u. Here I shall focus mainly on the neutral system $B_d - \bar{B}_d$, and sometimes refer to it as $B - \bar{B}$. But before I discuss some of the asymmetries and the importance of their measurement, let me say a few words about the "Asymmetric B-Factories", mainly for pedagogical reasons.

Why Factory?: Almost all the hadronic decay modes of the B meson could be called "rare" modes because observations show that each of them has a decay rate which is quite small compared to the total rate. For instance, according to the 1992 data of the CLEO group [1] we have the branching fractions (BF)

$$BF(B_d \to J/\psi + K_S) = (1.02 \pm 0.43 \pm 0.17) \times 10^{-3}, \tag{2}$$

$$BF(B_d \to \pi^+\pi^-) \lesssim 4.8 \times 10^{-5}, (90\% CL); \tag{3}$$

the first decay mode is CKM-allowed and the second one is CKM-suppressed. This kind of numbers is perhaps not unexpected: in the decay of a meson of 5 GeV mass, kinematics would admit many distinct hadronic channels and, treating all of them on an equal footing, each gets only a tiny fraction of the total width. To increase the statistics therefore one would have to start with a large sample of B's and hence the need for a "B-Factory".

The Factory is nothing but a high luminosity e^+e^- collider which is tuned to produce the well-known onium-state $\Upsilon(4S)$ at mass 10.580 GeV. This $b\bar{b}$ resonance is produced with a cross section of 1.15 nb, and lies only about 23 MeV above the $B\bar{B}$ threshold. It decays into the two hadronic modes, B^+B^- and $B_d^0\bar{B}_d^0$, each with a BF very close to 50%;

$$e^+e^- \to \Upsilon(4S) \to B\bar{B}. \tag{4}$$

To get an idea of the peak luminosities which are being talked about we quote the design value for the Cornell Factory CESR-B [2], $L(peak) = 3 \times 10^{33} cm^{-2}s^{-1}$;

for comparison, recall the CESR 1992 peak value $2.5 \times 10^{32} cm^{-2}s^{-1}$, which is the present world record for the storage rings. This means an integrated luminosity of about 100 fb^{-1} in about 3 years of running, at an average L equal to a third of the peak value (the integrated value of 1991 was 1.2 fb^{-1}). Note that values like 100 $(fb)^{-1}$ are needed to study CP violation as they yield typically about 10^8 events of interest.

The second feature in B-Physics experiments pertains to the measurements of small decay times which lie in the picosecond range. The B meson lifetimes determined from the 1992 LEP (averages of the ALEPH and DELPHI) data, are [3]

$$\begin{aligned} \tau(B_d) &= 1.44 \pm 0.18\, ps, \\ \tau(B_u) &= 1.33 \pm 0.19\, ps, \\ \tau(B_s) &= 1.05 \pm 0.31\, ps. \end{aligned} \tag{5}$$

As the B_d has a speed $\beta^* = 0.065$ in the $\Upsilon(4S)$ rest frame, it travels a very short distance before decaying; the average decay length with the lifetime quoted above (ignoring the error) is

$$d^* = \gamma^* \beta^* c\tau = 28\, \mu m. \tag{6}$$

When the colliding e^+ and e^- have unequal energies it corresponds to an 'Asymmetric' B-Factory in which the Υ moves. The B's from such a moving source can then travel larger distances before they decay. Note that it is the increased speed of the B that helps here, and not so much the Lorentz time-dilatation.

To be specific, the Cornell Factory CESR-B [2], will be a two-ring collider with 8 GeV e^- hitting 3.5 GeV e^+. The Upsilon will be produced with a velocity $\beta = 0.39$ along the direction of the e^-. The resulting B's would have boost factors in the range

$$\beta\gamma = 0.36 - 0.50, \tag{7}$$

as the emission angles vary, and hence they would have larger decay distances

$$d = 153 - 215\, \mu m. \tag{8}$$

Measurements of such lengths are within easy reach of the present day vertex detectors using silicon micro-strips.

The intervals of time, or corresponding lengths, which one might be interested in resolving at the Factory could be smaller still. The actual structure of the colliding bunch typically will have a small transverse cross section of ($50\, \mu m \times 50\, \mu m$) but a large longitudinal dimension of about $1.5\, cm$. This kind of a needle-shaped bunch makes it hard to locate the point of creation of the Υ resonance with the needed accuracy at the asymmetric Factory; nonetheless one could ascertain easily the 'gap' between the two decay-instants t and t' of the boosted $B\bar{B}$ pair. Hence one expects to have a good knowledge of the relative time

$$\tau = t - t'. \tag{9}$$

As this time-difference could take values much smaller than a picosecond, one would have to determine the corresponding gaps between decay vertices which are smaller than 150 μm.

For measuring certain asymmetries we do not need the value of τ; it is enough if we can ensure that it is positive or negative. Note that at an asymmetric collider, event reconstruction will be easier than at the symmetric one because the separation between decay vertices of the two B's is increased by the boost. On the other hand too much boost can be couter-productive because it focuses or squeezes all the produced particles into a narrow cone and vertexing becomes a technical challenge.

Bottom Tagging: How do we ascertain whether it was a B that decayed and not a $\bar{B}$? Recall that in semileptonic decays the W couplings that govern the decay are: $b \to W^- c(u)$ and $\bar{b} \to W^+ \bar{c}(\bar{u})$. Since the charge of the emitted lepton has to be the same as that of the emitted W, the B flavour at the instant of decay can be tagged by the charge of the primary lepton (say, a muon with at least 1.5 GeV in the $\Upsilon(4S)$ rest frame):

$$\begin{aligned} B(\bar{b}d) &\to \ell^+ + \ldots, \\ \bar{B}(b\bar{d}) &\to \ell^- + \ldots . \end{aligned} \tag{10}$$

The backgrounds for such events arise mainly from the charm and tau decays, e.g., through the chains $B \to D \to \ell$ and $B \to \tau \to \ell$, and they are estimated by Monte Carlo simulations.

As for hadron tags the inclusive mode with a charged kaon (e.g., due to the cascade decays $B \to \bar{D} + \ldots \to K^+ + \ldots$) may be promising because of its large $BF = (85 \pm 11)\%$. Monte Carlo studies indicate that background K's from lighter hadrons might be manageable; e.g. below 3 GeV/c, if a K is detected with 100% efficiency the B flavour could be determined with about 40% efficiency which is 3 - 4 times larger than the lepton-tagging efficiency.

An important feature of the Upsilon factory is the possibility of measuring an observable which is odd with respect to time; such measurements are obviously not possible with the "conventional" beams of B's. In the $\Upsilon(4S)$ rest frame let us suppose we have identified the left-moving bottom meson to be a $\bar{B}$ at time t_L because the lepton tag was ℓ^-. Then its companion moving towards the right must be a B at time t_L. However the time t_R, the instant of decay of the right mover, could be either later or earlier than t_L. In this case therefore we say that the right-moving B can 'evolve' forward or backward in time before its decay, because the zero of time ($t = 0$) at the Factory is decided by the tag-time.

Summarizing, the Asymmetric B-Factory is designed to produce copious numbers of correlated $B\bar{B}$ pairs, each pair travelling at considerable (constant) speed. Information would be available on the relative time difference between the two decays of a given $B\bar{B}$ pair, wherein one of the decays serves as the bottom-tag.

My plan is to talk briefly about the following specific topics:

(a) CP violating asymmetries to test the SM; Winstein ambiguity,

(b) Data on dilepton charge asymmetry and CP violation,

(c) Asymmetry for determining the parameter y ,

(d) Validity of the $\Delta B = \Delta Q$ rule.

First, we mention of the notation and present some general conclusions based on the box-diagram of the Standard Model:

Notation: Let the propagating states $B_k (k = 1,2)$ which have complex masses $(m_k - \frac{i}{2}\Gamma_k)$ be defined as

$$\begin{aligned} B_1 &= pB + q\bar{B}, \\ B_2 &= pB - q\bar{B}. \end{aligned} \tag{11}$$

The phase convention is $(CP)\ B = +\bar{B}$. We define the following parameters

$$\begin{aligned} g &= q/p, \\ x &= (m_2 - m_1)/\Gamma, \\ y &= (\Gamma_2 - \Gamma_1)/(\Gamma_2 + \Gamma_1), \\ \Gamma &= (\Gamma_2 + \Gamma_1)/2. \end{aligned} \tag{12}$$

Let us consider the B decays into CP eigenstates f where

$$\bar{f} \equiv (CP)f = \pm f, \tag{13}$$

and denote the decay amplitudes by

$$A_f = \text{Ampl}(B \to f), \tag{14}$$
$$\bar{A}_f = \text{Ampl}(\bar{B} \to f). \tag{15}$$

We define the (phase-convention independent) complex parameter

$$u_f = g\,\frac{\bar{A}_f}{A_f}, \tag{16}$$

which, if CP invariance holds, would take the value +1 (−1) for CP-even (CP-odd) eigenstates f.

Lore of the "BOX": A study of the standard box-diagram with the exchange of W-pair, and assuming the number of quark generations to be 3, leads us to expect the ratio

$$\frac{|\Gamma_{12}|}{|M_{12}|} \simeq \frac{O(m_c^2)}{O(m_t^2)} \lesssim 10^{-3} \tag{17}$$

to be quite small (see e.g., Ref 4). Here, as per the usual notation, the off-diagonal mass-matrix element $[M_{12} - i\Gamma_{12}/2]$ is essentially p^2; we recall that Γ_{12} gets contributions only from the physically allowed b quark decays. Thus we expect the mixing ratio g mainly to be a phase factor,

$$g = \frac{q}{p} = \left[\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right]^{1/2} \simeq \left[\frac{M_{12}^*}{M_{12}}\right]^{1/2}, \tag{18}$$

and we can set

$$|g| \simeq 1, \tag{19}$$

and, because y is determined by $Re(\Gamma_{12})$,

$$y \simeq 0. \tag{20}$$

Although not strictly necessary, for the sake of displaying formulas which are simple, we shall also assume that

$$|u_f| \simeq 1. \tag{21}$$

2. CP Violating Asymmetries

Let us start with pure states B and $\bar{B}$ at proper time $t = 0$. The time-evolved states $B(t)$ and $\bar{B}(t)$ would decay into a common channel f with the rates given by (see Refs. [4-8] and references therein)

$$\frac{d\Gamma}{dt}\left[\begin{array}{c} B(t) \to f \\ \bar{B}(t) \to f \end{array}\right] \sim |A_f|^2 e^{-\Gamma t}\left[1 \mp Im(u_f)\,\sin(x\Gamma t)\right]. \tag{22}$$

One constructs the time dependent asymmetry

$$A(t) \equiv \frac{d\Gamma(B \to f) - d\Gamma(\bar{B} \to f)}{d\Gamma(B \to f) + d\Gamma(\bar{B} \to f)} \tag{23}$$

$$= -Im(u_f)\,\sin(x\Gamma t). \tag{24}$$

Note that if we had not assumed eq.(21) we would have obtained an extra term $(1 - |u_f|^2)\cos(x\Gamma t)$, which could be isolated by the time-dependence characteristic of the cosine factor.

It is however useful to consider the "time-integrated asymmetry" which is formed by the rates that are integrated over all positive time, from 0 to ∞,

$$a_f = -\left(\frac{x}{1+x^2}\right)\; Im\,(u_f). \tag{25}$$

This is a neat formula because it depends only on the experimentally determinable quantities: the mixing parameter x and the CKM-matrix phase that determines $Im\,(u_f)$.

The present value of x (averaging over the data of ARGUS and CLEO groups) [3] is

$$x = 0.67 \pm 0.10. \tag{26}$$

A comment in regard to the sign of x is relevant here. In Eq.(12) we defined $x = (m_2 - m_1)/\Gamma$, but the mass-eigenstates B_1 and B_2 were not identified. If the parameter y were not zero one could, e.g., identify B_2 as the long-lived state. But in the limit of $y = 0$, we have to obviously look for a criterion other than the lifetime difference: One may, for instance, define the state B_2 to be that which decays dominantly into CP-odd states, assuming the CP violations to be "small". To be specific, we shall define the state B_2 to be that which decays to the CP-odd state $J/\psi K_S$. Methods suggested [9] to fix the sign of x need the timing of kaon decay in the decays $B_d \to J/\psi K_S$ at a B-Factory. Here the idea is to exploit the strangeness oscillations to serve as 'analysers' of bottom oscillations to decide the sign of x.

As for the factor $Im\,(u_f)$ in Eq.(25) we turn to the Standard Model with 3 generations of quarks. The unitarity of the 3×3 CKM-matrix implies the condition (column 1 with column 3*)

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$

This triangle condition can be recast in simpler notation as follows

$$W_u + W_c + W_t = 0, \quad (27)$$
$$W_q \equiv V_{qd}V_{qb}^*, \quad (q = u, c, t).$$

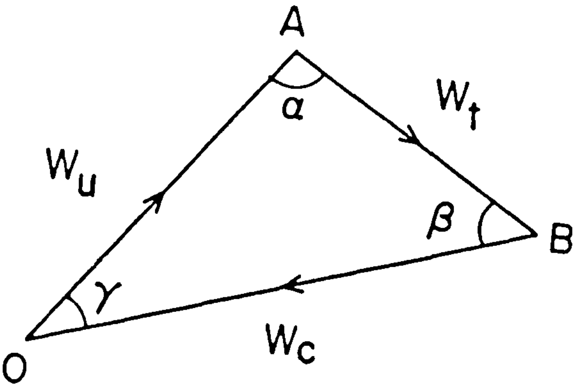

Figure 1. CKM Unitarity Triangle

The three angles of the triangle OAB are given by

$$\alpha = \arg\,[-W_t/W_u]\,,\ \ \beta = \arg\,[-W_c/W_t],\ \ \gamma = \arg\,[-W_u/W_c]. \quad (28)$$

The area of the triangle is a measure of the CP violation in the Standard Model framework with 6 quarks.

Needless to say, it is important to establish this triangle by all possible experiments. One has to make sure that it is not the degenerate case of a straight line, and that the lengths of the sides are such that the triangle does close. Customarily one chooses to orient the unitarity triangle so that W_c lies along the horizontal axis in the Argand plane. This amounts to choosing a phase convention. In the convention adopted by the Wolfenstein parameterization of the CKM matrix, $W_c \simeq -A\lambda^3$ is real and the side OB of the triangle is along the ρ axis.

To keep track of the several phases that arise in writing u_f, we adopt the approach of Nir and Silverman [10] and write

$$u = \xi\,\frac{X}{X^*}\cdot\frac{Y}{Y^*}\cdot\frac{Z}{Z^*}\,, \quad (29)$$

where the first factor ξ is the CP-eigenvalue (± 1) of the state f. Each of the factors X, Y, Z turns out to be a product of two CKM-matrix elements: X comes from the two V's at the two ends of the W in the quark-level diagram of the decay $B \to f$; for instance,

$$X[\bar{b} \to \bar{c}c\bar{s}] \;=\; V_{cb}V_{cs}^*, \quad (30)$$
$$X[\bar{b} \to \bar{u}u\bar{d}] \;=\; V_{ub}V_{ud}^*$$
$$\;=\; W_u^*. \quad (31)$$

The factor Y arises from the dominant box-diagram (with virtual $t\bar{t}$ pair) describing $B\bar{B}$ mixing; it does not depend on state f; it denotes the possibility that by virtue of mixing, the decay of $B(d\bar{b})$ arises from an initial $\bar{B}(b\bar{d})$ state:

$$\begin{aligned} Y[\bar{B}(b\bar{d}) \to (t\bar{t}\,\text{box}) \to B(d\bar{b})] &= V_{tb}^* V_{td} \\ &= W_t. \end{aligned} \tag{32}$$

The factor Z is similar to Y; it arises from $K\bar{K}$ mixing when the final state f contains a K_S or a K_L meson. Obviously we ought to set $Z = 1$ if there is no neutral kaon in the state f. For evaluating Z we read off the $K\bar{K}$-mixing phase from the box-diagram involving only c-quarks (note that the 'box-mechanism' is relied upon only to the extent of getting the phase). If we start with a neutral K with positive strangeness in the primary decay of B, we then have

$$Z[K(\bar{s}d) \to (\bar{c}c\,\text{box}) \to \bar{K}(\bar{d}s)] = V_{cs} V_{cd}^* . \tag{33}$$

The neutral B decay mode that enjoys maximum attention and is regarded as the "show-case sample" for the decays in B physics, is the two-body mode

$$B \to J/\psi + K_S . \tag{34}$$

It has the distinctive lepton-pair of the J/ψ for its triggering. The final hadronic state has $CP = -1$ because the K has to be emitted in a relative p-wave. The decay is governed by the CKM-favoured transition $\bar{b} \to \bar{c}c\bar{s}$. Substituting for X, Y, and Z in Eq.(29) the argument of u is given by

$$\begin{aligned} \arg(u) &= -2\arg(XYZ) \\ &= -2\arg[W_c^* W_t] \\ &= 2\beta, \end{aligned}$$

and hence the imaginary part of u is

$$Im\,[u(B_d \to J/\psi K_S)] = +\sin(2\beta). \tag{35}$$

A measurement of the time-integrated asymmetry in this channel can therefore determine the angle β by means of eq.(25). The asymmetry in respect of the decay $B_d \to D^+D^-$ also gives information on the same angle.

Next, consider the CKM-suppressed transition $\bar{b} \to \bar{u}u\bar{d}$ which underlies the B decay into the CP-even state of two pions,

$$B_d \to \pi^+\pi^- . \tag{36}$$

As there is no kaon in the final state we immediately set $Z = 1$, and using Eqs.(31) and (32) obtain

$$\begin{aligned} \arg(u) &= +2\arg[W_u^* W_t] \\ &= -2\arg[W_u/W_t] \\ &= -2(\pi - \alpha); \end{aligned}$$

we thus have

$$Im\,[u(B_d \to \pi^+\pi^-)] = +\sin(2\alpha). \tag{37}$$

It should however be pointed out that this determination of α is obscured by what is usually called the "rescattering pollution". This complication arises because the 2π final state can also be reached from a differnt decay mode by final state rescattering; for instance,

(1) $B(\bar{b}d) \rightarrow \bar{u}(u\bar{d})\, d \rightarrow \pi^+\pi^-$,
(2) $B(\bar{b}d) \rightarrow \bar{c}(c\bar{d})\, d \rightarrow D^+D^- \rightarrow \pi^+\pi^-$.

Here the last step $D^+D^- \rightarrow \pi^+\pi^-$ proceeds by strong interactions (say, by the exchange of a D^* meson). The decay amplitudes can be written as

$$\begin{aligned} A_{2\pi} &= W_u|A_1|e^{i\delta_1} + W_c|A_2|e^{i\delta_2}, \\ \bar{A}_{2\pi} &= W_u^*|A_1|e^{i\delta_1} + W_c^*|A_2|e^{i\delta_2}, \end{aligned}$$

where the δ's are strong interaction phases. We obtain Eq.(37) if we can show that $|A_2| \ll |A_1|$. Unfortunately a reliable estimate of A_2 does not exist because one does not know how to calculate the "long-range" contributions to the reaction $B \rightarrow \bar{D}D$. Perturbative QCD calculations of Penguin diagrams are not meaningful here as the Q-value involved in the decay is not large, $(m_B - 2m_D) \simeq 1.5$ GeV.

The haze due to rescattering pollution is however minimal or absent in determining the angle β from the decay (34). This is because the important "second" mechanism involving rescattering

$$B \rightarrow D_s^+D^- \rightarrow J/\psi K_S,$$

also has the <u>same</u> weak phase as the first mechanism.

The "third angle" γ will perhaps be the most difficult to determine as one needs to measure the asymmetry in the B_s decays. The decay that is usually suggested is $B_s \rightarrow \rho + K_S$ for which

$$Im\,[u(B_s \rightarrow \rho^0 K_S) = +\sin(2\gamma). \qquad (38)$$

Asymmetries in B_s decays in general are expected to be quite small as the arguments based on the box-diagram imply a very large $B_s - \bar{B}_s$ mixing (say, $|x_s| \gtrsim 10$); i.e., the $B_s - \bar{B}_s$ oscillations would be so rapid that whatever may be the initial state it quickly ends up as a near equal mixture of B_s and $\bar{B}_s$, and thus any asymmetry in the decays gets washed out [4]. Further this means one needs a B_s-Factory which is a e^+e^- machine tuned for the $\Upsilon(5S)$ at 10.865 GeV; this resonance has a small production cross section ($\simeq 0.16$ nb) and a small branching fraction ($\lesssim 0.1$) for decay into $B_s\bar{B}_s$ state.

The present phenomenological analyses (based on the available data on $B_d\bar{B}_d$ mixing, $|V_{ub}/V_{cb}|$, etc.) cannot ascertain even whether the angle AOB is acute or obtuse, that is, whether the apex A lies in the 1st or the 2nd quadrant in the $\rho\eta$-plane. For a discussion of the B decay asymmetries outside the Standard Model framework, see Ref. 11.

Winstein Ambiguity: This interesting ambiguity [12] stems from the possibility that the two measured asymmetries $a(J/\psi K_S)$ and $a(\pi^+\pi^-)$ are equal and opposite such that the following relation is satisfied

$$\sin(2\beta) = -\sin(2\alpha). \qquad (39)$$

This relation in fact is the prediction of the Super Weak (SW) model, which was originally proposed by Wolfenstein in 1964 to account for the data on CP violation in neutral kaons. It should be noted that the model is yet to be falsified by experiments: Based on the analyses of their full data the experimental groups have annonced at the 1991 Lepton-Photon Symposium, Geneva,

$$\begin{aligned} Re\left(\frac{\epsilon'}{\epsilon}\right) &= (2.3 \pm 0.7) \times 10^{-3}, \ (NA31) \\ &= (0.60 \pm 0.69) \times 10^{-3}, \ (E731). \end{aligned}$$

This ratio ought to vanish in SW. According to the SW scenario all the CP violation effects arise only from the transitions involving Δ (flavour) $= 2$; in regard to the off-diagonal element in the $K\bar{K}$ mass-matrix the SW transitions make the parameter M_{12} complex but keep the Γ_{12} real. A consequence of the SW model is that the asymmetries for CP-odd and CP-even decay states should be equal and opposite as implied by Eq.(39).

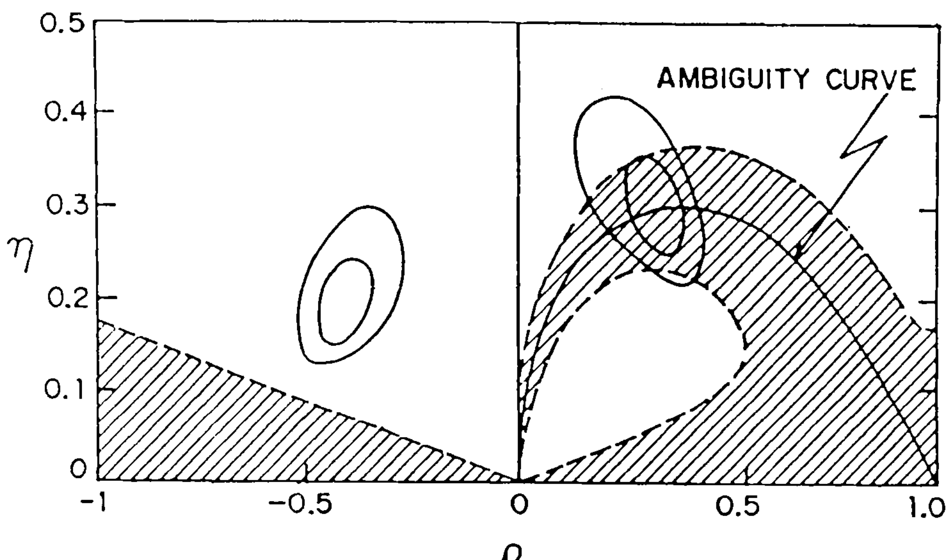

Figure 2. Winstein's Ambiguity

Since the measurements of asymmetries will be having some errors, one could in practice test only the consistency of the relation (39). Hence it is appropriate to display the implications between the CKM parameters, not as a curve but, as a region in the parameter space. Winstein's analysis shows (see Fig. 2) that for realistic errors on the measurements there exists a sizable region in the $\rho\eta$-plane wherein we will not be able to distinguish between the SM and SW; for further details see Ref. 13. Note however that an observation of CP violation in the $B^{\pm}$ sector would at once rule out the SW option, as there is no mixing in the case of charged mesons.

3. CP Asymmetry in Like-Sign Dilepton Events

I am talking about this because as of now this is the only asymmetry relating to CP violation in the B system on which an experimental number is being mentioned. The extraction of this number of course does not need any timing information, or reconstruction of a particular exclusive mode. It is a measurement, at a $\Upsilon(4S)$ machine, of the difference in the total numbers of like-sign dilepton events of both kinds, $N(++)$ and $N(--)$.

Following the $\Delta B = \Delta Q$ rule of the SM, we define the two allowed amplitudes for decay into an exclusive semileptonic mode $(\ell^+ X)$ and its CPT-conjugate $(\ell^- \bar{X})$ as

$$\begin{aligned} A(\ell^+) &= \langle \ell^+ X | T | B \rangle, \\ A(\ell^-) &= \langle \ell^- \bar{X} | T | \bar{B} \rangle; \end{aligned} \tag{40}$$

these are related by CPT invariance:

$$A(\ell^-) = [A(\ell^+)]^* e^{2i\delta},$$

where δ is the eigenchannel scattering phase due to electro-weak interactions in the final state. Due to mixing, the time-evolved states $B(t)$ and $\bar{B}(t)$ do lead to 'wrong-sign' leptons. If all the physically connected final channels are summed over, one gets effectively,

$$|A(\ell^+)| = |A(\ell^-)|.$$

We now sum over the final spins and momenta, and finally sum over the channel index X to pass to the inclusive limit. In this way, without making the assumptions of eqs. (19) and (20), we obtain the same overall constant (which we suppress) in the following four time-dependent rates, written for brevity as a matrix product:

$$\frac{d\Gamma}{dt} \begin{bmatrix} B \to \ell^+ \\ \bar{B} \to \ell^- \\ B \to \ell^- \\ \bar{B} \to \ell^+ \end{bmatrix} \sim e^{-\Gamma t} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ |g|^2 & -|g|^2 \\ |g|^{-2} & -|g|^{-2} \end{bmatrix} \begin{bmatrix} \cosh(y\Gamma t) \\ \cos(x\Gamma t) \end{bmatrix}. \tag{41}$$

Consider the semileptonic B decays starting with $\Upsilon(4S) \to B\bar{B}$. Taking the electric charge of the lepton that tags the B to be the first index (say), we see that the last two equations above lead to the class of events with likesign dileptons. From these rates the dilepton charge asymmetry [14] is (independent of time)

$$a(\ell\ell) = \frac{N(++) - N(--)}{N(++) + N(--)} = \frac{1 - |g|^4}{1 + |g|^4}. \tag{42}$$

A preliminary value quoted by the CLEO group [1], on the basis of about 200 events containing likesign dileptons, is

$$a(\ell\ell) = (-0.58 \pm 9.9 \pm 3.8)\ \% \,. \tag{43}$$

This means, at 90% C.L., the dilepton charge asymmetry (originating from the CP violation in $B\bar{B}$ mixing) is less than 14%; an alternative way of stating this is

$$Re(\epsilon) < 4\%, \tag{44}$$

where ϵ is given by $g = (q/p) = (1-\epsilon)/(1+\epsilon)$. Note that in the case of K_L decays the celebrated SLAC leptonic charge-asymmetry implied $Re(\epsilon_K) \simeq 0.16\%$.

The LEP groups are expected [3] to provide soon a number for the asymmetry $a(\ell\ell)$, which is a factor 2 better than the above. However its measurement at the level of 10^{-3} (which is the estimate from the models [15]), requires about 10^{10} upsilons and several years of running of a B Factory (note that the number goes inversely as the square of the asymmetry).

4. Asymmetry to Obtain y

An asymmetry which would eventually shed light on the small parameter y in eq.(12), can be constructed [9] from the time dependence of the B_d and $\bar{B}_d$ decays into the same CP eigenstate f. If t denotes the time of decay $B/\bar{B} \to f$ and t' the lepton-tag time, then the relative time $\tau = (t-t')$ can be either positive or negative, whereas the sum $T = (t+t')$ must be greater than $|\tau|$. Hence the probability that one of the B's decays into ℓ^+ or ℓ^- and the companion B decays to the state f after a time τ can be worked out exactly:

$$\begin{aligned} W_\ell^{(f)}(\tau) &= \int_{|\tau|}^{\infty} d(t+t')\,\{D[\ell^+(t'),\, f(t)] + D[\ell^-(t'),\, f(t)]\} \\ &\sim e^{-\Gamma|\tau|}\{(1+|u_f|^2)\,\cosh(y\Gamma\tau) + 2Re(u_f)\,\sinh(y\Gamma\tau) \\ &\qquad -\Omega\,(1-|u_f|^2)\,\cos(x\Gamma\tau) + 2\Omega\, Im(u_f)\sin(x\Gamma\tau)\}, \end{aligned} \tag{45}$$

where the 'overlap' parameter Ω is defined as

$$\Omega = \frac{1-|g|^2}{1+|g|^2}. \tag{46}$$

The corresponding time-asymmetry we construct is

$$A_f(\tau) = \frac{W_\ell^{(f)}(\tau) - W_\ell^{(f)}(-\tau)}{W_\ell^{(f)}(\tau) + W_\ell^{(f)}(-\tau)} \tag{47}$$

$$= \frac{2Re(u_f)\sinh(y\Gamma\tau) + 2\Omega\, Im(u_f)\sin(x\Gamma\tau)}{(1+|u_f|^2)\cosh(y\Gamma\tau) - \Omega(1-|u_f|^2)\cos(x\Gamma\tau)}. \tag{48}$$

Here the effects due to CP violations enter through the nonvanishing of the parameters Ω, $Im(u_f)$ and $(1-|u_f|^2)$. In order to go to the limit of CP conservation therefore we set these parameters zero, and use the identity

$$\frac{2Re\, u}{1+|u|^2} = \pm 1 \;\mp \frac{|1\mp u|^2}{1+|u|^2}.$$

Since we must have $u_f = \pm 1$ according as the state f is CP-even or CP-odd, we obtain for the asymmetry

$$A_f(\tau) = \pm\tanh(y\Gamma\tau), \quad \text{where CP } (f) = \pm(f). \tag{49}$$

This is a useful formula. The corrections to it due to violations of CP-symmetry arise from terms which are of *second- and higher-order smallness*, and hence ignorable. For small enough $(y\Gamma\tau)$ and, say, for an odd CP state such as $J/\psi K_S$, we see

$$A_f(\tau) = -\tanh(y\Gamma\tau) \simeq (\Gamma_1 - \Gamma_2)\,\tau; \tag{50}$$

hence the sign of y gets determined if we regard the state B_2 to be dominantly CP odd.

Note that a measurement of the asymmetry $A_f(\tau)$ at a B Factory needs no special experimental effort; one needs to reanalyse the same data which will be collected for studying CP violation in the decays of B into CP eigenstates, such as in (34) and (36). This is because for the CP-asymmetries one looks at the difference in the rates of ℓ^+ and ℓ^- emission in correlation with a CP-eigenstate f, while for $A_f(\tau)$ one looks at the corresponding sum as in eq.(45). Secondly, as the formula (50) is the same for all decay states having the same CP-eigenvalue, we can easily enlarge the event sample.

It should be mentioned that if we start by defining eq. (47) in terms of the time-integrated rates $\int_0^\infty d\tau W(\pm\tau)$, the corresponding time-integrated asymmetry will be given by

$$A_f = (\pm)\, y,$$

where the sign $(\pm)$ refers to the CP signature of the state f.

5. Test of the $\Delta B = \Delta Q$ Rule

Recently Kobayashi and Sanda [16] have suggested ways for testing CPT invariance at a B-Factory. Their program to examine the mass-matrix CPT violation $[(q_1/p_1) \neq (q_2/p_2)]$ supposes that both CPT invariance and the $\Delta B = \Delta Q$ rule are respected by the amplitudes of the semileptonic B decays.

We shall take a different stand [17]: We take CPT invariance as sacrosanct and explore the breakdown of the Standard Model through violations of the $\Delta B = \Delta Q$ rule in neutral B decays. According to this selection rule the transition to an exclusive semileptonic state will be classified as 'allowed' and 'suppressed' as follows:

Allowed	Suppressed
$B \to R\ell^+ : A$	$\bar{B} \to R\ell^+ : \rho A$
$\bar{B} \to \bar{R}\ell^- : \bar{A}$	$B \to \bar{R}\ell^- : \bar{\rho}\bar{A}$

here the symbol R, stands for the remaining part of the final state including the appropriate neutrino, e.g., R could stand for $D^{*-}\nu_\ell$. The two complex parameters ρ and $\bar{\rho}$ (both of which ought to vanish in SM) are related by CPT invariance

$$\bar{\rho} = \rho^*. \tag{51}$$

Let us consider (at an asymmetric B Factory) the "timed" unlike-sign dilepton events from exclusive decays of the B_d. We denote their number as $N[R\ell^+(t'), \bar{R}\ell^-(t)]$. Defining the variables $T = (t + t')$ and $\tau = (t - t')$, we integrate over T from $|\tau|$ upto ∞, and obtain (suppressing an irrelevant overall factor)

$$N[R\ell^+, \bar{R}\ell^-; (\tau)] \sim e^{-\Gamma|\tau|}\left[|\gamma|^2 e^{y\Gamma\tau} + e^{-y\Gamma\tau} + 2\,Re(\gamma e^{ix\Gamma\tau})\right] \tag{52}$$

$$N[R\ell^+, \bar{R}\ell^-; (-\tau)] \sim e^{-\Gamma|\tau|}\left[|\gamma|^2 e^{-y\Gamma\tau} + e^{y\Gamma\tau} + 2\,Re(\gamma e^{-ix\Gamma\tau})\right] \tag{53}$$

the two rates differ by the sign of τ; the parameter γ is

$$\gamma = \frac{1+(\bar{\rho}/g)}{1-(\bar{\rho}/g)}\,\frac{1-\rho g}{1+\rho g}. \tag{54}$$

The asymmetry that should vanish according to the $\Delta B = \Delta Q$ rule can therefore be constructed

$$A_{BQ}(\tau) = \frac{N[R\ell^+, \bar{R}\ell^-;\,(\tau)] - N[R\ell^+,\,\bar{R}\ell^-(-\tau)]}{N[R\ell^+, \bar{R}\ell^-;\,(\tau)] + N[R\ell^+,\,\bar{R}\ell^-(-\tau)]}. \tag{55}$$

Integrating the N's over τ from 0 to ∞, the corresponding time-integrated asymmetry becomes

$$a_{BQ} = \frac{1}{2+x^2-y^2}\left[-Im(\gamma)x(1-y^2) - \frac{1}{2}(1-|\gamma|^2)\,(1+x^2)\,y\right]. \tag{56}$$

Assuming CPT invariance, we substitute eq.(51) in eq.(54) and observe that the only phase which controls the phase of γ is that of $(g\rho)$ which is a phase-convention-independent combination. For small values of rho, since

$$\gamma \simeq 1 - 4i\,Im(g\rho), \tag{57}$$

we see that, by ignoring terms which are second and higher order in ρ, eq.(57) becomes

$$a_{BQ} \simeq \left[\frac{4x(1-y^2)}{2+x^2-y^2}\right] Im(g\rho). \tag{58}$$

The first factor is harmless because its value is about 1.1 for $x \simeq 0.67$ and $y \simeq 0$. Thus this asymmetry directly measures the quantity $Im(g\rho)$. This essentially is $Im(\rho)$ if the CP violation at the "structure-level" turns out to be negligibly small, $g \simeq 1$, as is indicated by several model estimates [15]. In any case as the selection rule $\Delta B = \Delta Q$ is an integral part of the Standard Model and plays a crucial role in B Physics, an experimental test of its validity is obviously important.

To conclude, after the pion Factories LAMPF and TRIUMF, the recently-funded Frascati Phi-Factory (DAϕNE) and the Spain's Tau-Charm Factory — the next step seems to be a B-Factory. The 1987 discovery of the ARGUS group that there is sizable mixing in B_d mesons, generated great interest in B-physics. The B's may indeed be the heaviest quark mesons which permit an experimental study of the origin of CP violation (the top bound states would decay extremely rapidly by the kinematically allowed transition $t \to bW^+$). Against such a background it is difficult to explain why, as of now, none of the proposals (CESR-B, KEK-B, PEP-II, CERN-ISR, ...) has been funded. However in view of the tremendous enthusiasm and the world-wide support that this project seems to enjoy, it will be fair to predict that a B-Factory will be built — the only question is, where and when.

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Discussion

B.Ananthanarayan : How do these asymmetries help resolve the Winstein ambiguity?

K.V.L. Sarma: Winstein's ambiguity arises because the asymmetries to be measured might obey certain relationship; hence we will not be able to resolve the ambiguity between SM and SW on the basis of these asymmetries.

B.Ananthanarayan : What is the way out?

K.V.L. Sarma: The superweak model asserts that all CP violation is confined to the transitions involving two units of flavour change. There are many ways to rule it out : if CP violation is detected at any level in the decays of the charged mesons B^+ and B^-, or if the ratio (ϵ/ϵ') in neutral kaon decays is shown to be nonzero, etc.